

# Tight-binding approach to study the coupling effects in metamaterials

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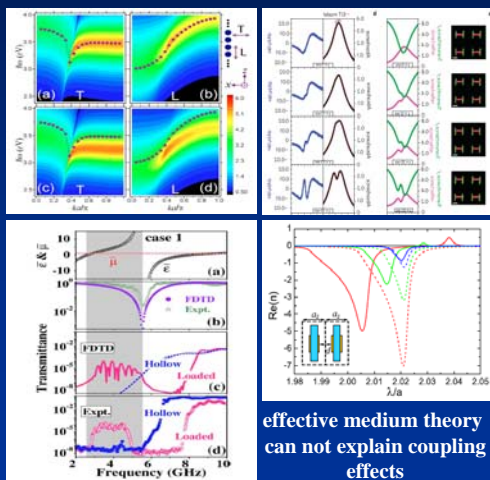
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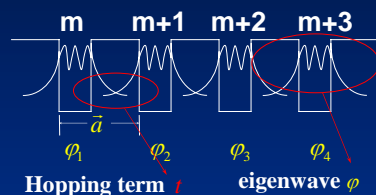
## Abstract

We established a generalized tight-binding method (TBM) to study the coupling effects in metamaterials. All parameters involved in our theory can be calculated from *first principles*, and the theory is applicable to general photonic systems with both dielectric and magnetic materials. As the illustration, we applied the theory to study cutoff waveguides loaded with resonant electric/magnetic metamaterials. We not only accurately computed the coupling strengths between two resonant metamaterials, but also revealed a number of interesting coupling-induced phenomena. Microwave experiments and full-wave numerical simulations were performed to successfully verify all predictions drawn from the TBM.

## Previous work



## Establish a theory to study coupling



$$t = \langle \varphi_m | \sum_{i \neq n} V_i | \varphi_n \rangle / \langle \varphi_1 | \varphi_1 \rangle$$

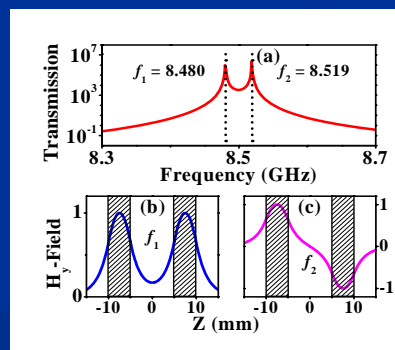
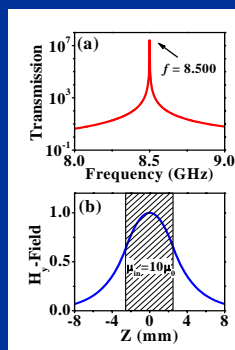
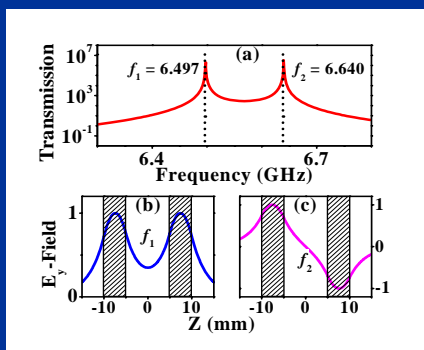
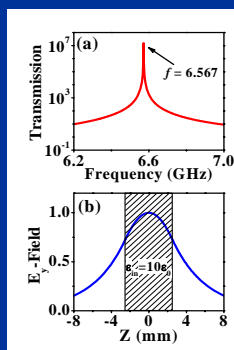
$$\hat{V}_i = \begin{pmatrix} 0 & i\Delta \frac{1}{\epsilon(\vec{r})} \nabla \times \\ -i\Delta \frac{1}{\mu(\vec{r})} \nabla \times & 0 \end{pmatrix}$$

Tight-binding method (TBM) seems to be a good approach to study the coupling effects in such systems

## TE+ $\epsilon(\vec{r})$

## Benchmark

## TE+ $\mu(\vec{r})$

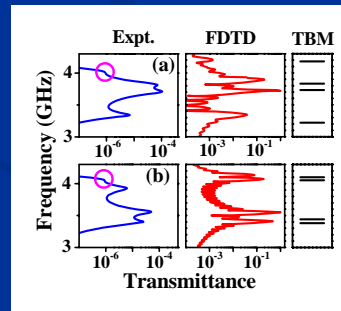
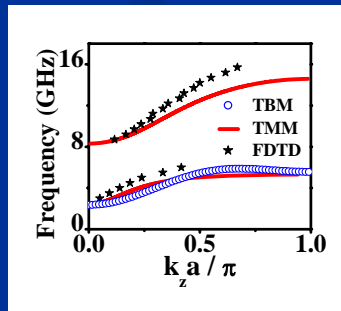
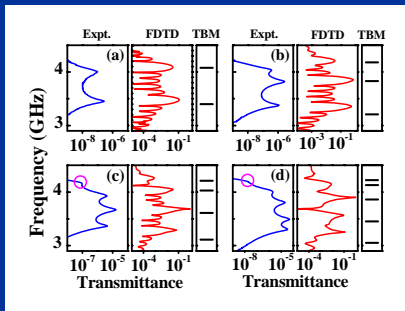


## Realistic system

Peaks' positions  
2, 3, 4, 5 layers case  
FDTD VS. TBM

Dispersion relationship  
FDTD VS. TMM VS. TBM

Peaks' positions  
nonperiodic cases  
FDTD VS. TBM



we apply the TBM to study the realistic systems – a WR90 waveguide loaded with multiple slabs of electric resonant metamaterials. The metamaterial that we designed is a metallic cross structure deposited on a dielectric substrate, with geometry shown in figure.

Experimental (left) and FDTD (middle) results of the transmission spectrum for the waveguides loaded with (a) 2, (b) 3, (c) 4, and (d) 5 metamaterial slabs. Right panels depict the positions of the eigenmodes predicted by the TBM. FDTD results and experimental spectra are generally in good agreements with the TBM prediction.

EM wave dispersion relations inside the loaded waveguides calculated by TBM, TMM and FDTD. Results calculated by TMM and FDTD are in good agreements with the TBM results. Such good agreements reinforced the notion that the TBM works quite well for such metamaterial systems.

TBM can be employed to study the systems with slabs arranged *not periodically*. Experimental (left) and FDTD (middle) results of the transmission spectrum for two different kinds of the waveguides loaded with 4 metamaterial slabs non-periodically. Right panels depict the positions of the eigenmodes predicted by the TBM.

- we have established a generalized tight binding method to study the coupling effects in photonic metamaterials.
- The crucial advantages of our theory are that it can determine all involved parameters from first principles, and can be applied to general photonic systems with both dielectric and magnetic materials.
- Taking the cutoff waveguides loaded with resonant metamaterials as explicit examples, we applied the TBM to study various interesting phenomena induced by the coupling effects in metamaterials.
- Our results showed that the TBM can provide not only quantitative measures on the coupling strengths between different resonant units, but also other information such as the symmetries of the eigenmodes, the resonant peak positions and the dispersion relations in periodic and even non-periodic systems.
- Microwave experiments and FDTD simulations on realistic systems were performed to verify all the predictions drawn from the TBM.