Improved transfer matrix method without numerical instability

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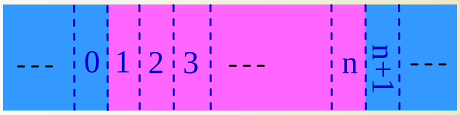
What's transfer matrix (TMM) ?



$$\Phi(n+1) = T_{n+1} T_C^{\,n} T_0 \Phi(0) \qquad (1)$$

TMM transfer wave function from one site to another.
TMM is widely used in: Electronic Transport; Photonics; Electromagnetics; Acoustics; Seismology.
But the numerical instability seriously limit its further application!

New improved TMM

Our method is to eliminate the divergence originating term $\exp(-ik_j nx + \theta nx)$. Induce new variables $\{\nu_p\}$

$$\begin{pmatrix} \vdots \\ \{\nu_j\} e^{ik_j x - \theta x} \end{pmatrix} = \widetilde{T}_0 \begin{pmatrix} \{t\} \\ 0 \end{pmatrix} \qquad (4a)$$

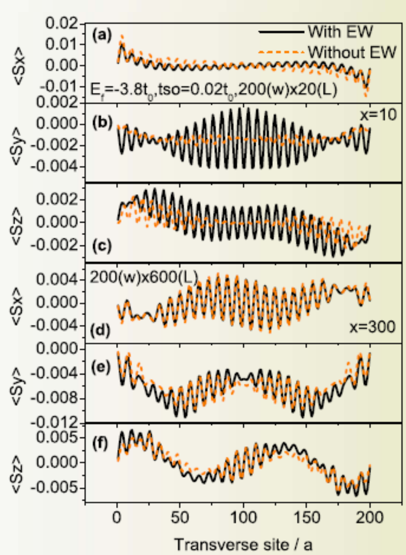
$$\begin{pmatrix} In \\ \{r\} \end{pmatrix} = \widetilde{T_{n+1}} \begin{pmatrix} \vdots \\ \{\nu_j\} \end{pmatrix} \qquad (4b)$$

From eq. (4a), (4b), we get an expanded equation set

$$A' \begin{Bmatrix} \{t_i\} \\ \{\nu_j\} \end{Bmatrix} = b' \qquad (5)$$

The elements of matrix A' are finite now. $\{t_i\},\{r_i\}$ can be got without numerical instability.

III. The effect of evanescent wave is considered exactly.



Numerical Instability of TMM

Transform eq.(1) to k-space:

$$\psi(n+1) = \widetilde{T_{n+1}} \widetilde{T_C}^{\,n} \widetilde{T_0} \psi(0)$$

$$\begin{pmatrix} In \\ \{r\} \end{pmatrix} = \widetilde{T_{n+1}} \begin{pmatrix} \{e^{ik_j x}\} & 0 & 0 & 0 \\ 0 & \{e^{ik_j x - \theta x}\} & 0 & 0 \\ 0 & 0 & \{e^{-ik_j x}\} & 0 \\ 0 & 0 & 0 & \{e^{-ik_j x + \theta x}\} \end{pmatrix}^n \widetilde{T}_0 \begin{pmatrix} \{t\} \\ 0 \end{pmatrix} \qquad (2)$$

$$\left|\{e^{ik_j x}\}\right| = 1; \left|\{e^{ik_j x - \theta x}\}\right| < 1; \left|\{e^{-ik_j x}\}\right| = 1; \left|\{e^{-ik_j x + \theta x}\}\right| > 1.$$

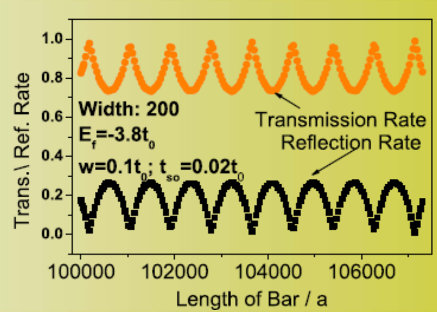
Reconstruct eq.(2) to get eq.(3) to solve $\{t\}$ and $\{r\}$.

$$A\{t\} = b; \{t\} = A^{-1} b \qquad (3)$$

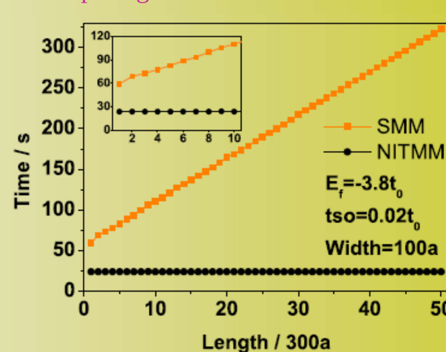
when the transfer step number **n** is large, the inverse of matrix A will meet the problem of numerical overflow due to the term: $\exp(-ik_j nx + \theta nx)$.

Examples of Application

I. Capability to deal with extreme long length.



II. The computing time is the zeroth order of length.



Reference:
HuiQiong Yin and Ruibao Tao, EPL 84(2008), 57006.
And all the references therein.