Experimental evidence for a human counterpart of the theory of fluctuations

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1. Introduction

According to the theory of fluctuations in statistical mechanics, the fluctuation of an extensive quantity of a physical system is directly proportional to the number of constructive units inside the system, which we call the *principle of proportionality*. Nevertheless, such a system is a natural system whose constructive units are molecules without the adaptability to environmental changes due to the lack of learning ability. Here we attempt to investigate a social system whose constructive units are humans with the adaptability to environmental changes because of the presence of learning ability. For this purpose, we take a resource-allocation system as a model system. The system originates from the minority game, but can handle both unbiased and biased distributions of two resources, M_1 and M_2. We investigate four cases with different numbers and different types of participants, and find that the fluctuation of the extensive quantity can also satisfy the principle of proportionality under a certain condition.

2. Background

In statistical physics, a macroscopic quantity describing a system is the average of the relevant microscopic quantity, A, over all possible microstates, <A>, under given macroscopic conditions.

$$\langle A\rangle = \sum_s \rho_s A_s \qquad (1)$$

where ρ_s is the probability when the system lies in the s-th microstate, and A_s is the value of A at the s-th microstate. Then, the fluctuation of A is defined as

$$\sigma_0^2 = \left\langle \left(A_s - \langle A\rangle\right)^2 \right\rangle = \sum_s \rho_s \left(A_s - \langle A\rangle\right)^2 \qquad (2)$$

According to the theory of fluctuations, it is known that the fluctuation (σ_0^2) of an extensive quantity of a physical system is directly proportional to the number (n_0) of constructive units inside the system, i. e.,

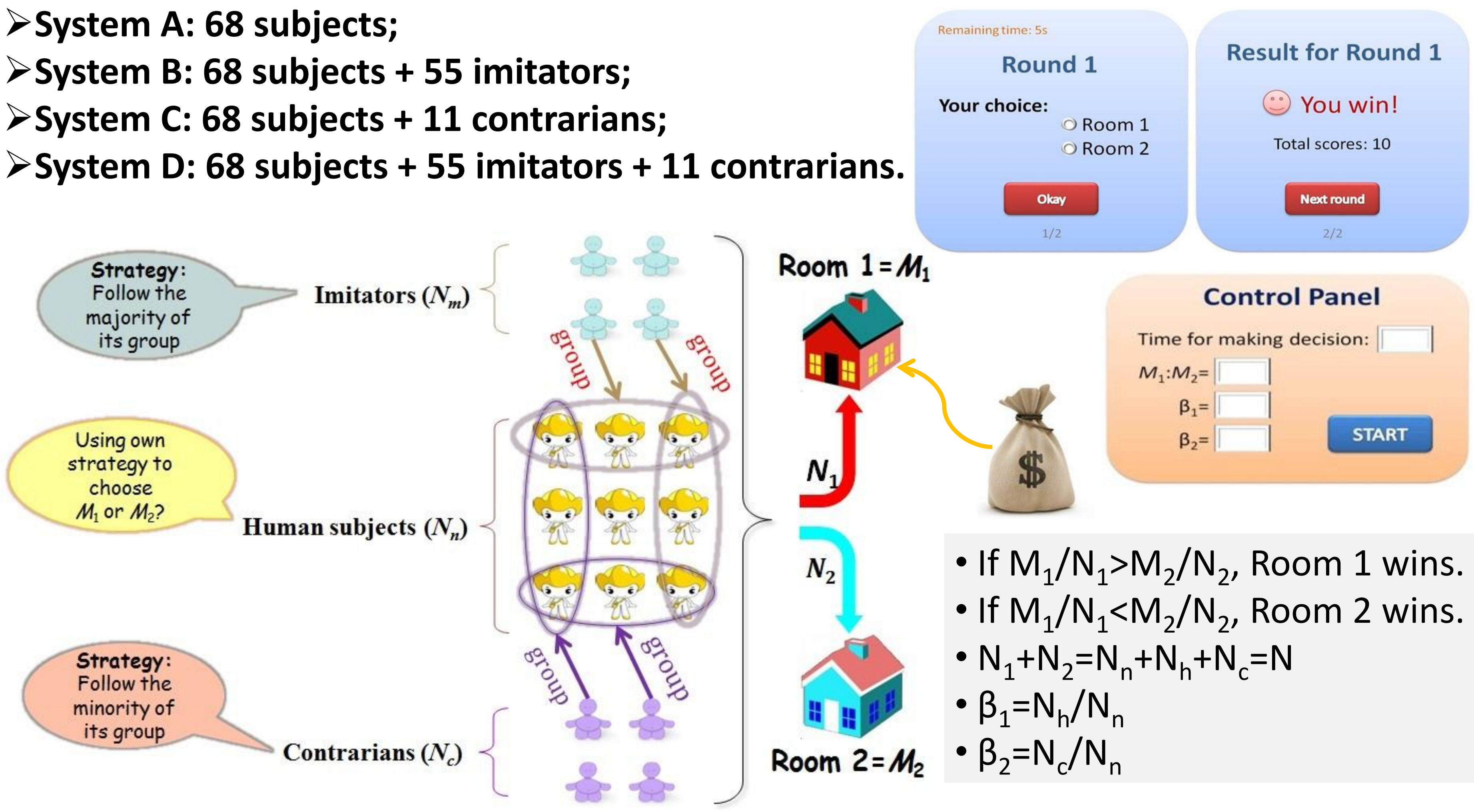
$$\frac{\sigma_0^2}{n_0} = c_0 \qquad (3)$$

