

# A theoretical study on the conversion efficiencies of gradient meta-surfaces

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## Motivations:

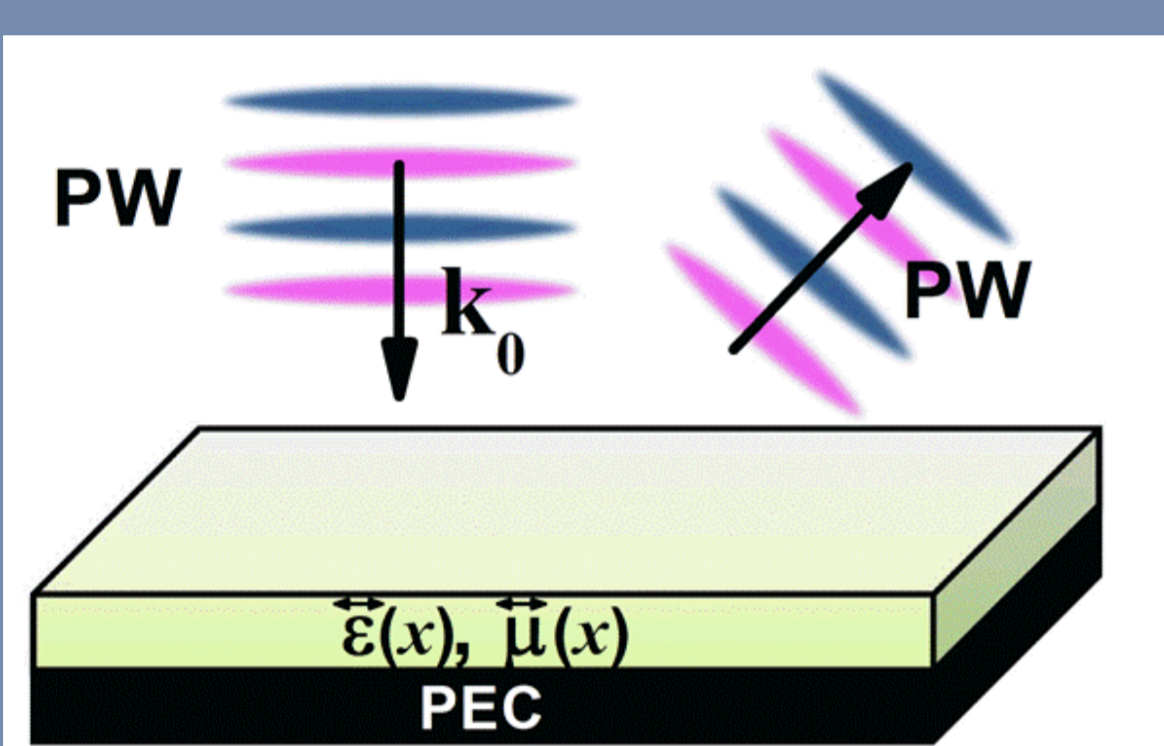
In a recent work (Sun S. et al., Nat. Mater., 11 (2012) 426), nearly 100% efficiency of the conversion of an incident propagating wave (PW) to an obliquely outgoing PW or even a surface wave (SW) is demonstrated in an **ideal** gradient meta-surface (GM).

However, **practical systems** might have **non-equal** and  $\epsilon_{\parallel}(x)$  and  $\mu_{\parallel}(x)$  and profiles and sometimes use **supercells** to truncate the profiles to avoid using too large values of  $\epsilon$  and  $\mu$ .

Here, based on non-ideal GM systems, we systematically studied the factors that influence the efficiencies of such conversion processes (both PW-PW and PW-SW).

