Experimental evidence for a human counterpart of the theory of fluctuations

Yuan Liang, Kenan An, and Jiping Huang

Department of Physics, Fudan University, Shanghai, 200433, China

Introduction

According to the theory of fluctuations in statistical mechanics, the fluctuation of an extensive quantity of a physical system is directly proportional to the number of constructive units inside the system, which we call the principle of proportionality. Nevertheless, such a system is a natural system whose constructive units are molecules without the adaptability to environmental changes due to the lack of learning ability. Here we attempt to investigate a social system whose constructive units are humans with the adaptability to environmental changes because of the presence of learning ability. For this purpose, we take a resource-allocation system as a model system. The system originates from the minority game, but can handle both unbiased and biased distributions of two resources, M_1 and M_2. We investigate four cases with different numbers and different types of participants. As a result, we find that the fluctuation of the extensive quantity, namely, the number of participants choosing M_1 or M_2, can also satisfy the principle of proportionality under a certain condition. As revealed, the underlying mechanism lies in the spontaneous sub-group cooperation arising from self-adaptive preference adjustment of selfish participants. It reveals a kind of universality between nature and society.

In statistical physics, a macroscopic quantity describing a system is the average of the relevant microscopic quantity, A, over all possible microstates, <A>, under given macroscopic conditions. Namely, this average <A> is given by

$$\langle A\rangle = \sum_s \rho_s A_s \tag{1}$$

where ρ_s is the probability when the system lies in the s-th microstate, and A_s is the value of A at the s-th microstate. Then, the fluctuation of A is defined as

$$\sigma_0^2 = \left\langle \left(A_s - \langle A\rangle\right)^2 \right\rangle = \sum_s \rho_s \left(A_s - \langle A\rangle\right)^2 \tag{2}$$

According to the theory of fluctuations, it is known that the fluctuation (σ_0^2) of an extensive quantity (say, volume or mass) of a physical system is directly proportional to the number (n_0) of constructive units inside the system, i. e.,

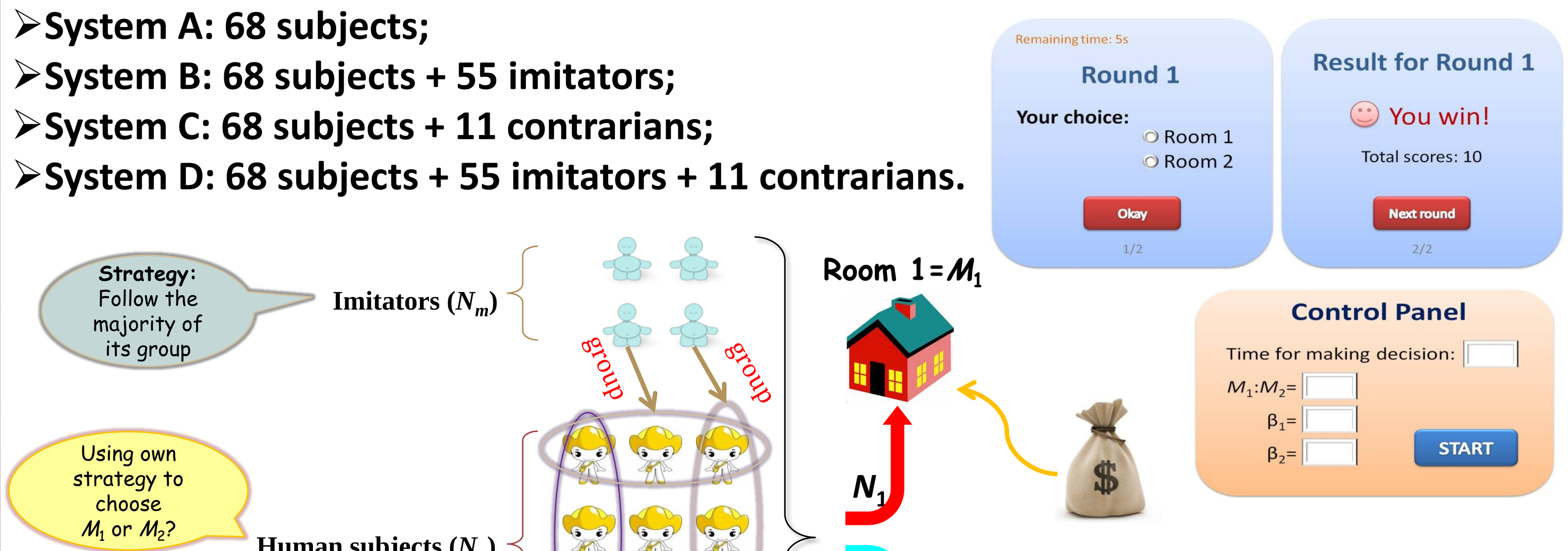
$$\frac{\sigma_0^2}{n_0} = c_0 \tag{3}$$