where c_0 is a non-zero constant. We call Eq. (3) the principle of proportionality. Here we attempt to raise a question: does this equation have a counterpart in social systems? We experimentally investigate four resource-allocation systems involving different numbers and different types of participants[1-3].

3. Method

- **Subjects:** human players from Fudan university
- **Imitator & Contrarian:** Agents generated by computer

- ➢**System A: 68 subjects;**
- ➢**System B: 68 subjects + 55 imitators;**
- ➢**System C: 68 subjects + 11 contrarians;**
- ➢**System D: 68 subjects + 55 imitators + 11 contrarians.**



- If $M_1/N_1 > M_2/N_2$, Room 1 wins.
- If $M_1/N_1 < M_2/N_2$, Room 2 wins.
- $N_1+N_2=N_n+N_h+N_c=N$
- $\beta_1=N_h/N_n$
- $\beta_2=N_c/N_n$

The most important parameter of the system is N_i (i=1 or 2). Because N_i is directly proportional to the total number of participants in the system, it is naturally an extensive quantity. We define the fluctuation, σ^2, of N_i according to Eq. (2) as

$$\sigma^2 = \left\langle \left(N_1 - \overline{N}_1\right)^2 \right\rangle = \left\langle \left(N_2 - \overline{N}_2\right)^2 \right\rangle \equiv \frac{1}{2}\sum_{i=1}^{2} \left\langle \left(N_i - \overline{N}_i\right)^2 \right\rangle \qquad (4)$$