## I. Efficiency issues of model GMs

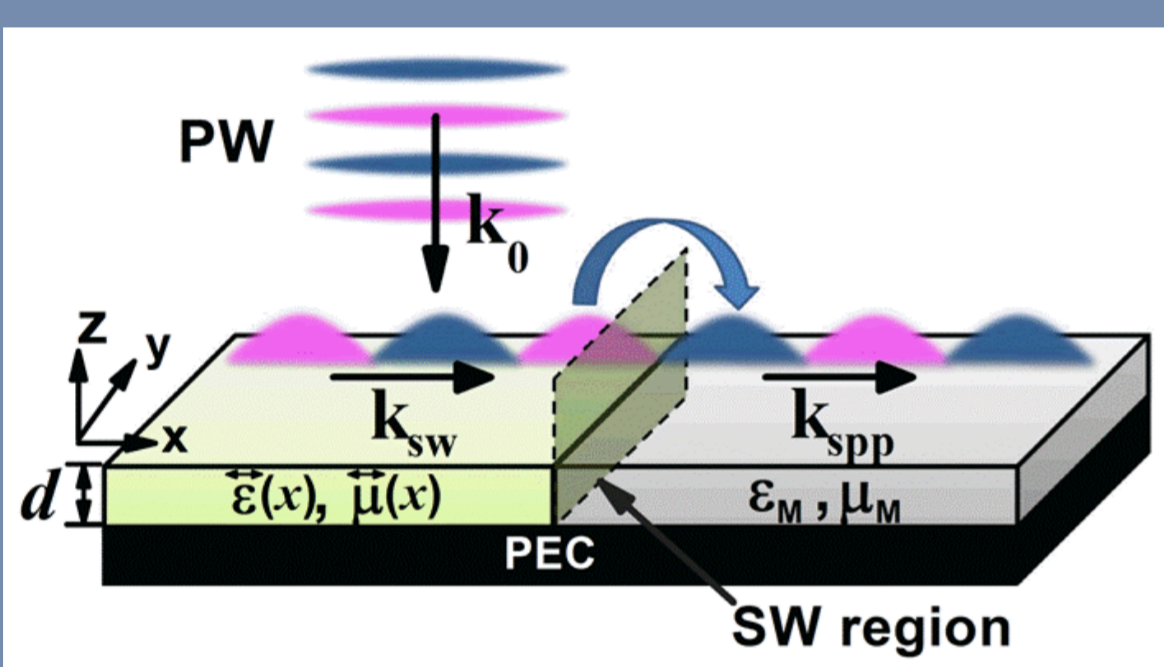
### (A) Methods



- To study the conversion efficiency for the **PW-PW process**, we calculate the reflectance:

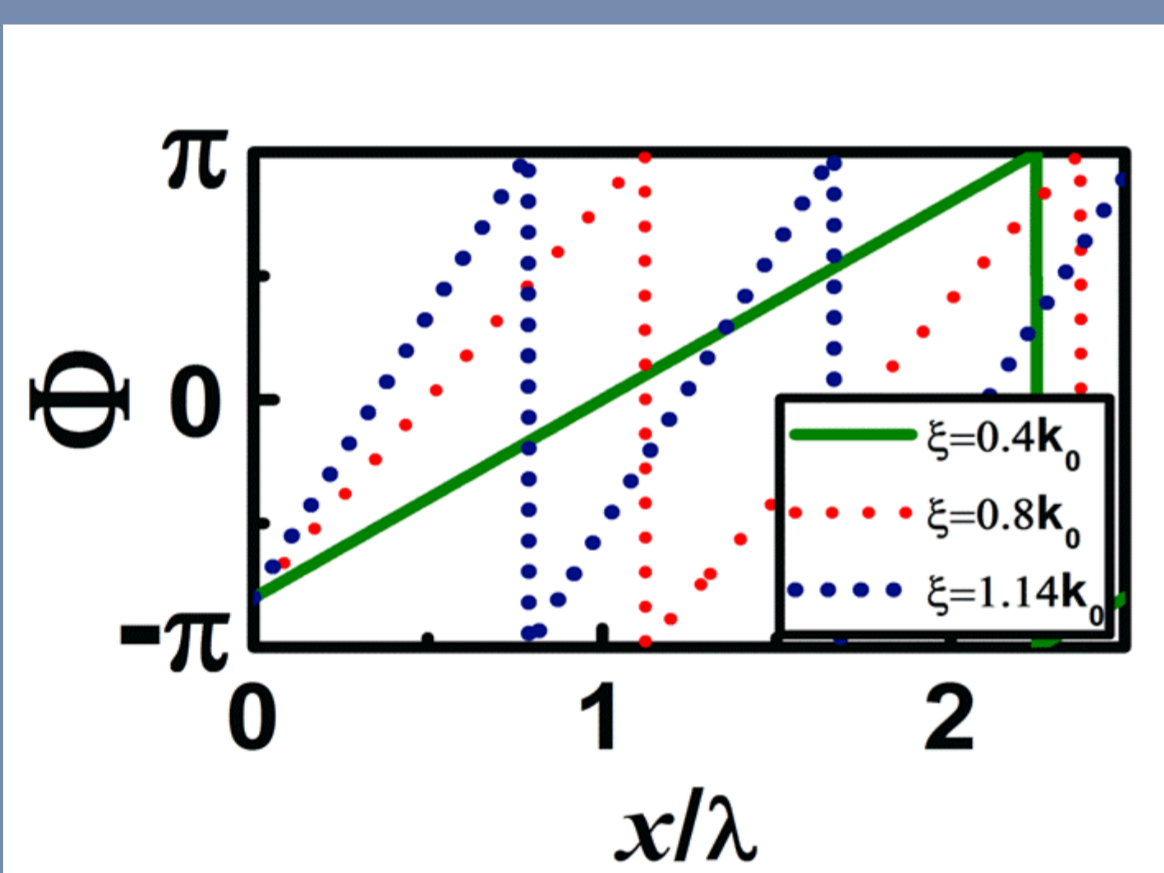
$$R_{\text{PW-PW}} = |\rho_{k_x}|^2 \cos \theta_r / \cos \theta_i$$