where c_0 is a non-zero constant. We call Eq. (3) the principle of proportionality. Nevertheless, this system is a natural system whose constructive units are molecules without the adaptability to environmental changes. The interactions between such molecules can be described by classical forces. Here we attempt to raise a question: does this equation have a counterpart in social systems? We experimentally investigate four resource-allocation systems involving different numbers and different types of participants[1-3].

Method

- **Subjects:** human players from Fudan university
- **Imitator & Contrarian:** Agents generated by computer

- **System A: 68 subjects;**
- **System B: 68 subjects + 55 imitators;**
- **System C: 68 subjects + 11 contrarians;**
- **System D: 68 subjects + 55 imitators + 11 contrarians.**



- If $M_1/N_1>M_2/N_2$, Room 1 wins.
- If $M_1/N_1<M_2/N_2$, Room 2 wins.
- $N_1+N_2=N_n+N_h+N_c=N$
- $\beta_1=N_h/N_n$
- $\beta_2=N_c/N_n$

The most important parameter of the system is N_i (i=1 or 2). Because N_i is directly proportional to the total number of participants in the system, it is naturally an extensive quantity. We define the fluctuation, σ^2, of N_i according to Eq. (2) as

$$\sigma^2 = \left\langle \left(N_1 - \widetilde{N}_1\right)^2 \right\rangle = \left\langle \left(N_2 - \widetilde{N}_2\right)^2 \right\rangle \equiv \frac{1}{2}\sum_{i=1}^{2}\left\langle \left(N_i - \widetilde{N}_i\right)^2 \right\rangle \tag{4}$$