We obtain the fluctuation per participant as

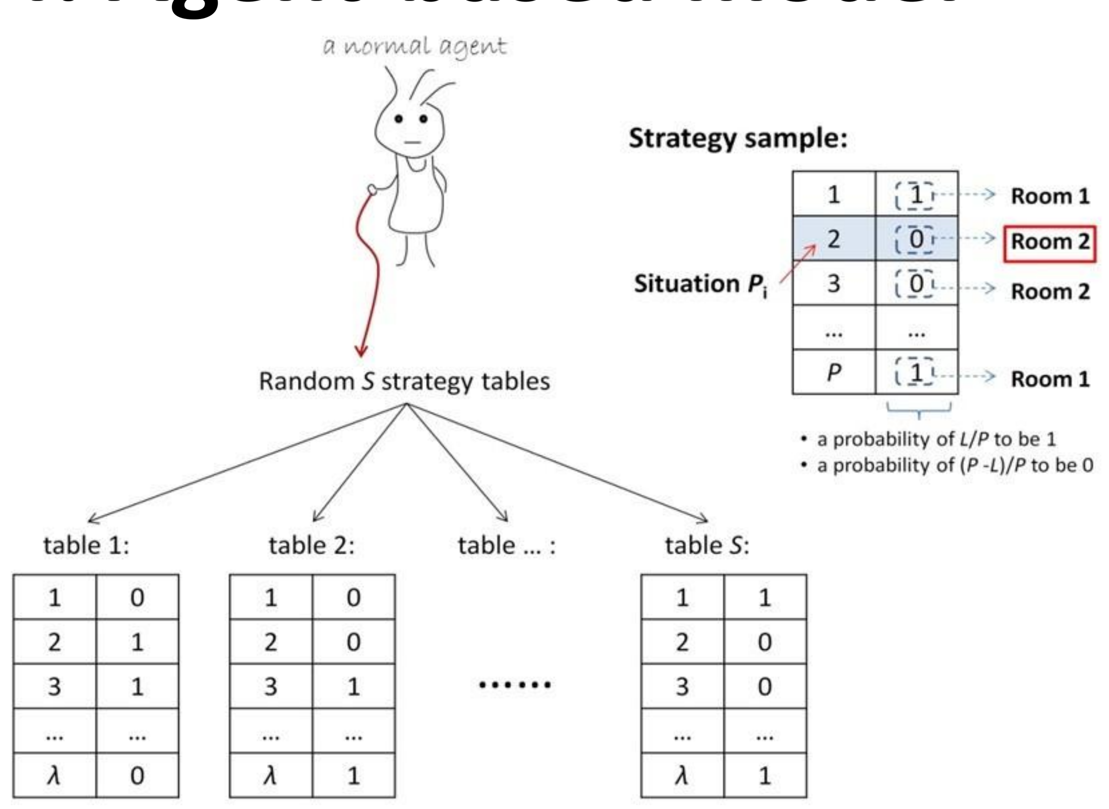
$$\frac{\sigma^2}{N} = \frac{1}{2N}\sum_{i=1}^{2} \left\langle \left(N_i - \overline{N}_i\right)^2 \right\rangle \qquad (5)$$

Definition 1: $\overline{N}_i = \langle N_i \rangle$

Definition 2: $\frac{\langle N_1\rangle}{\langle N_2\rangle} = \frac{M_1}{M_2}$ (6), then $\overline{N}_i = \frac{M_i}{M_1+M_2}N$ (7)

4. Agent-based model

- Each normal agent has S strategies.
- L is randomly chosen from 0~P.



Parameters for simulations in Figure 1: (c)-(e) N_n=68, (c)-(f) S=5, (c)-(e) P=40 for M_1/M_2=1, (c)-(d) P=15 for M_1/M_2=3, and (f) P= 30, 60, 100, 130 from left to right for M_1/M_2=1 and P=15, 25, 35, 50 from left to right for M_1/M_2=3. The simulation for each set of parameters lasts 400 rounds, and only the last 200 rounds are used for statistics based on Eq. (1).