$$= |\rho_{k_x}|^2 \sqrt{1 - (\xi / k_0)^2}$$



- For the **PW-SW process**,  $k_{\text{SW}} = \xi$ ,  $\xi > k_0$ .

Use an eigen-SPP guide material with  $k_{\text{SPP}} = k_{\text{SW}}$ , and calculate the **power flow ratio** as the PW-SW conversion efficiency.



- How to generate the systems we want to study:

We assume

$$\epsilon_{\parallel}(x) = 1 + \alpha \cdot \xi x / 2k_0 d$$

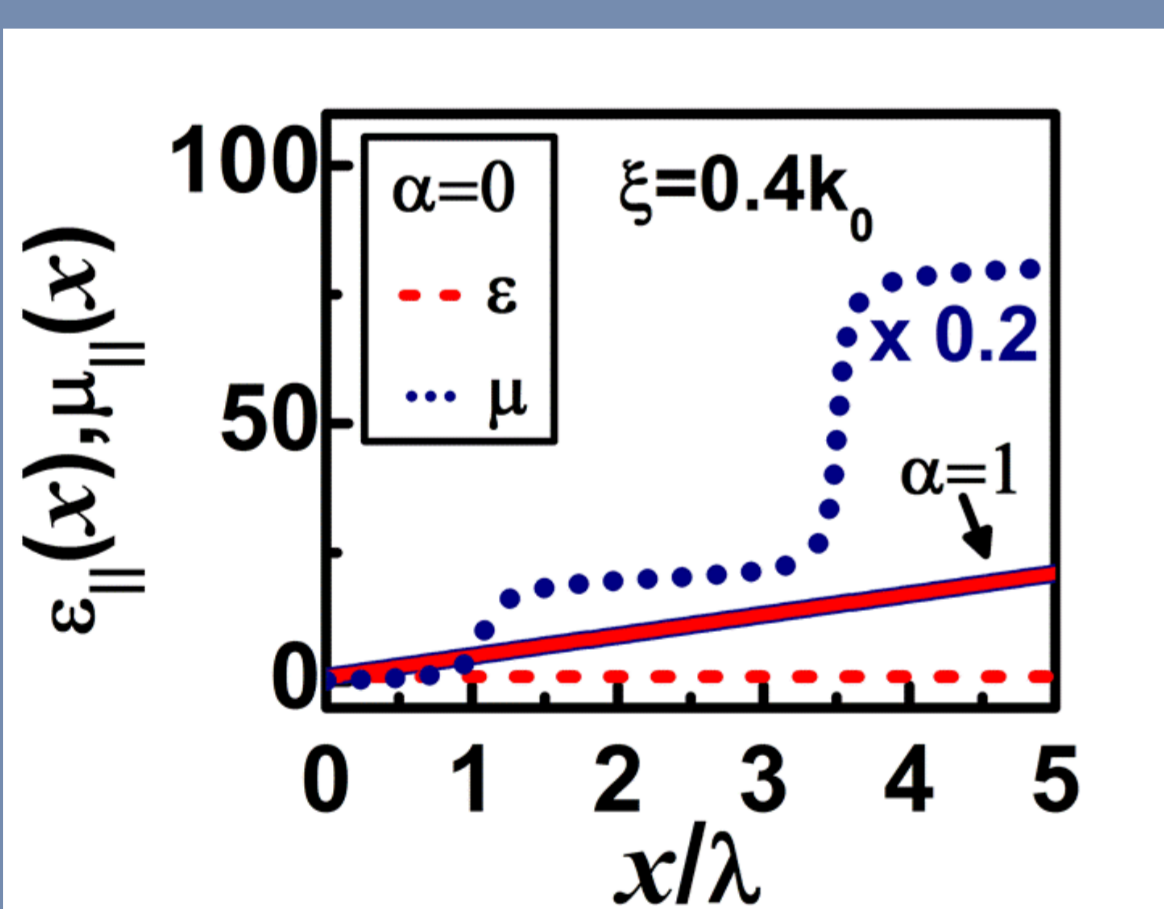
and then retrieve  $\mu_{\parallel}(x)$  by letting the calculated reflection phase

