Analogy between rotation and density for Dirac fermions in a magnetic field

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Abstract

We analyze the energy spectra of Dirac fermions in the presence of rotation and magnetic field. We find that the Landau degeneracy is resolved by rotation. A drastic change in the energy dispersion relation leads to the "rotational magnetic inhibition" that is a novel phenomenon analogous to the inverse magnetic catalysis in a magnetic system at finite chemical potential.

Introduction

One of the most essential and established changes of quark matter driven by strong magnetic fields is the inevitable breaking of chiral symmetry, which is called the magnetic catalysis. The magnetic catalysis could be affected by other controlling parameters even though the magnetic field is strong. For instance, in a dense system at large chemical potential, the magnetic field would not enhance but suppress the chiral condensate, which is called the inverse magnetic catalysis. In this work we will pay attention to the competition between rotation and the magnetic catalysis.

Dirac equation in a rotating frame

- Dirac equation in curved spacetime

$$\left[i\gamma^\mu(D_\mu+\Gamma_\mu)-m\right]\psi=0, \tag{1}$$

where

$$\begin{aligned}\Gamma_\mu &= -\frac{i}{4}\omega_{\mu ij}\sigma^{ij}\,,\\ \omega_{\mu ij} &= g_{\alpha\beta}e_i^\alpha(\partial_\mu e_j^\beta+\Gamma^\beta_{\nu\mu}e_j^\nu)\,,\\ \sigma^{ij} &= \frac{i}{4}[\gamma^i,\gamma^j]\,.\end{aligned} \tag{2}$$

- We choose angular velocity $\boldsymbol{\Omega}=\Omega\hat{z}$, the metric reads

$$g_{\mu\nu}=\begin{pmatrix}1-(x^2+y^2)\Omega^2 & y\Omega & -x\Omega & 0\\ y\Omega & -1 & 0 & 0\\ -x\Omega & 0 & -1 & 0\\ 0 & 0 & 0 & -1\end{pmatrix}. \tag{3}$$

- Vierbein

$$e_0^t=e_1^x=e_2^y=e_3^z=1\,,\quad e_0^x=y\Omega\,,\quad e_0^y=-x\Omega\,, \tag{4}$$

- Symmetry gauge

$$A_i=(0,By/2,-Bx/2,0), \tag{5}$$

- The dispersion relation

$$\left[E+\Omega(\ell+s_z)\right]^2=p_z^2+(2n+1-2s_z)eB+m^2\,. \tag{6}$$

- To justify the quantization and to maintain the causality, condition should be imposed

$$1/\sqrt{eB}\ll R\leq 1/\Omega\,. \tag{7}$$

NJL model with rotation and magnetic field

- NJL modein curved spacetime

$$\begin{aligned}\mathcal{L}=&\bar\psi\left[i\gamma^\mu(D_\mu+\Gamma_\mu)-m_{\rm current}\right]\psi\\ &+\frac{G}{2}\left[(\bar\psi\psi)^2+(\bar\psi\gamma_5\psi)^2\right].\end{aligned} \tag{8}$$

- the effective thermodynamic potential in mean-field approximation

$$\begin{aligned}V_{\rm eff}(m)=&\frac{(m-m_{\rm current})^2}{2G}-\frac{T}{S}\sum_{q=\pm}\int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\sum_{n,\ell,s_z}\\ &\times\left\{\frac{\beta(\varepsilon+q\Omega j)}{2}+\ln\left[1+e^{-\beta(\varepsilon+q\Omega j)}\right]\right\},\end{aligned} \tag{9}$$

where $\varepsilon\equiv\sqrt{p_z^2+(2n+1-2s_z)eB+m^2}$ is the energy dispersion relation without rotation.

- Gap equation

$$\frac{\partial V_{\rm eff}(m)}{\partial m}=0 \tag{10}$$

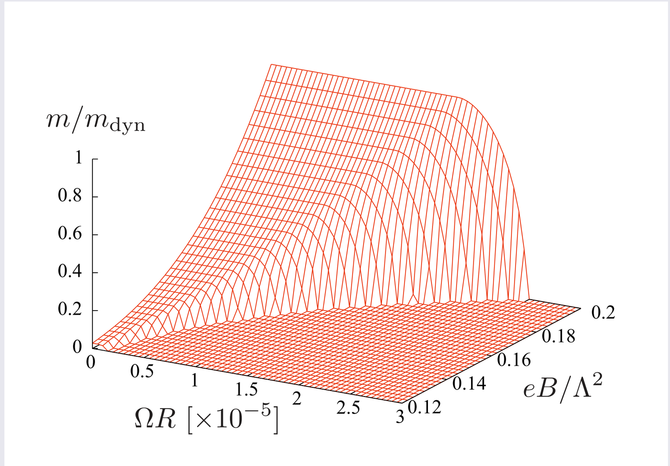
- A smoothened cutoff function

$$f(p_z,n;\Lambda)=\frac{\sinh(\Lambda/\delta\Lambda)}{\cosh[\tilde\varepsilon(p_z,n)\Lambda/\delta\Lambda]+\cosh(\Lambda/\delta\Lambda)} \tag{11}$$

