# Experimental evidence for a human counterpart of the theory of fluctuations

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## Introduction

According to the theory of fluctuations in statistical mechanics, the fluctuation of an extensive quantity of a physical system is directly proportional to the number of constructive units inside the system, which we call the principle of proportionality. Nevertheless, such a system is a natural system whose constructive units are molecules without the adaptability to environmental changes due to the lack of learning ability. Here we attempt to investigate a social system whose constructive units are humans with the adaptability to environmental changes because of the presence of learning ability. For this purpose, we take a resource-allocation system as a model system. The system originates from the minority game, but can handle both unbiased and biased distributions of two resources, $M_1$ and $M_2$. We investigate four cases with different numbers and different types of participants. As a result, we find that the fluctuation of the extensive quantity, namely, the number of participants choosing $M_1$ or $M_2$, can also satisfy the principle of proportionality under a certain condition. As revealed, the underlying mechanism lies in the spontaneous sub-group cooperation arising from self-adaptive preference adjustment of selfish participants. It reveals a kind of universality between nature and society.

In statistical physics, a macroscopic quantity describing a system is the average of the relevant microscopic quantity, A, over all possible microstates, <A>, under given macroscopic conditions. Namely, this average <A> is given by

$$\langle A\rangle=\sum_s \rho_s A_s \qquad (1)$$

where $\rho_s$ is the probability when the system lies in the s-th microstate, and $A_s$ is the value of A at the s-th microstate. Then, the fluctuation of A is defined as

$$\sigma_0^2=\left\langle \left(A_s-\langle A\rangle\right)^2\right\rangle=\sum_s \rho_s\left(A_s-\langle A\rangle\right)^2 \qquad (2)$$

According to the theory of fluctuations, it is known that the fluctuation ($\sigma_0^2$) of an extensive quantity (say, volume or mass) of a physical system is directly proportional to the number ($n_0$) of constructive units inside the system, i. e.,

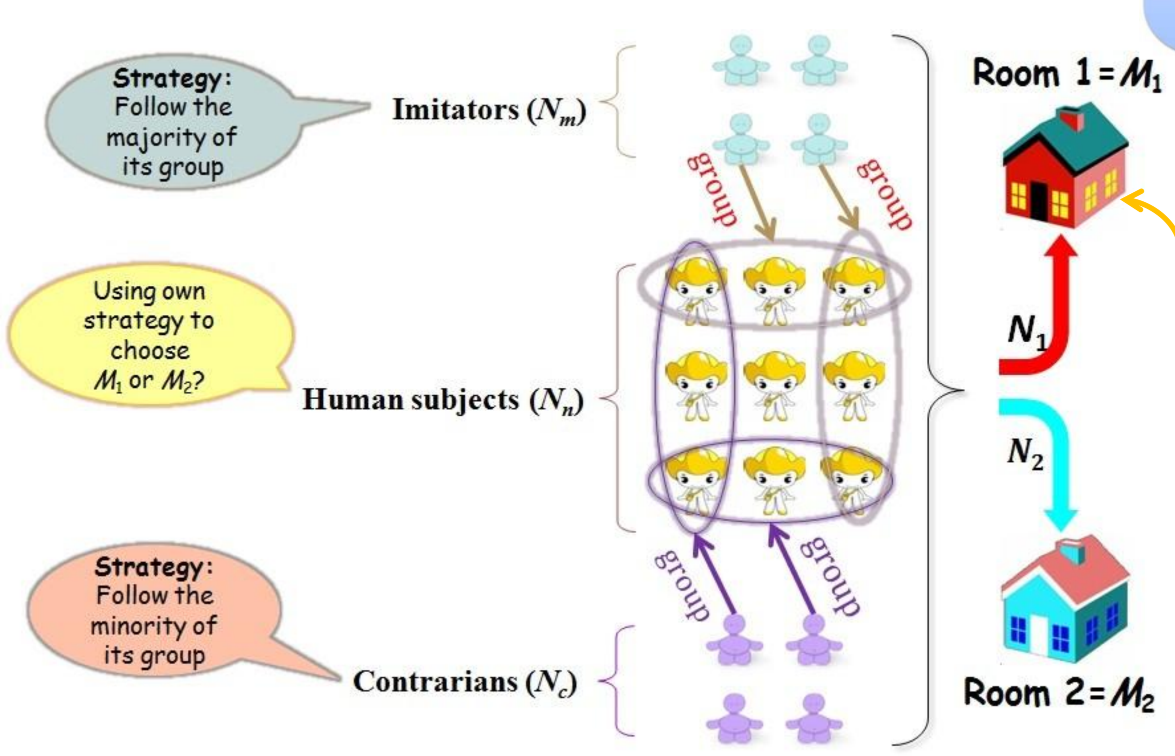
$$\frac{\sigma_0^2}{n_0}=c_0 \qquad (3)$$

