Renormalized Phonon in FPU-β Model

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Introduction

Model Hamiltonian:

$$H = \sum_{n=1}^{N}\left[\frac{P_n^2}{2M} + \frac{1}{2}(x_{n+1}-x_n)^2 + \frac{\beta}{4}(x_{n+1}-x_n)^4\right]$$

N: Particle number

M: Particle mass x: Displacement P: Momentum

β: Strength of nonlinearity

Anomalous heat conduction[1]:

(heat conductivity) $\kappa \propto N^{\alpha}$ $0 < \alpha < 1$

Mechanism and heat carrier ?

Effective Phonon Theory[2]:
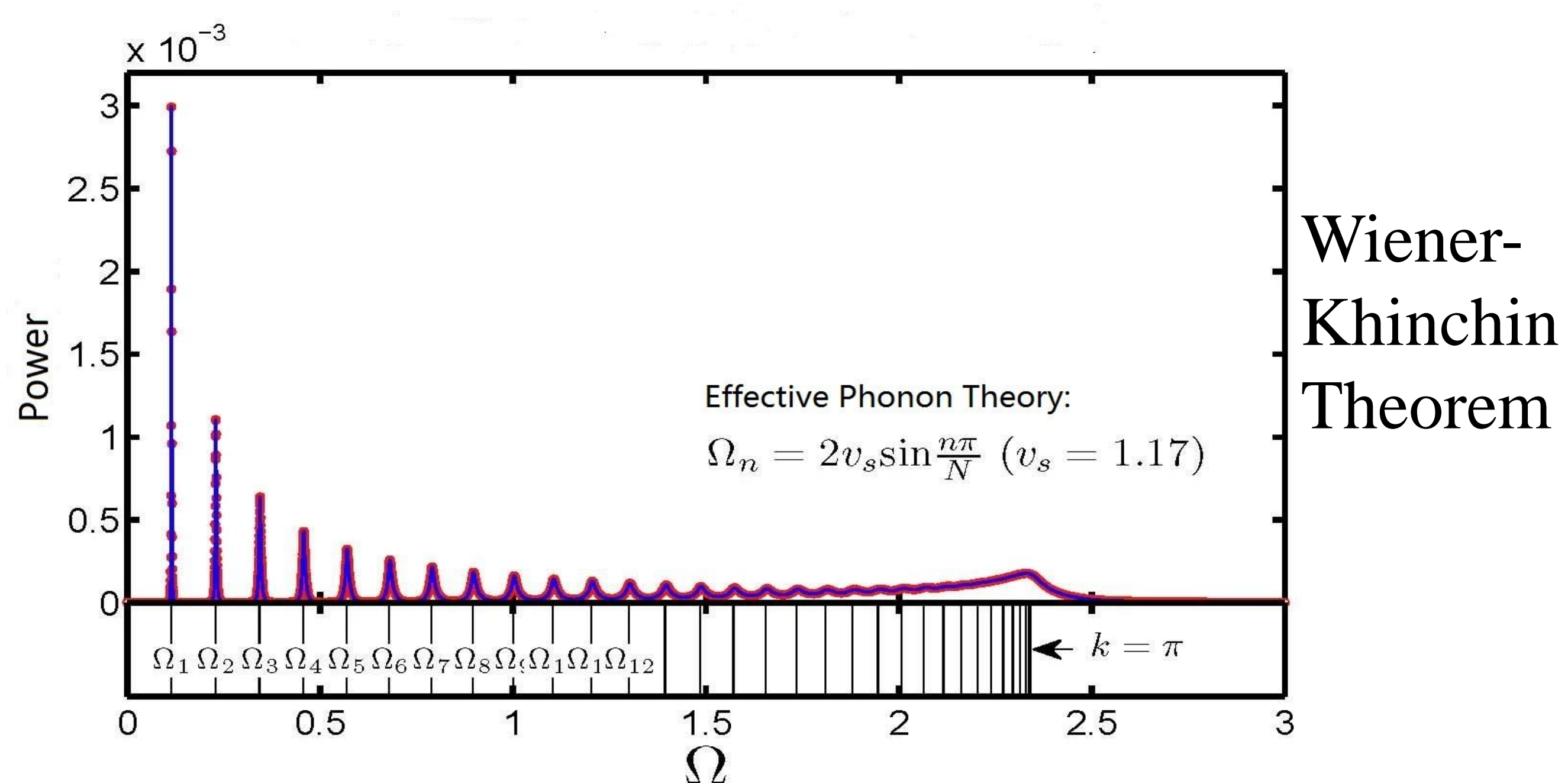
Heat carrier is the renormalized phonon with dispersion