We obtain the fluctuation per participant as

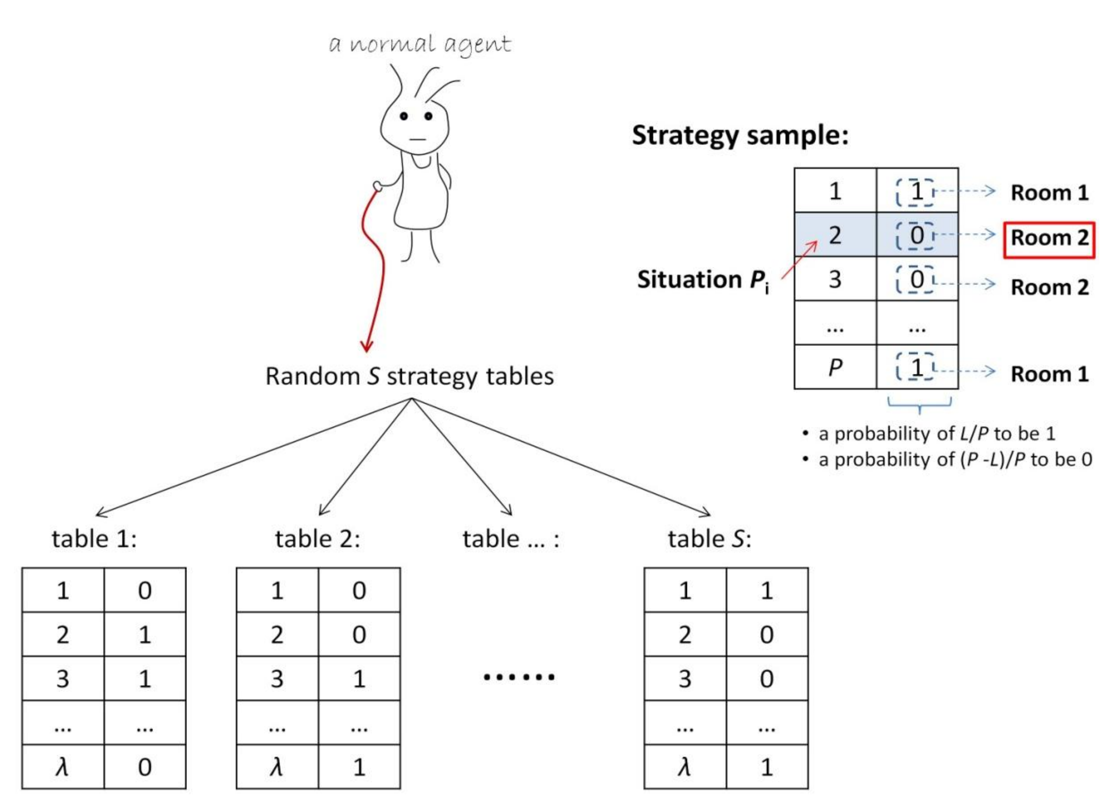
$$\frac{\sigma^2}{N} = \frac{1}{2N}\sum_{i=1}^{2}\left\langle \left(N_i - \widetilde{N}_i\right)^2 \right\rangle \tag{5}$$

Definition 1: $\widetilde{N}_i = \langle N_i \rangle$

Definition 2: $\dfrac{\langle N_1\rangle}{\langle N_2\rangle} = \dfrac{M_1}{M_2}$ (6), $\widetilde{N}_i = \dfrac{M_i}{M_1+M_2}N$ (7)

Agent-based model

- Each normal agent has S strategies.
- L is randomly chosen from 0~P.



Parameters for simulations in Figure 1: (c)-(e) N_n=68, (c)-(f) S=5, (c)-(e) P=40 for M_1/M_2=1, (c)-(d) P=15 for M_1/M_2=3, and (f) P= 30, 60, 100, 130 from left to right for M_1/M_2=1 and P=15, 25, 35, 50 from left to right for M_1/M_2=3. The simulation for each set of parameters lasts 400 rounds, and only the last 200 rounds are used for statistics based on Eq. (1).