$$\Phi(x) = \cos^{-1} \frac{-\epsilon_{\parallel} + \mu_{\parallel} \tan^2(\sqrt{\epsilon_{\parallel} \mu_{\parallel}} k_0 d)}{\epsilon_{\parallel} + \mu_{\parallel} \tan^2(\sqrt{\epsilon_{\parallel} \mu_{\parallel}} k_0 d)}$$

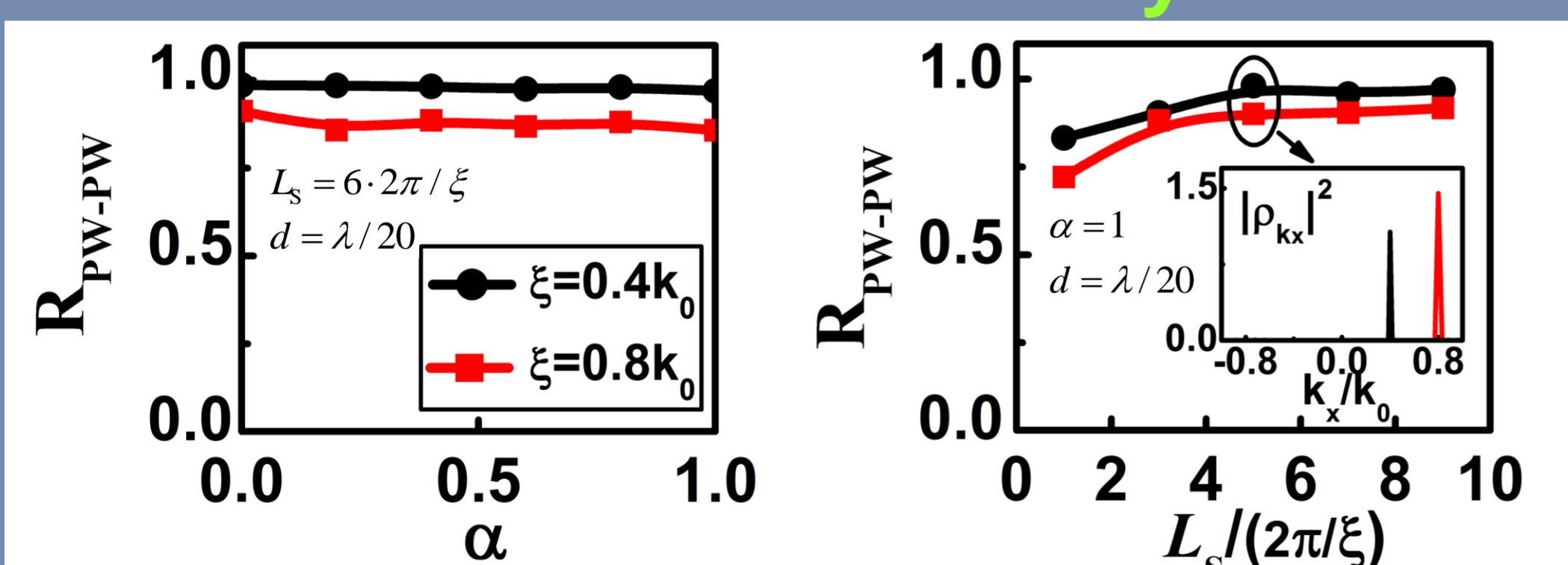
satisfy the given

$$\Phi(x) = \Phi_0 + \xi x$$

$\alpha \in [0, 1]$ , a parameter to measure the degree of impedance mismatch: from **completely impedance-mismatched** to **impedance-matched**