$$\Omega_k = \sqrt{\lambda}\,\omega_k,$$

with $\lambda = 1 + \frac{\beta\langle(x_{n+1}-x_n)^4\rangle}{\langle(x_{n+1}-x_n)^2\rangle}$ and $\omega_k = \frac{2}{\sqrt{M}}\left|\sin\frac{k}{2}\right|$.
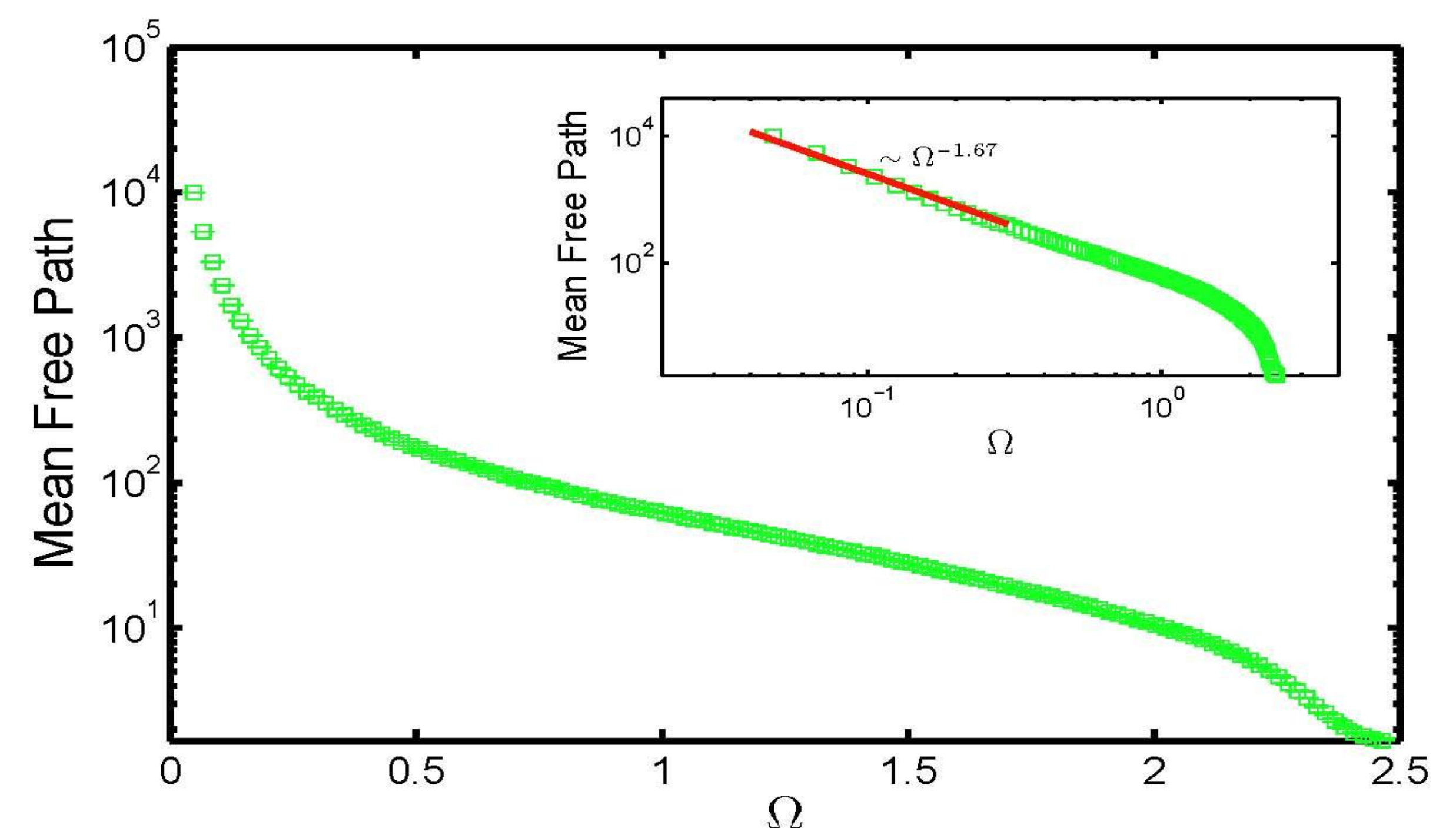
Does renormalized phonon exist ?

