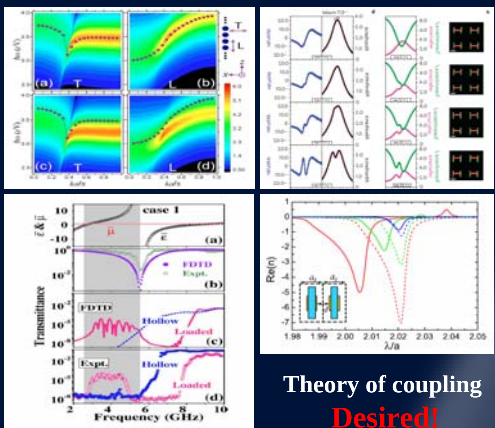
Tight-binding analysis of coupling effects in metamaterials

Hao Xu[1], Qiong He, Shiyi Xiao, Bin Xi, Jiaming Hao, Lei Zhou[2]

Physics Department, Fudan University, Shanghai, 200433, China

We established a generalized tight-binding method (TBM) to study the coupling effects in metamaterials. All parameters involved in our theory can be calculated from *first principles*, and the theory is applicable to general photonic systems with both dielectric and magnetic materials. As an illustration, we applied the theory to study the cutoff waveguides loaded with resonant electric / magnetic metamaterials. We not only accurately computed the coupling strengths between two resonant metamaterials, but also revealed a number of interesting coupling-induced phenomena. Microwave experiments and full-wave numerical simulations were performed to successfully verify all predictions drawn from the TBM.

Background and motivations



Theory of coupling

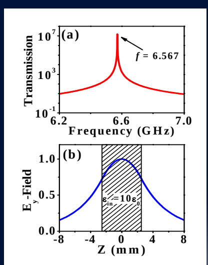
Desired!

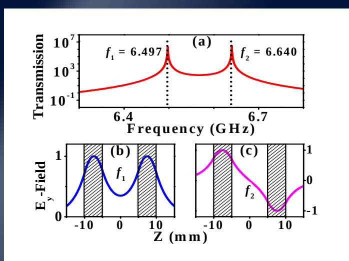
Benchmark tests

TE+ $\varepsilon(\vec r)$

$t_{NN} < 0$

$$\hat H = \begin{pmatrix} \omega_0 & t_{NN} \\ t_{NN} & \omega_0 \end{pmatrix} \qquad \omega = \begin{cases} \omega_0 + t_{NN}, & |\psi^+\rangle = |\varphi_1\rangle + |\varphi_2\rangle \ \text{even} \\ \omega_0 - t_{NN}, & |\psi^-\rangle = |\varphi_1\rangle - |\varphi_2\rangle \ \text{odd} \end{cases}$$

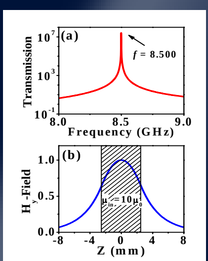


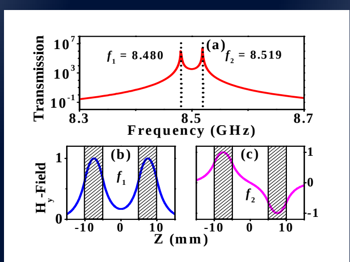


TM+ $\mu(\vec r)$

$t_{NN} < 0$

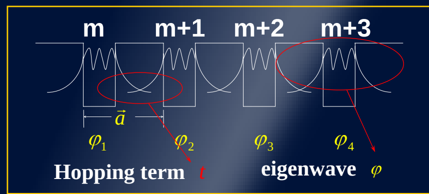
$$\hat H = \begin{pmatrix} \omega_0 & t_{NN} \\ t_{NN} & \omega_0 \end{pmatrix} \qquad \omega = \begin{cases} \omega_0 + t_{NN}, & |\psi^+\rangle = |\varphi_1\rangle + |\varphi_2\rangle \ \text{even} \\ \omega_0 - t_{NN}, & |\psi^-\rangle = |\varphi_1\rangle - |\varphi_2\rangle \ \text{odd} \end{cases}$$





Our approach: TBM for photons! Theoretical framework

$$\hat H = \begin{pmatrix} 0 & i\frac{1}{\varepsilon}\nabla\times \\ -i\frac{1}{\mu}\nabla\times & 0 \end{pmatrix}, \quad i\frac{\partial}{\partial t}|\varphi\rangle = \hat H|\varphi\rangle, \quad |\varphi\rangle = \begin{pmatrix} \vec E \\ \vec H \end{pmatrix}$$

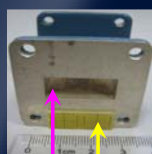


$$(\hat H_0 + \hat V_i)|\varphi_i(\vec r)\rangle = \omega_0|\varphi_i(\vec r)\rangle, \quad \hat H_0 = \begin{pmatrix} 0 & i\frac{1}{\varepsilon_{ref}(\vec r)}\nabla\times \\ -i\frac{1}{\mu_{ref}(\vec r)}\nabla\times & 0 \end{pmatrix}$$

$$\hat V_i = \begin{pmatrix} 0 & i\Delta\frac{1}{\varepsilon(\vec r)}\nabla\times \\ -i\Delta\frac{1}{\mu(\vec r)}\nabla\times & 0 \end{pmatrix}, \quad t = \langle\varphi_m|\sum_{i\neq n} V_i|\varphi_n\rangle / \langle\varphi_1|\varphi_1\rangle$$

Applications to realistic systems

Realistic system

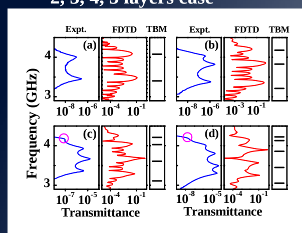


Waveguide 10.16×22.86

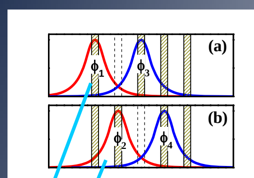
E-resonance

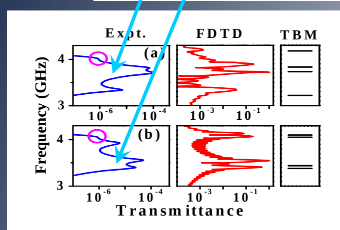
$$\varepsilon_r^{in} = 1 + \frac{280}{5.08^2 - f^2} + \frac{1950}{15.95^2 - f^2}$$

Peaks' positions
2, 3, 4, 5 layers case

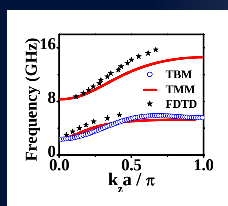


Peaks' positions
non-periodic cases





Dispersion relationship



- Establish a theory to study the couplings in photonic systems
- Parameters determined form first principle
- Applicable to both dielectric and magnetic systems
- We applied our theory to realistic systems, excellent agreement between theory and experiments

Reference: Hao Xu, Qiong He, Shiyi Xiao, Bin Xi, Jiaming Hao, and Lei Zhou, J. Appl. Phys. 109, 023103 (2011).

1. xh@fudan.edu.cn 2. phzhou@fudan.edu.cn