where $c_0$ is a non-zero constant. We call Eq. (3) the principle of proportionality. Nevertheless, this system is a natural system whose constructive units are molecules without the adaptability to environmental changes. The interactions between such molecules can be described by classical forces. Here we attempt to raise a question: does this equation have a counterpart in social systems? We experimentally investigate four resource-allocation systems involving different numbers and different types of participants[1-3].

## Method

- **Subjects:** human players from Fudan university
- **Imitator & Contrarian:** Agents generated by computer

- **System A: 68 subjects;**
- **System B: 68 subjects + 55 imitators;**
- **System C: 68 subjects + 11 contrarians;**
- **System D: 68 subjects + 55 imitators + 11 contrarians.**


- If $M_1/N_1>M_2/N_2$, Room 1 wins.
- If $M_1/N_1<M_2/N_2$, Room 2 wins.
- $N_1+N_2=N_n+N_h+N_c=N$
- $\beta_1=N_h/N_n$
- $\beta_2=N_c/N_n$

The most important parameter of the system is $N_i$ (i=1 or 2). Because $N_i$ is directly proportional to the total number of participants in the system, it is naturally an extensive quantity. We define the fluctuation, $\sigma^2$, of $N_i$ according to Eq. (2) as

$$\sigma^2=\left\langle\left(N_1-\overline{N}_1\right)^2\right\rangle=\left\langle\left(N_2-\overline{N}_2\right)^2\right\rangle\equiv\frac{1}{2}\sum_{i=1}^{2}\left\langle\left(N_i-\overline{N}_i\right)^2\right\rangle \qquad (4)$$