5. Results

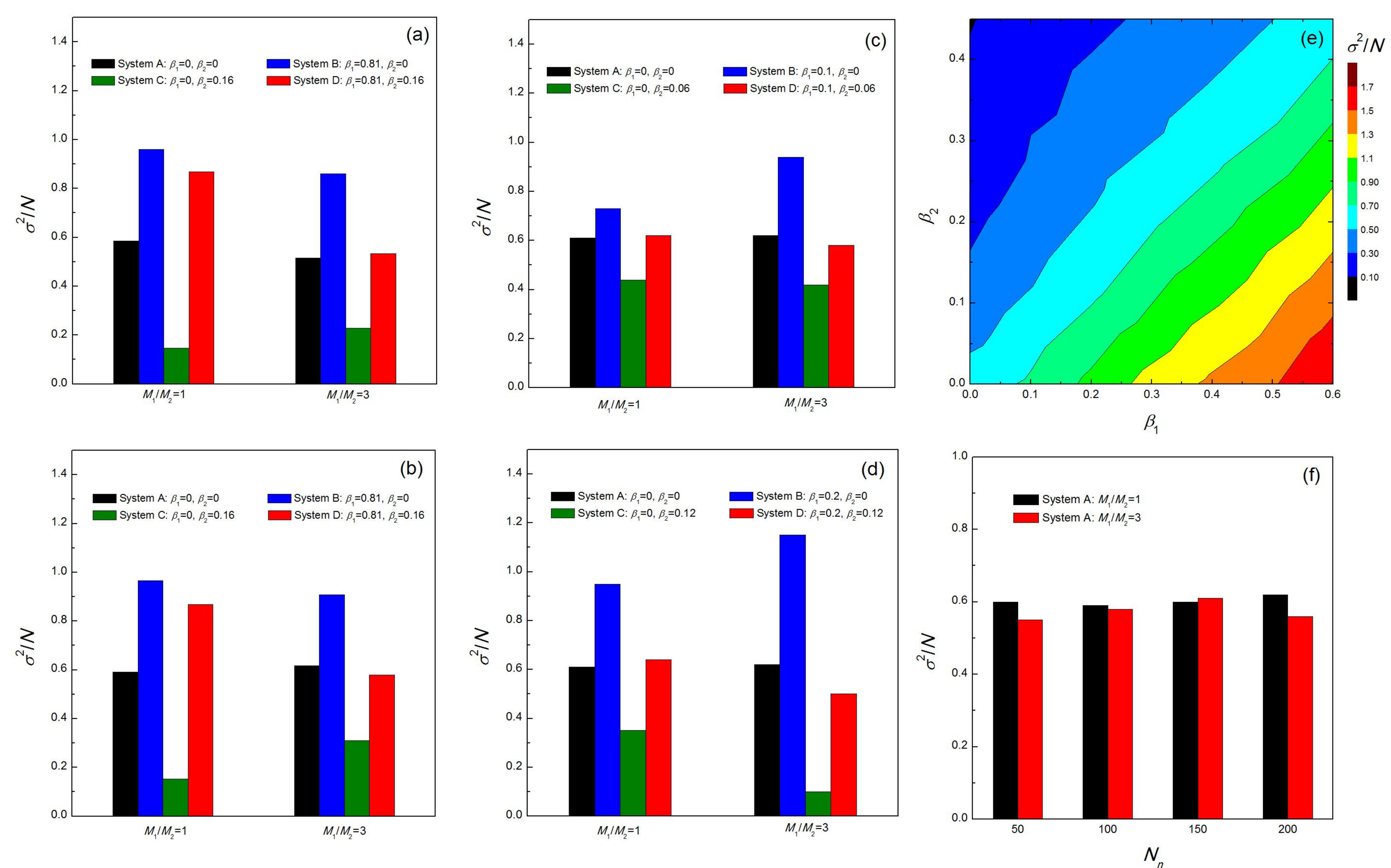
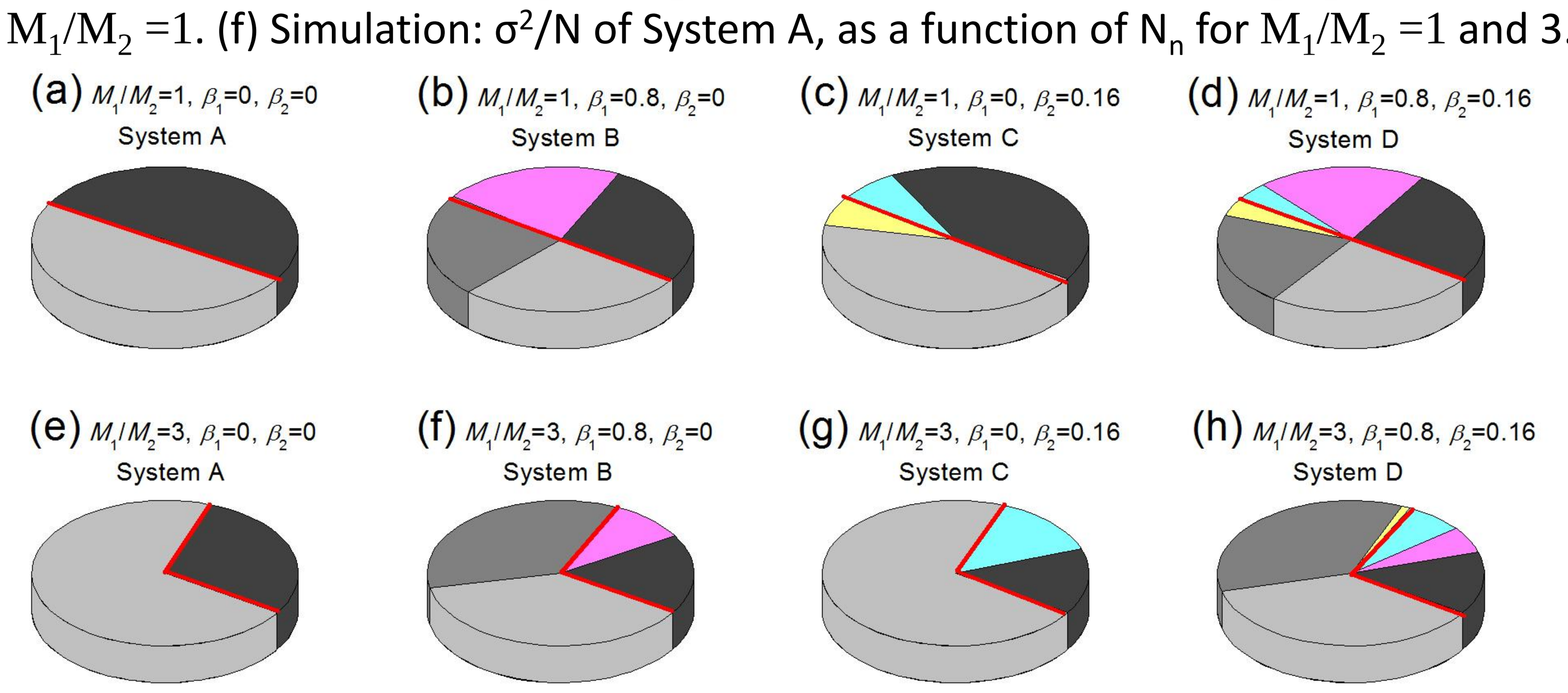