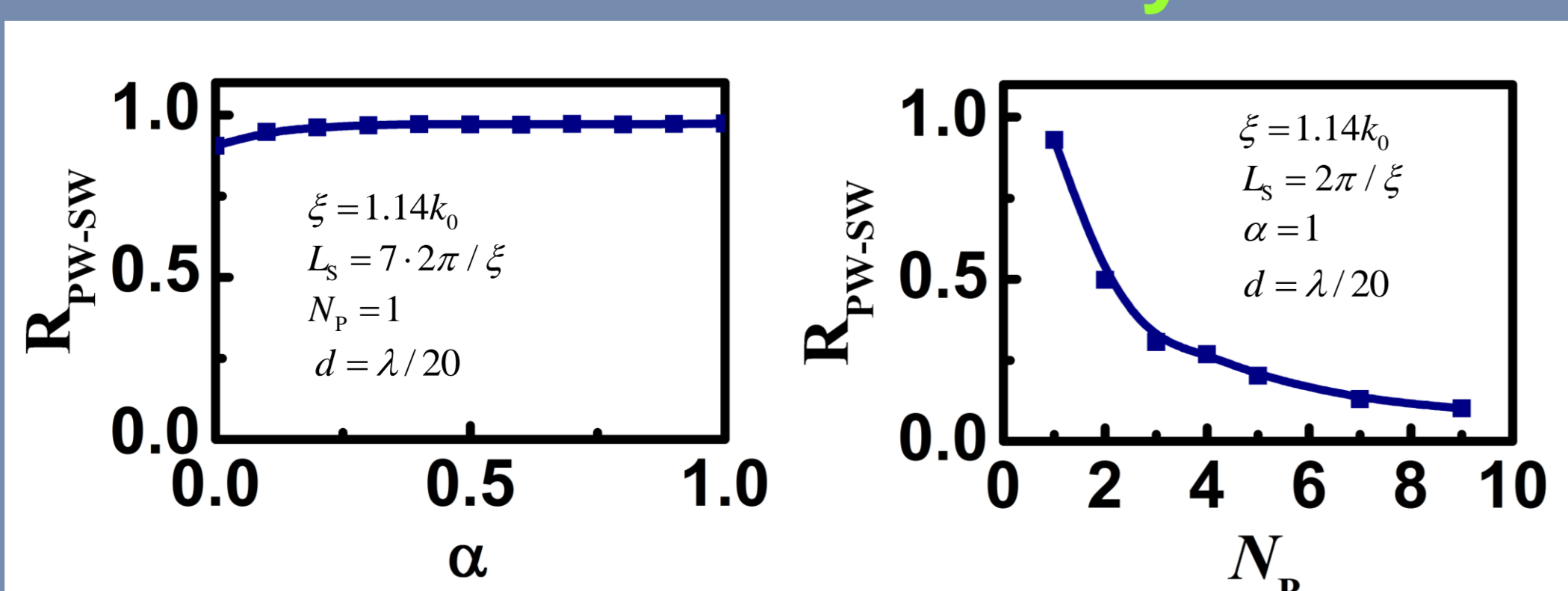


### (B) PW-PW conversion efficiency



$L_s$ : the periodicity of one supercell

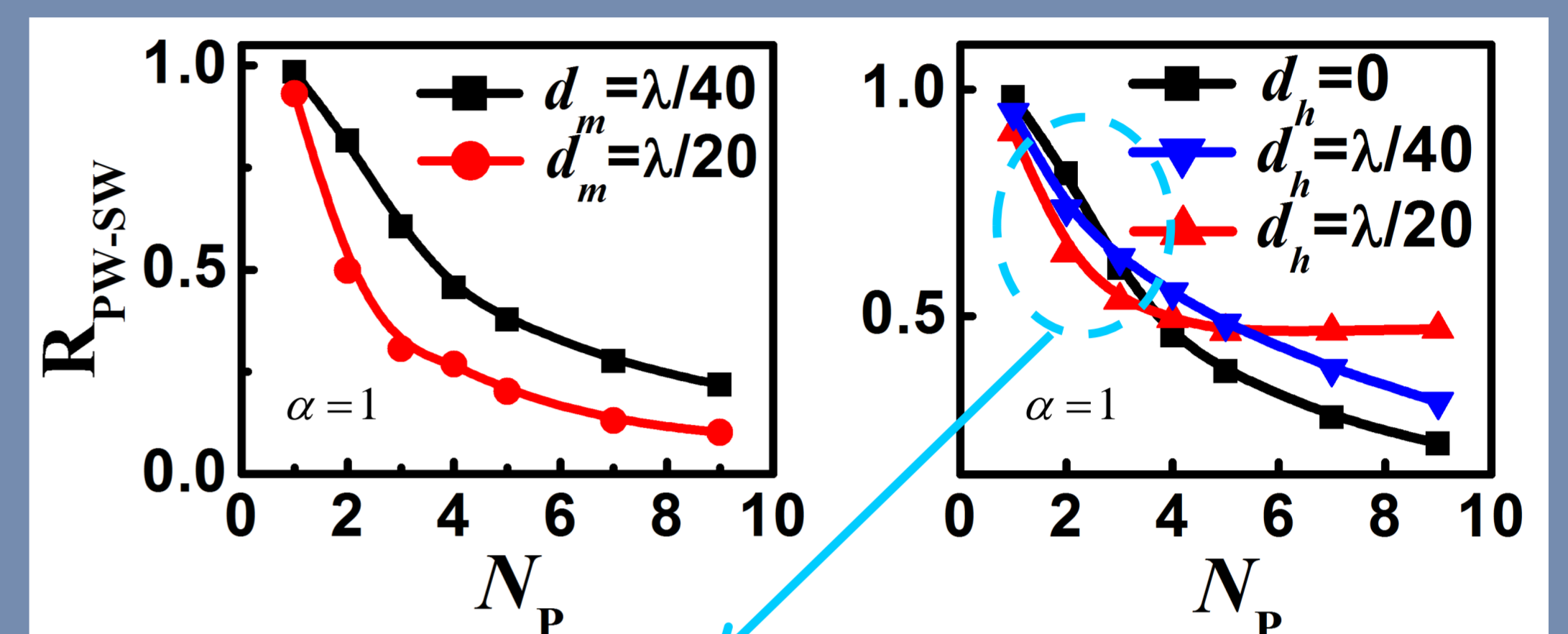
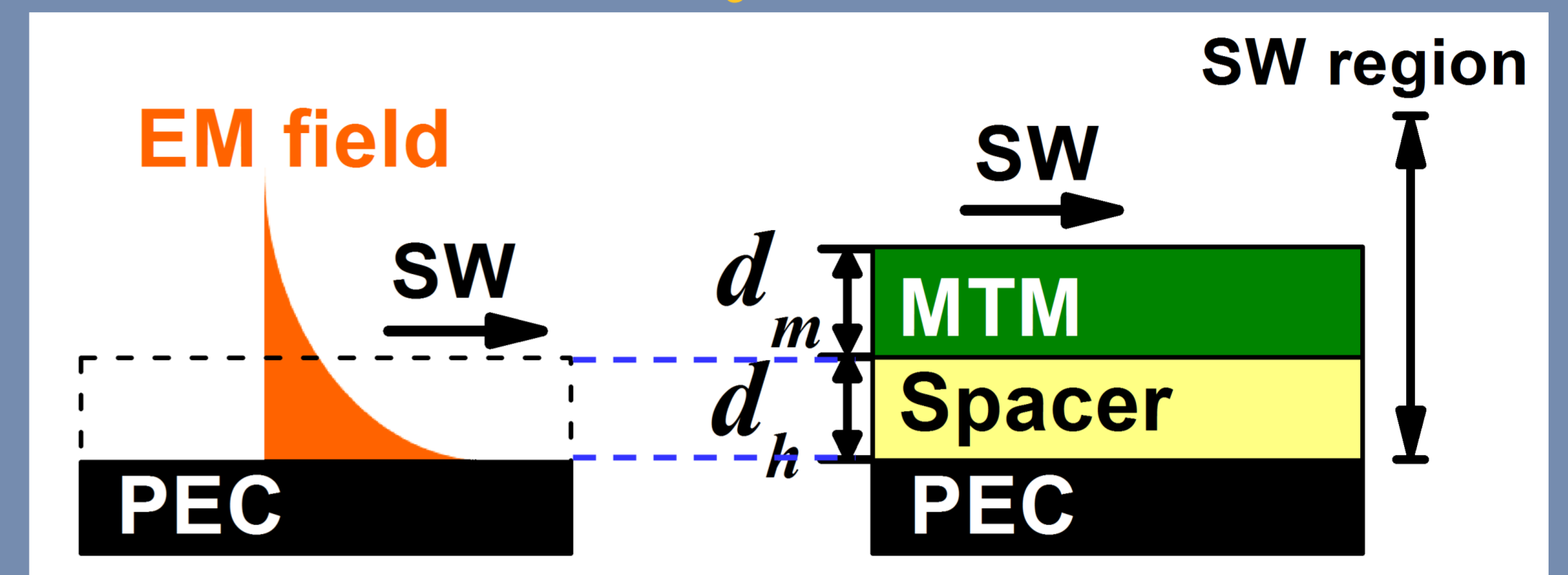
### (C) PW-SW conversion efficiency



$N_p$ : the total number of supercells inside the device

We found that while intra-supercell impedance-mismatch can hardly affect the conversion efficiencies, the scatterings caused by inter-supercell discontinuities can have non-negligible effects on the PW-SW conversion efficiency.