We obtain the fluctuation per participant as

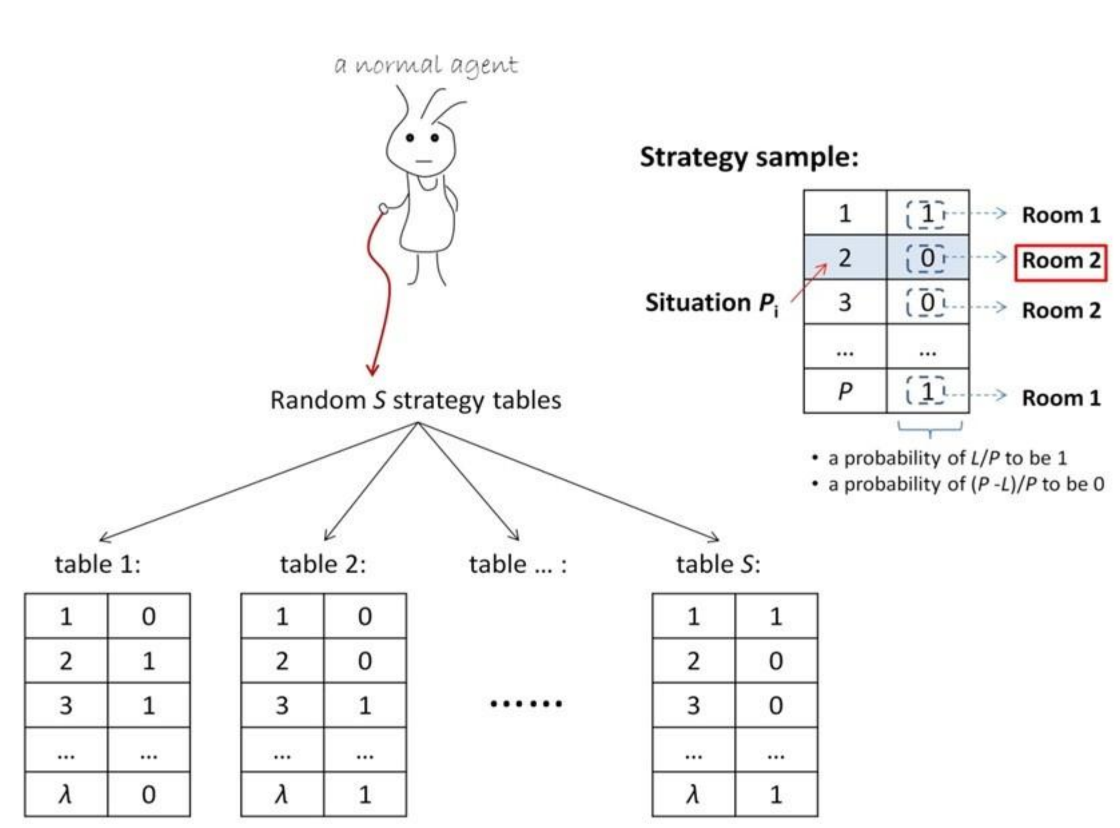
$$\frac{\sigma^2}{N}=\frac{1}{2N}\sum_{i=1}^{2}\left\langle\left(N_i-\overline{N}_i\right)^2\right\rangle \qquad (5)$$

Definition 1: $\overline{N}_i=\langle N_i\rangle$

Definition 2: $\frac{\langle N_1\rangle}{\langle N_2\rangle}=\frac{M_1}{M_2}$ (6), then $\overline{N}_i=\frac{M_i}{M_1+M_2}N$ (7)

## Agent-based model

- Each normal agent has S strategies.
- L is randomly chosen from 0~P.




**Parameters for simulations in Figure 1:** (c)-(e) $N_n$=68, (c)-(f) S=5, (c)-(e) P=40 for $M_1/M_2$=1, (c)-(d) P=15 for $M_1/M_2$=3, and (f) P= 30, 60, 100, 130 from left to right for $M_1/M_2$=1 and P=15, 25, 35, 50 from left to right for $M_1/M_2$=3. The simulation for each set of parameters lasts 400 rounds, and only the last 200 rounds are used for statistics based on Eq. (1).

