An interacting pairing model in two dimension

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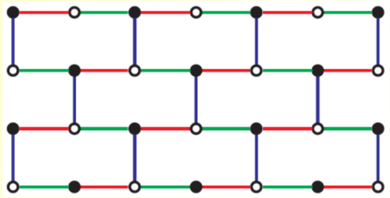
We study interacting pairing states of electrons on a brick-wall lattice. By applying Jordan-Wigner transformation on the electron operators, we find that there are Z_2 vertices in this model. These vortices can be either Abelian or non-Abelian anyons, depending on the model parameters.

I. Hamiltonian

We first consider the system on a brick-wall lattice. In order to do the mapping later, we will really work on a honeycomb lattice. The brick-wall lattice is a good guide for our eyes in that sites therein can be conveniently labeled.



$$\mathcal{H} = H_k + H_{inter-orbit} + H_p,$$

$$\begin{aligned}
H_k = & \; iJ_{3e} \sum_{\sigma,n+m=\text{even}} \left[a^{(1)\dagger}_{m,n,s,\sigma} a^{(2)}_{m,n+1,s,\sigma} + a^{(2)\dagger}_{m,n-1,s,\sigma} a^{(1)}_{m,n,s,\sigma} - h.c.\right] \\
& -iJ_{3o} \sum_{\sigma,n+m=\text{even}} \left[a^{(1)\dagger}_{m,n,p,\sigma} a^{(2)}_{m,n+1,p,\sigma} + a^{(2)\dagger}_{m,n-1,p,\sigma} a^{(1)}_{m,n,p,\sigma} - h.c.\right] \\
& +i \sum_{\sigma,n+m=\text{even}} \Big[J_{z1} \left((a^{(1)\dagger}_{m,n,s,\sigma} + a^{(1)}_{m,n,s,\sigma})(a^{(2)\dagger}_{m+1,n,s,\sigma} + a^{(2)}_{m+1,n,s,\sigma})\right) a^{(1)\dagger}_{m,n,s,\sigma} a^{(2)}_{m+1,n,s,\sigma} \\
& +J_{z2} \left((a^{(1)\dagger}_{m,n,p,\sigma} + a^{(1)}_{m,n,p,\sigma})(a^{(2)\dagger}_{m+1,n,p,\sigma} + a^{(2)}_{m+1,n,p,\sigma})\right) a^{(1)\dagger}_{m,n,p,\sigma} a^{(2)}_{m+1,n,p,\sigma} - h.c.\Big],
\end{aligned}$$

$$\begin{aligned}
H_{inter-orbit} = & \; i \sum_{\sigma,n+m=\text{even}} \left[J_{y1} a^{(1)\dagger}_{m,n,p,\sigma} a^{(2)}_{m,n+1,s,\sigma} + J_{y2} a^{(2)\dagger}_{m,n-1,p,\sigma} a^{(1)}_{m,n,s,\sigma} - h.c.\right] \\
& - \sum_{\sigma,n+m=\text{even}} \left[J_{x1} a^{(1)\dagger}_{m,n,s,\sigma} a^{(1)}_{m,n,p,\sigma} + J_{x2} a^{(2)\dagger}_{m+1,n,s,\sigma} a^{(2)}_{m+1,n,p,\sigma} + h.c.\right],
\end{aligned}$$

$$\begin{aligned}
H_p = & - \sum_{\sigma,n+m=\text{even}} \left[J_{x1} a^{(1)\dagger}_{m,n,s,\sigma} a^{(1)\dagger}_{m,n,p,\sigma} + J_{x2} a^{(2)\dagger}_{m+1,n,s,\sigma} a^{(2)\dagger}_{m+1,n,p,\sigma} + h.c.\right] \\
& -i \sum_{\sigma,n+m=\text{even}} \left[J_{y1} a^{(1)\dagger}_{m,n,p,\sigma} a^{(2)\dagger}_{m,n+1,s,\sigma} + J_{y2} a^{(2)\dagger}_{m,n-1,p,\sigma} a^{(1)\dagger}_{m,n,s,\sigma} - h.c.\right] \\
& -i \sum_{\sigma,n+m=\text{even}} \Big[J_{z1} \left((a^{(1)\dagger}_{m,n,s,\sigma} + a^{(1)}_{m,n,s,\sigma})(a^{(2)\dagger}_{m+1,n,s,\sigma} + a^{(2)}_{m+1,n,s,\sigma})\right) a^{(1)\dagger}_{m,n,s,\sigma} a^{(2)\dagger}_{m+1,n,s,\sigma} \\
& +J_{z2} \left((a^{(1)\dagger}_{m,n,p,\sigma} + a^{(1)}_{m,n,p,\sigma})(a^{(2)\dagger}_{m+1,n,p,\sigma} + a^{(2)}_{m+1,n,p,\sigma})\right) a^{(1)\dagger}_{m,n,p,\sigma} a^{(2)\dagger}_{m+1,n,p,\sigma} - h.c.\Big] \\
& -iJ_{3e} \sum_{\sigma,n+m=\text{even}} \left[a^{(1)\dagger}_{m,n,s,\sigma} a^{(2)\dagger}_{m,n+1,s,\sigma} + a^{(2)\dagger}_{m,n-1,s,\sigma} a^{(1)\dagger}_{m,n,s,\sigma} - h.c.\right] \\
& +iJ_{3o} \sum_{\sigma,n+m=\text{even}} \left[a^{(1)\dagger}_{m,n,p,\sigma} a^{(2)\dagger}_{m,n+1,p,\sigma} + a^{(2)\dagger}_{m,n-1,p,\sigma} a^{(1)\dagger}_{m,n,p,\sigma} - h.c.\right].
\end{aligned}$$

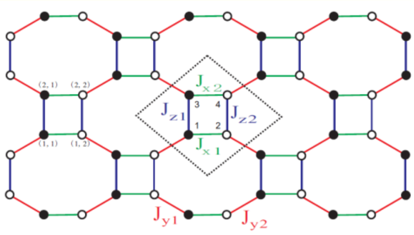
The subscripts k, p can distinguish between kinetic and spin-triplet pairing terms when omitting terms in brackets.