Results

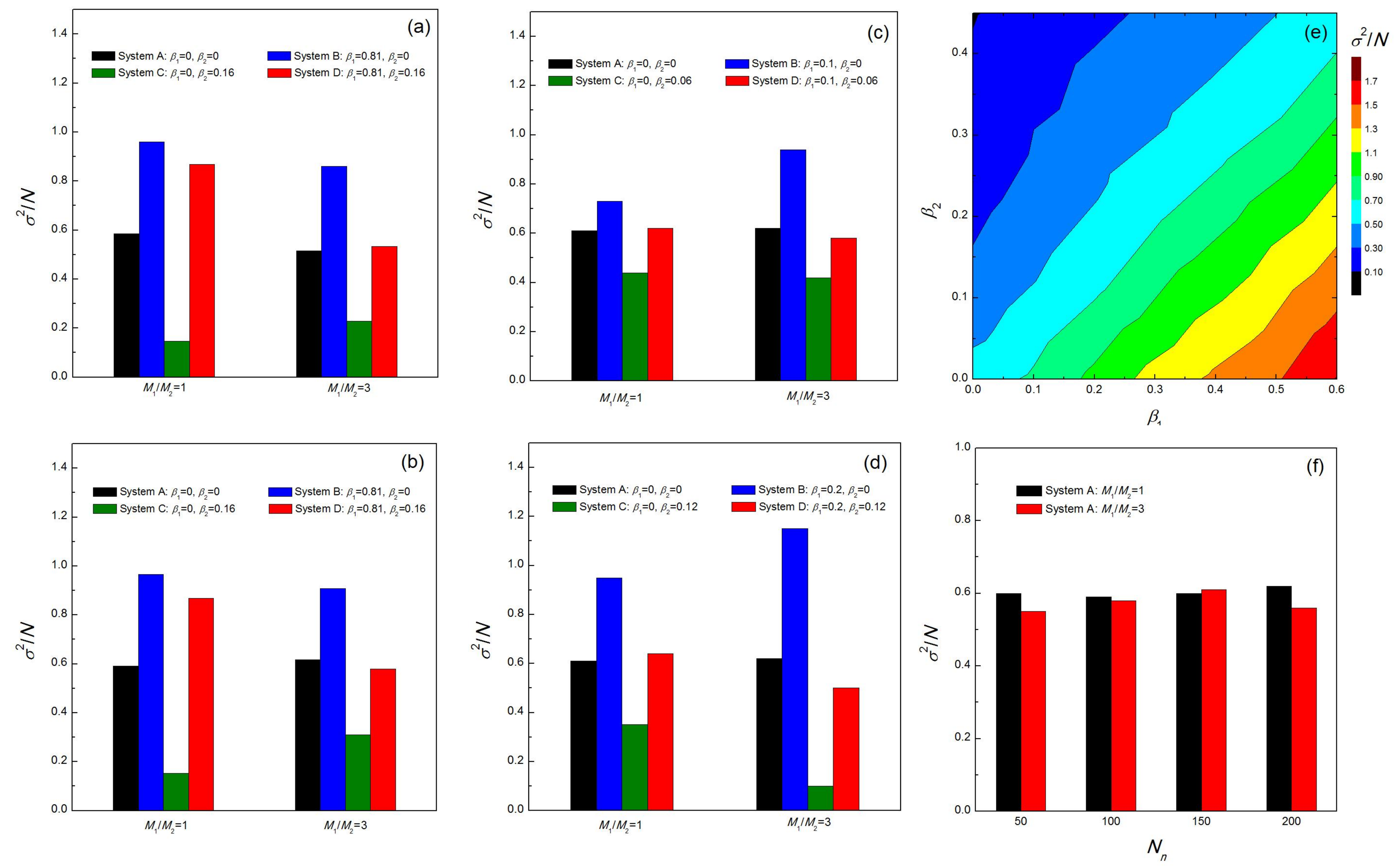
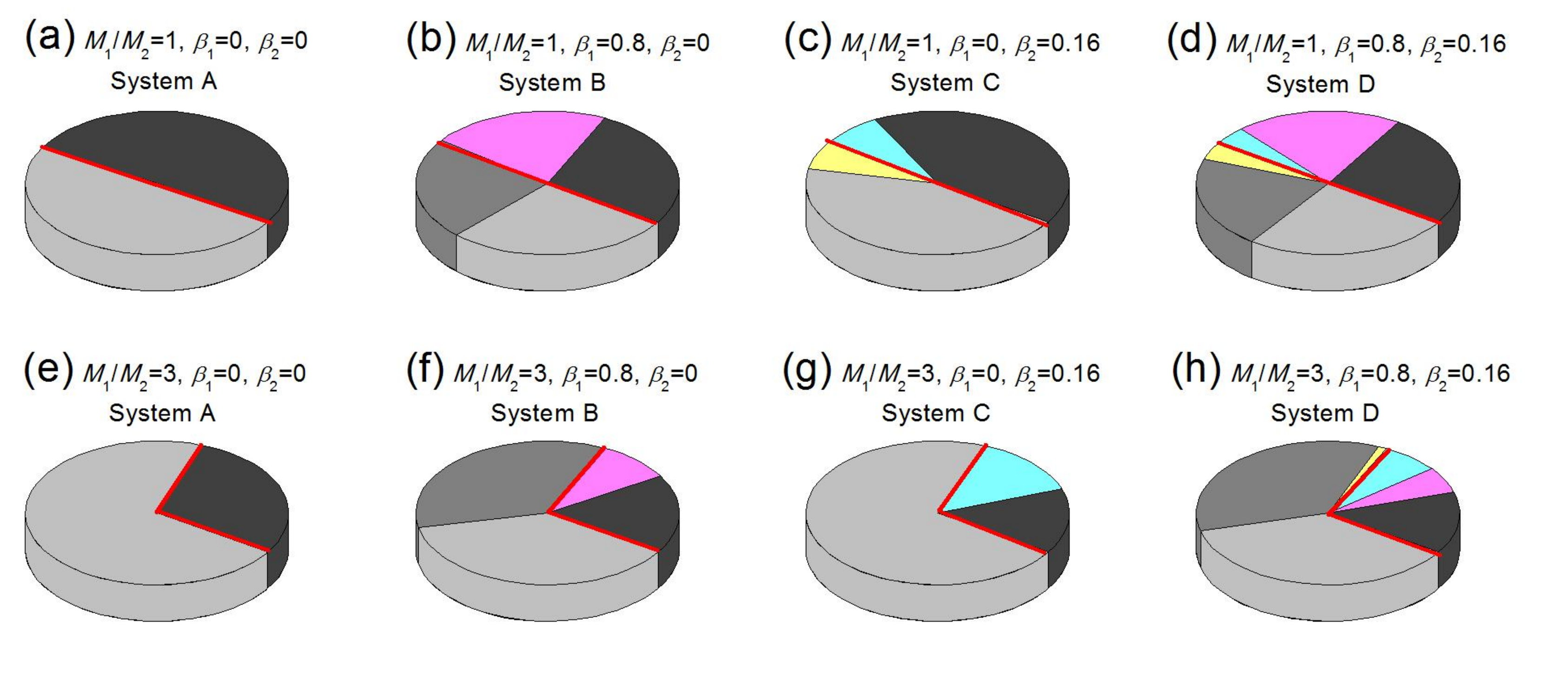