## II. An improved model GM with enhanced efficiency

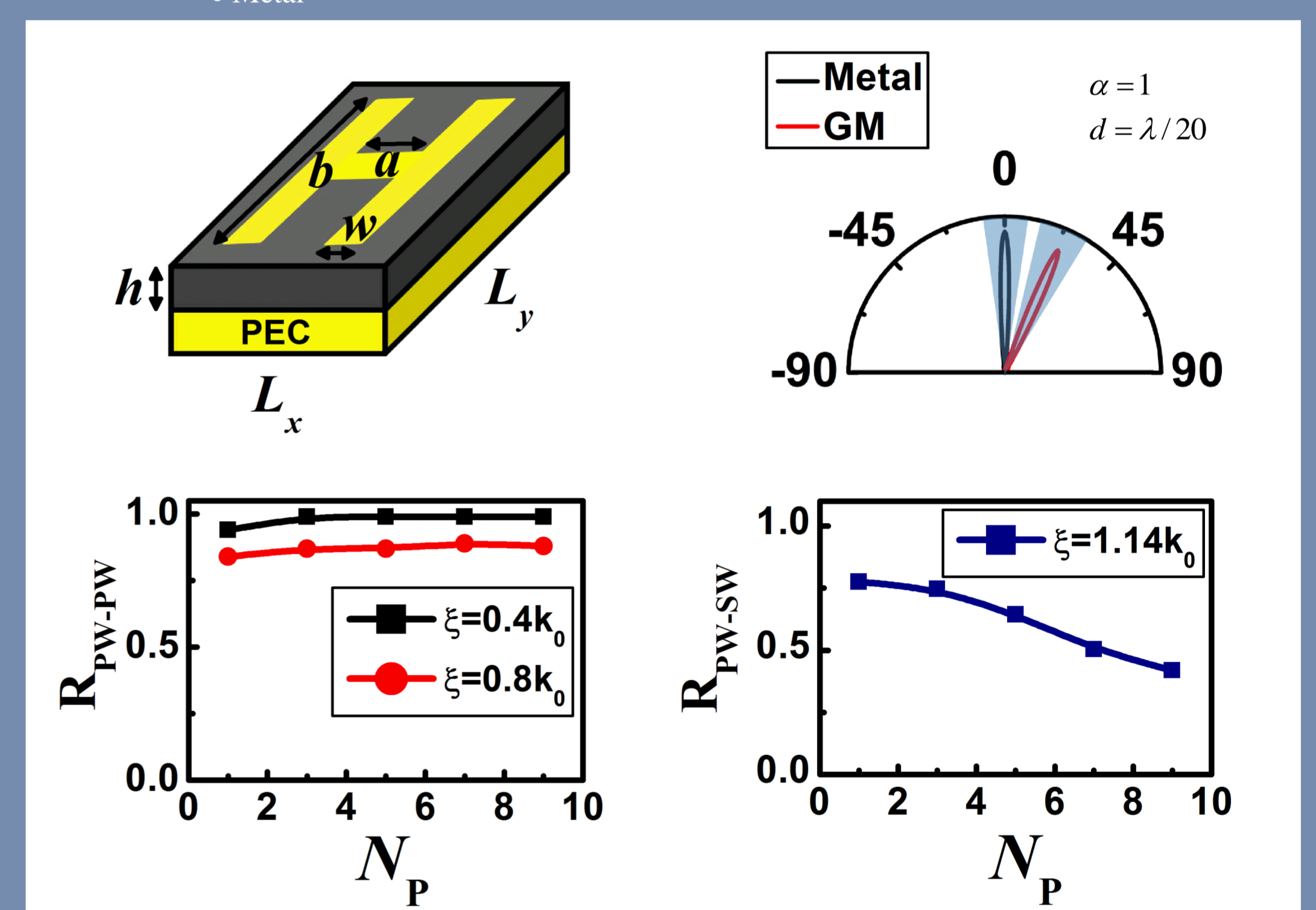


Less subwavelength GM region. There is a subtle balance between two mechanisms.