Numerical Simulation Results[3]
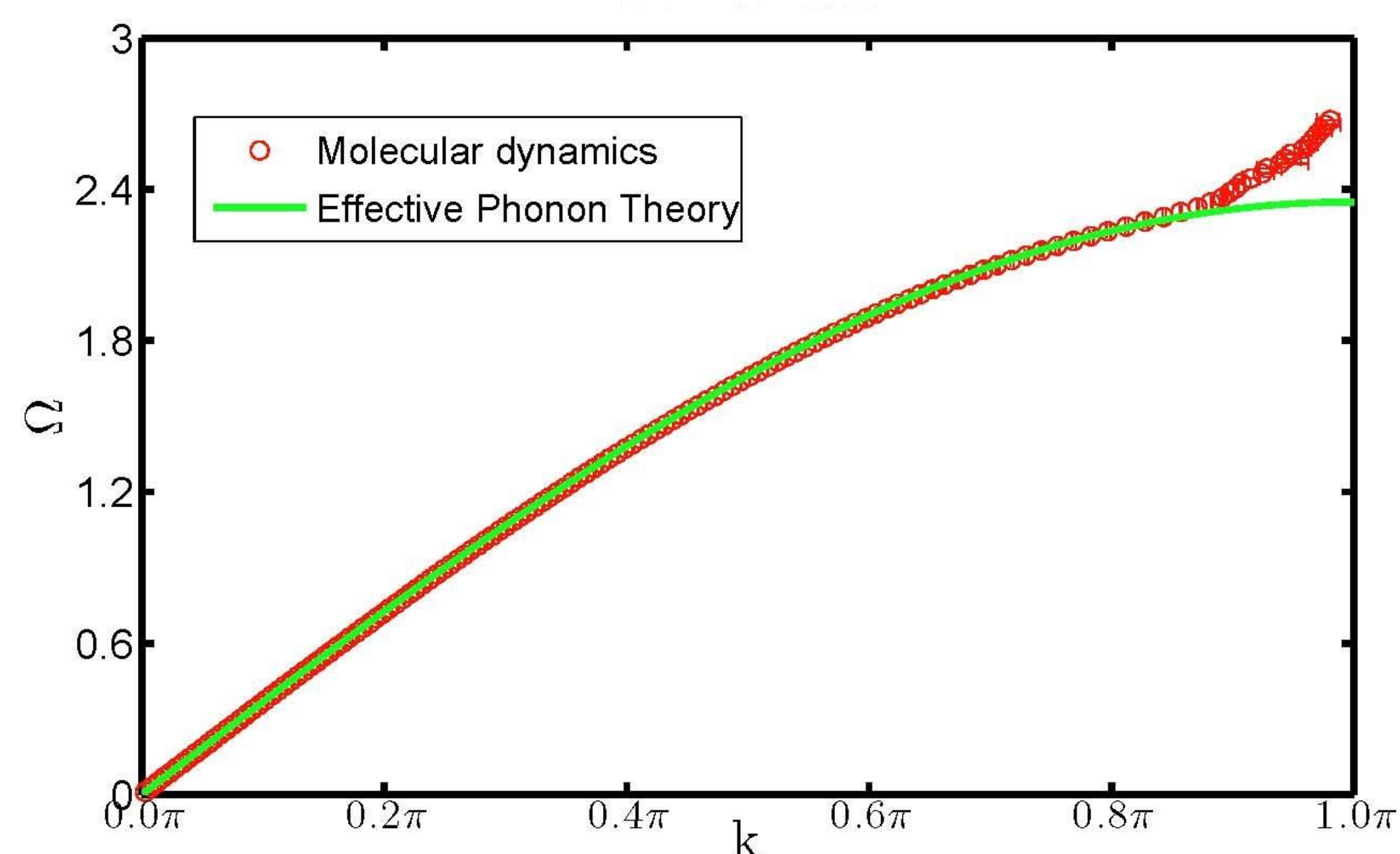
1. Power spectrum(N=64,T=0.2, β=1, M=1)

Wiener-Khinchin Theorem

Effective Phonon Theory:
$\Omega_n = 2v_s\sin\frac{n\pi}{N}$ ($v_s = 1.17$)

3. Mean free path(N=2048,T=0.2, β=1, M=1)

$\sim \Omega^{-1.67}$

Divergent MFP results in anomalous heat conduction!

2. Dispersion(N=2048,T=0.2, β=1, M=1)

o Molecular dynamics
— Effective Phonon Theory

Summary:

(1). Renormalized phonon does exist in FPU-β model.

(2). Anomalous heat conduction is induced by long-wavelength mode with divergent mean free path.

Reference:

[1]. A.Dhar, Adv.Phys. 57, 457(2008)

[2]. N.Li and B.Li, AIP Advances 2, 041408(2012)

[3]. S.Liu et al, unpublished