II. "Expand" the Hamiltonian

We will "expand" the system, i.e, we add a new site on the original site. This can be done by replacing the original column index *n as 2n - 1:*

$$(1, 2, 3, 4, 5, \cdots) \mapsto (1, 3, 5, 7, 9, \cdots)$$

$$\begin{aligned}
a^{(1)\dagger}_{m,1,s,\sigma} &\mapsto a^{(1)\dagger}_{m,1,\sigma}, \; a^{(1)\dagger}_{m,3,s,\sigma} \mapsto a^{(1)\dagger}_{m,5,\sigma}, \cdots \\
a^{(1)\dagger}_{m,1,p,\sigma} &\mapsto a^{(2)\dagger}_{m,2,\sigma}, \; a^{(1)\dagger}_{m,3,p,\sigma} \mapsto a^{(2)\dagger}_{m,6,\sigma}, \cdots \\
a^{(3)\dagger}_{m,1,s,\sigma} &\mapsto a^{(3)\dagger}_{m,1,\sigma}, \; a^{(3)\dagger}_{m,3,s,\sigma} \mapsto a^{(3)\dagger}_{m,5,\sigma}, \cdots \\
a^{(3)\dagger}_{m,1,p,\sigma} &\mapsto a^{(4)\dagger}_{m,2,\sigma}, \; a^{(3)\dagger}_{m,3,p,\sigma} \mapsto a^{(4)\dagger}_{m,6,\sigma}, \cdots
\end{aligned}$$



III. Mapping fermions to spins

By performing the Jordan-Wigner transformation on an open system

$$\begin{aligned}
\sigma^{+}_{m,n,\uparrow} &= 2a^{\dagger}_{m,n,\uparrow} \prod_{n'=1}^{N} \prod_{m'<m} \sigma^{z}_{m',n',\uparrow} \prod_{n''<n} \sigma^{z}_{m,n'',\uparrow}, \\
\sigma^{+}_{m,n,\downarrow} &= 2a^{\dagger}_{m,n,\downarrow} \prod_{n'=1}^{N} \prod_{m'<m} \sigma^{z}_{m',n',\downarrow} \prod_{n''<n} \sigma^{z}_{m,n'',\downarrow}
\end{aligned}$$

we have

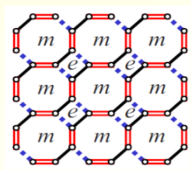
$$\mathcal{H} = \sum_{\tau=\uparrow,\downarrow} (H_\tau + H_{\tau,3})$$

where

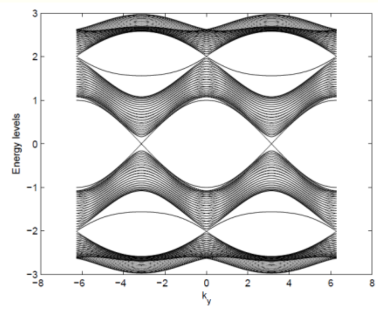
$$\begin{aligned}
H_\tau = & \; J_{x1} \sum_{x_1-\text{link}} \sigma^{x}_{m,n,\tau}\sigma^{x}_{m,n+1,\tau} + J_{x2} \sum_{x_2-\text{link}} \sigma^{x}_{m,n,\tau}\sigma^{x}_{m,n+1,\tau} \\
& -J_{y1} \sum_{y_1-\text{link}} \sigma^{y}_{m,n,\tau}\sigma^{y}_{m,n+1,\tau} - J_{y2} \sum_{y_2-\text{link}} \sigma^{y}_{m,n,\tau}\sigma^{y}_{m,n+1,\tau} \\
& +J_{z1} \sum_{z_1-\text{link}} \sigma^{z}_{m,n,\tau}\sigma^{z}_{m+1,n,\tau} + J_{z2} \sum_{z_2-\text{link}} \sigma^{z}_{m,n,\tau}\sigma_{m+1,n,\tau},
\end{aligned}$$

$$\begin{aligned}
H_{\tau,3} = & -J_{3e} \sum_{(x_i,y_j)-\text{link}} \sigma^{x}_{m,n,\tau}\sigma^{z}_{m,n+1,\tau}\sigma^{y}_{m,n+2,\tau} \\
& -J_{3o} \sum_{(y_i,x_j)-\text{link}} \sigma^{y}_{m,n,\tau}\sigma^{z}_{m,n+1,\tau}\sigma^{x}_{m,n+2,\tau},
\end{aligned}$$

here the spin model is a topological model.



In particular, the edge states in a ground state can be nontrivial



In fact, if

$$\left(a^{(1)\dagger}_{m,n,\sigma} + a^{(1)}_{m,n,\sigma}\right)\left(a^{(3)\dagger}_{m+1,n,\sigma} + a^{(3)}_{m+1,n,\sigma}\right)$$

is not 1, but equal to -1, there is an anyon in the model.

IV. Conclusions

We find an interacting pairing model in two dimension and the mapping from it to a spin model. This spin model is studied by existed papers, which shows that it can have both Abelian and non-Abelian anyons. This means that the fermion model here allows anyon excitations, too.

Reference: S. Yang, D. L. Zhou, and C. P. Sun, Phys. Rev. B 76, 180404 (2007).