Figure 1. Fluctuation per participant σ^2/N of Systems. (a) Experiment: σ^2/N of Systems A, B, C, D for M_1/M_2 =1 and 3. $\overline{N}_i$ was determined by Eq. (1). (b) Experiment: Same as (a), but $\overline{N}_i$ was determined by Eq. (7) instead. (c) Simulation: σ^2/N of Systems A, B, C, and D for M_1/M_2 =1 and 3. (d) Simulation: Same as (c), but β_1 and β_2 are different. (e) Simulation: β_1-β_2 contour plot for σ^2/N at M_1/M_2 =1. (f) Simulation: σ^2/N of System A, as a function of N_n for M_1/M_2 =1 and 3.



Preference	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)
Normal: Room 1	49%	28%	44%	25.6%	71.1%	37.6%	71.6%	36.9%
Normal: Room 2	51%	27%	42.1%	25.2%	28.9%	17.7%	14.5%	13.9%
Imitator: Room 1	-	23%	-	20.6%	-	35.4%	-	35.3%
Imitator: Room 2	-	22%	-	20.4%	-	9.3%	-	5.8%
Contrarian: Room 1	-	-	6.8%	3.9%	-	-	0.2%	1.1%
Contrarian: Room 2	-	-	7.1%	4.3%	-	-	13.7%	7.2%

Figure 2. The average preference of all kinds of participants for Systems A, B, C, D under two resource ratios. The red line on each pie chart is used to divide the preference to Room 1 and 2.

6. Conclusion

Using the human experiments, we have found that the fluctuation of the extensive quantity can also satisfy the principle of proportionality. As revealed by the preference analysis, the underlying mechanism lies in the spontaneous cooperation of imitators and contrarians arising from self-adaptive preference adjustment of subjects. Although the interactions between units in natural and social systems are different, the principle of proportionality that holds in natural systems may have a human counterpart under some conditions.

Refs: [1] Wang W, Chen Y, and Huang JP, Heterogeneous preferences, decision-making capacity and phase transitions in a complex adaptive system. *PNAS*, 106, 8423-8428 (2009)

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