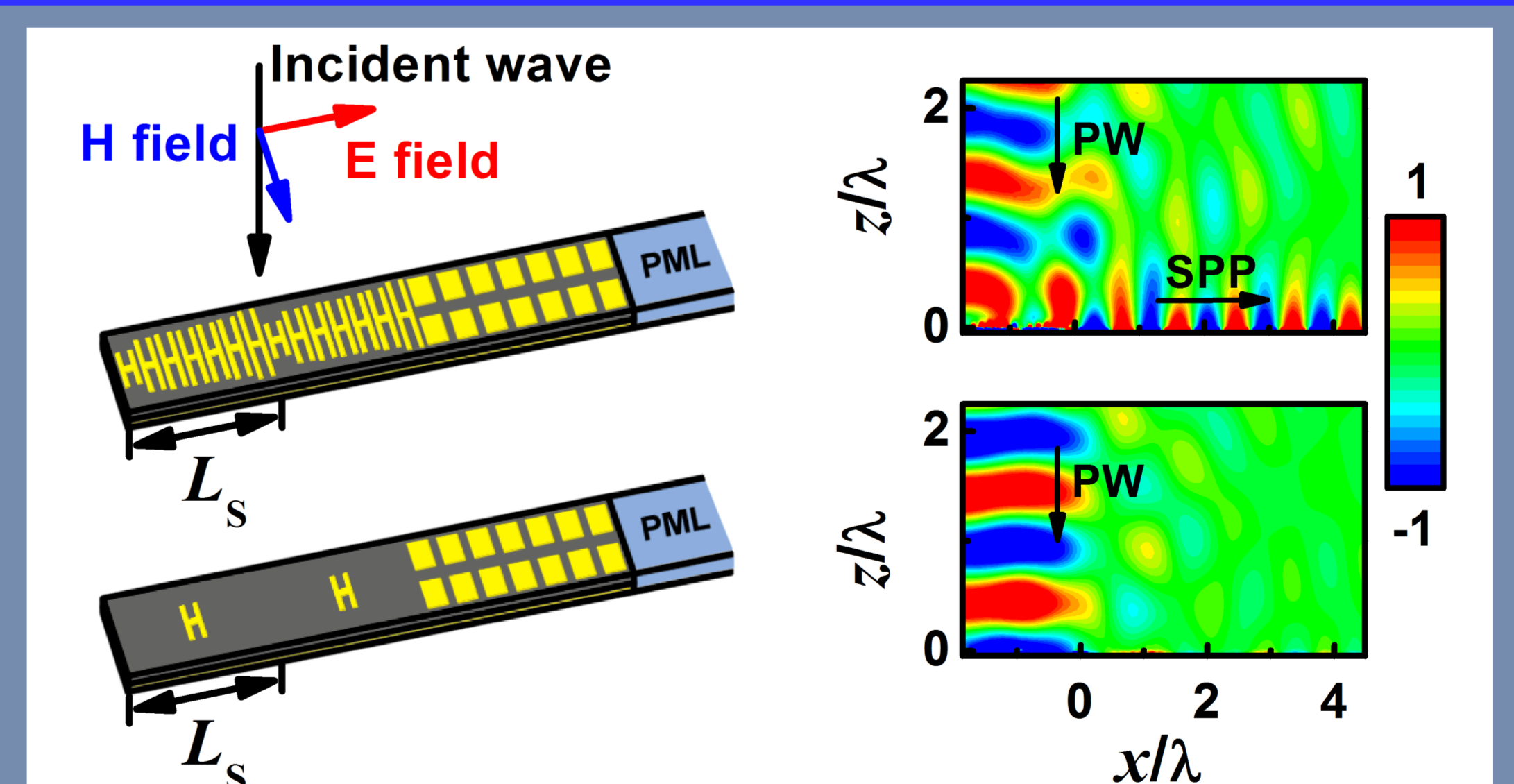
## III. Conversion efficiencies of realistic GM systems

The PW-PW conversion efficiency for a realistic GM

$$R_{\text{PW-PW}} = \frac{\int_{\text{GM}} P(\theta_r) d\Omega_r}{\int_{\text{Metal}} P(\theta_r) d\Omega_r}$$



The GM system working as a PW-SW converter can work even with a very small total length.



In contrast, a conventional grating coupler fails to work when its length is too short. The results show a 78% vs. 5.2% efficiency comparison. Because for the GM, every element contributes.

## Conclusions:

- We know the key factor affecting the conversion efficiency is the **super periodicity scattering** through the study of model GM.
- Our **improved model GM** can describe the realistic GM better.
- Probable application in **miniaturized situations** where a grating coupler is not suitable.

[1] Che Qu et al. Europhys. Lett. 101, 54002 (2013).

[2] Shulin Sun et al. Nat. Mater. 11, 426 (2012).