Figure 1. Fluctuation per participant σ^2/N of Systems. (a) Experiment: σ^2/N of Systems A, B, C, D for M_1/M_2 =1 and 3. N_i was determined by Eq. (1). (b) Experiment: Same as (a), but N_i was determined by Eq. (7) instead. (c) Simulation: σ^2/N of Systems A, B, C, and D for M_1/M_2 =1 and 3. (d) Simulation: Same as (c), but β_1 and β_2 are different. (e) Simulation: β_1-β_2 contour plot for σ^2/N at M_1/M_2 =1. (f) Simulation: σ^2/N of System A, as a function of N_n for M_1/M_2 =1 and 3.



Preference	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Normal: Room 1	49%	28%	44%	25.6%	71.1%	37.6%	71.6%	36.9%
Normal: Room 2	51%	27%	42.1%	25.2%	28.9%	17.7%	14.5%	13.9%
Imitator: Room 1	-	23%	-	20.6%	-	35.4%	-	35.3%
Imitator: Room 2	-	22%	-	20.4%	-	9.3%	-	5.8%
Contrarian: Room 1	-	-	6.8%	3.9%	-	-	0.2%	1.1%
Contrarian: Room 2	-	-	7.1%	4.3%	-	-	13.7%	7.2%

Figure 2. The average preference of all kinds of participants for Systems A, B, C, D under two resource ratios. The red line on each pie chart is used to divide the preference to Room 1 and 2.

References:

[1] Wang W, Chen Y, and Huang JP, Heterogeneous preferences, decision-making capacity and phase transitions in a complex adaptive system. *PNAS*, 106, 8423-8428 (2009)

[2] Liang Y, An KN, Yang G, and Huang JP. Contrarian behavior in a complex adaptive system. *Phys. Rev. E*, 87, 012809 (2013)

[3] Zhao L, Yang G, Wang W, Chen Y, Huang JP, Ohashi H, and Stanley HE. Herd behavior in a complex adaptive system. *PNAS*, 108, 15058-15063 (2011)