## Results

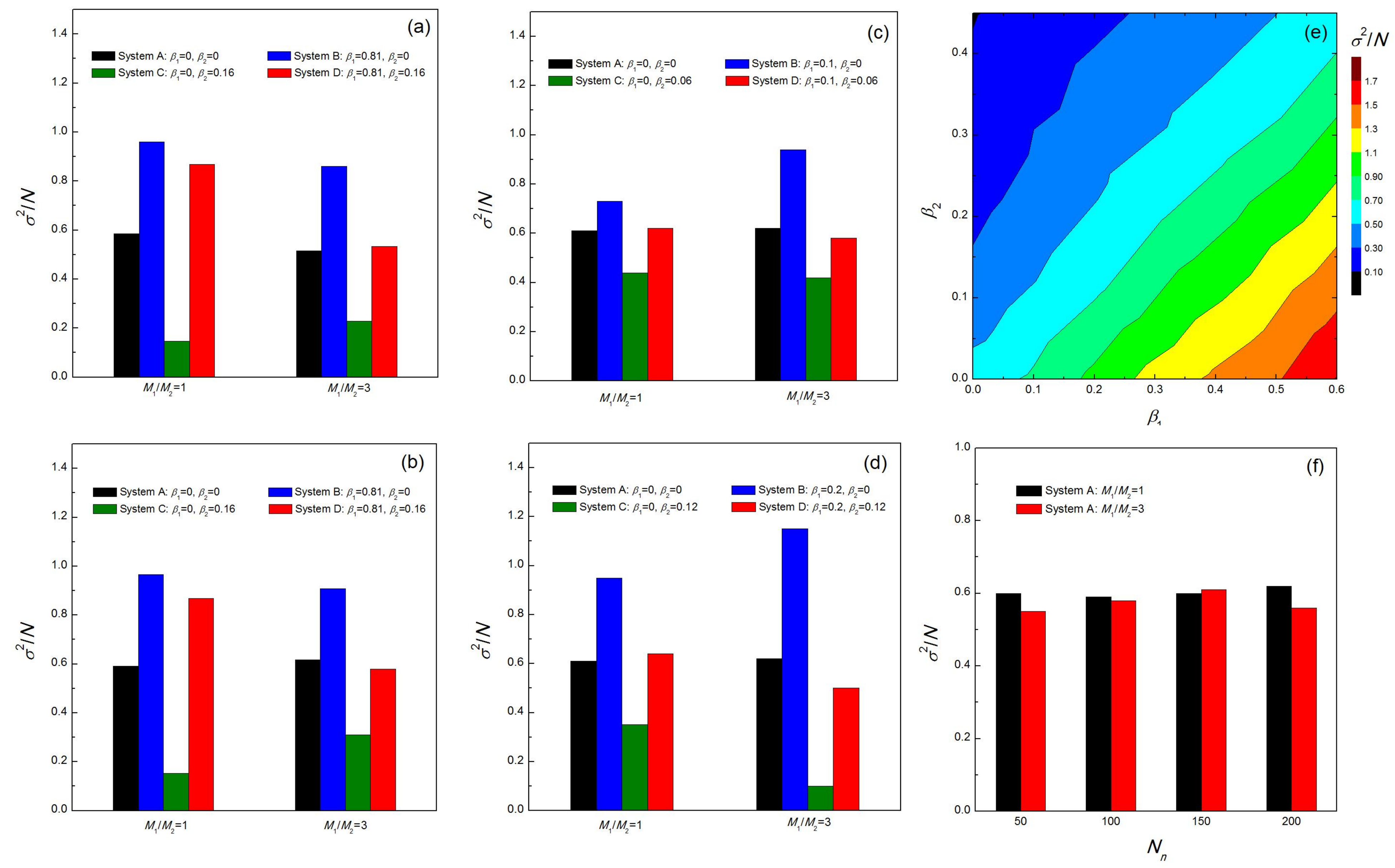
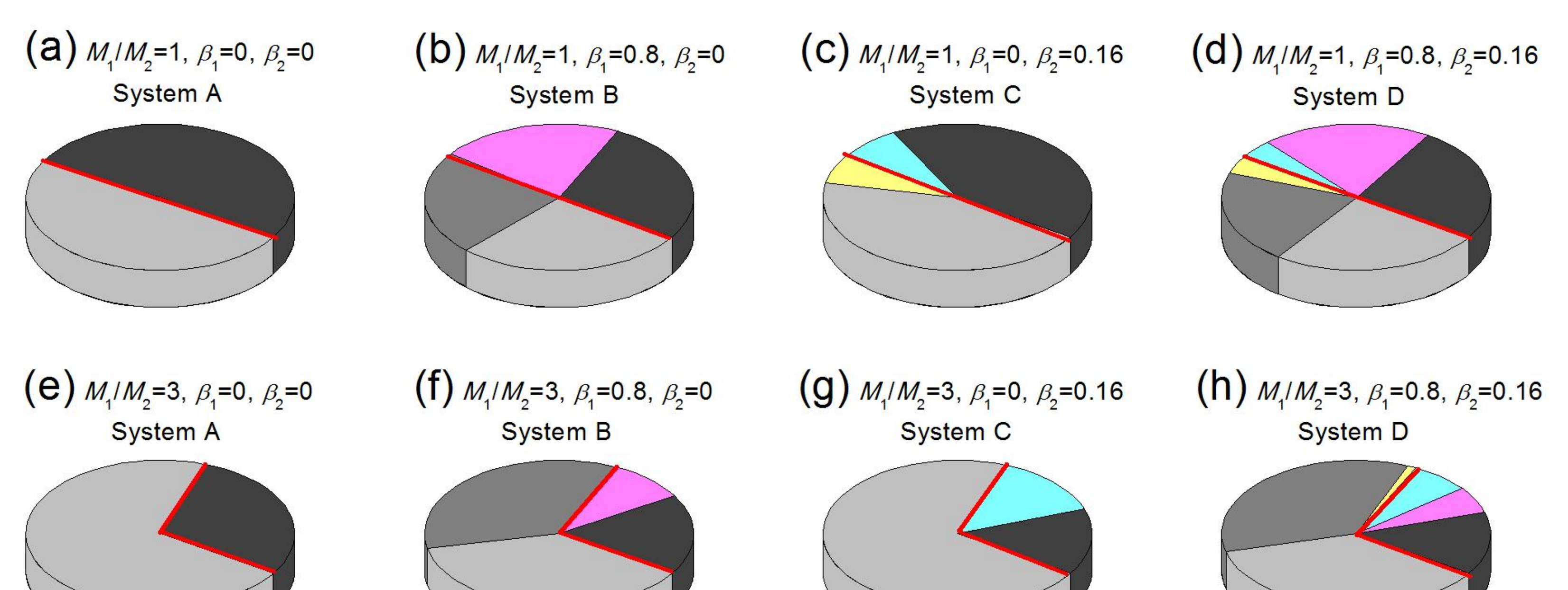