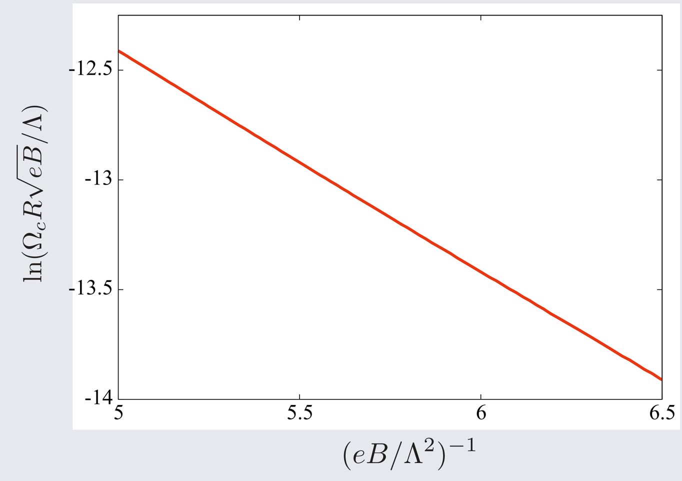
with $\tilde\varepsilon\Lambda\equiv\sqrt{p_z^2+2neB}$.

Dynamical mass at weak coupling ($G<G_c$)

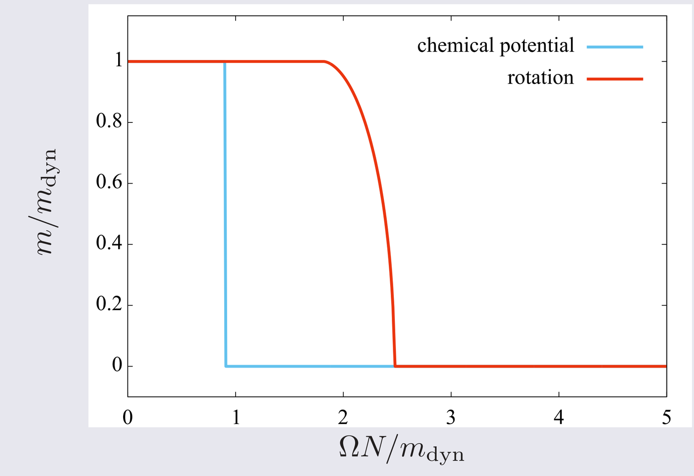
- At weak coupling, increasing eB leads to increasing dynamical mass (i.e. the magnetic catalysis)



- We can observe that there is a threshold for the dynamical mass with increasing Ω, above which $m=0$ and chiral symmetry is restored. This location of the critical Ω_c changes with eB,

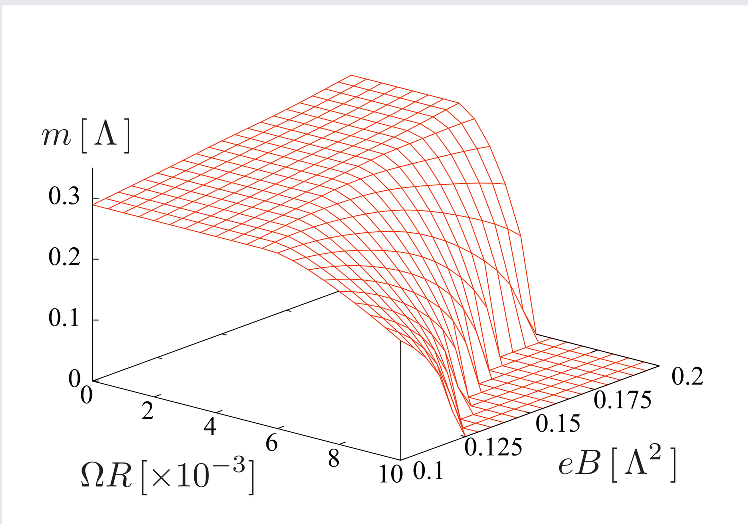


- When the angular velocity exceeds $\Omega\simeq m_{\rm dyn}/N$, the rotational effects become visible, but the damping of the dynamical mass starts slowly, while the mass suppression induced by finite chemical potential happens immediately. Such a difference stems from the ℓ-dependence of each mode.



Dynamical mass at strong coupling ($G>G_c$)

- For small angular velocity, the dynamical mass is almost independent of Ω and eB. With increasing Ω the dynamical mass is eventually suppressed by larger magnetic field, i.e. a counterpart of the finite-density inverse magnetic catalysis is manifested. We call this decreasing behavior of the mass for larger magnetic field the "rotational magnetic inhibition".
- In the weak coupling case only a small number of the Landau levels contribute to the gap equation, while many more Landau levels get involved as the coupling constant becomes larger. This is essential for the realization of the rotational magnetic inhibition as well as of the inverse magnetic catalysis at finite density.
- The QCD vacuum has a rich content with μ and eB, and for $G>G_c$, particularly, the de Haas-van Alphen oscillation may lead to several local minima of the gap equation. However, the figure here shows that this is not the case for the rotational magnetic inhibition.
- The rotational magnetic inhibition should be an observable effect in a table-top experiment.
- The rotational magnetic inhibition must be a sizable effect for the neutron star physics.



Reference and Acknowledgement

[1] H.-L. Chen, K. Fukushima, X.-G. Huang, and K. Mameda, Phys. Rev. D93, 104052 (2016)

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