Figure 1. Fluctuation per participant $\sigma^2/N$ of Systems. (a) Experiment: $\sigma^2/N$ of Systems A, B, C, D for $M_1/M_2$ =1 and 3. $\overline{N}_i$ was determined by Eq. (1). (b) Experiment: Same as (a), but $\overline{N}_i$ was determined by Eq. (7) instead. (c) Simulation: $\sigma^2/N$ of Systems A, B, C, and D for $M_1/M_2$ =1 and 3. (d) Simulation: Same as (c), but $\beta_1$ and $\beta_2$ are different. (e) Simulation: $\beta_1$-$\beta_2$ contour plot for $\sigma^2/N$ at $M_1/M_2$ =1. (f) Simulation: $\sigma^2/N$ of System A, as a function of $N_n$ for $M_1/M_2$ =1 and 3.




| Preference | (a) | (b) | (c) | (d) | (e) | (f) | (g) | (h) |
|---|---|---|---|---|---|---|---|---|
| Normal: Room 1 | 49% | 28% | 44% | 25.6% | 71.1% | 37.6% | 71.6% | 36.9% |
| Normal: Room 2 | 51% | 27% | 42.1% | 25.2% | 28.9% | 17.7% | 14.5% | 13.9% |
| Imitator: Room 1 | - | 23% | - | 20.6% | - | 35.4% | - | 35.3% |
| Imitator: Room 2 | - | 22% | - | 20.4% | - | 9.3% | - | 5.8% |
| Contrarian: Room 1 | - | - | 6.8% | 3.9% | - | - | 0.2% | 1.1% |
| Contrarian: Room 2 | - | - | 7.1% | 4.3% | - | - | 13.7% | 7.2% |

Figure 2. The average preference of all kinds of participants for Systems A, B, C, D under two resource ratios. The red line on each pie chart is used to divide the preference to Room 1 and 2.

**References:**

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[2] Liang Y, An KN, Yang G, and Huang JP. Contrarian behavior in a complex adaptive system. *Phys. Rev. E*, 87, 012809 (2013)

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