Gapped boundary theory of the twisted gauge theory model of three-dimensional topological orders

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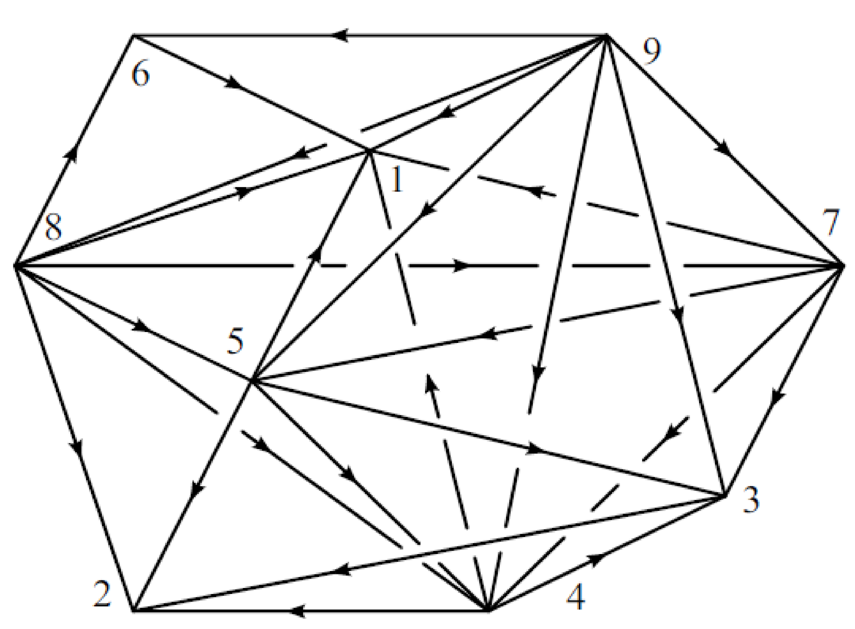
Introduction

The TGT model is an exactly solvable lattice Hamiltonian extension of the (3+1)-dimensional Dijkgraaf-Witten topological gauge theory with general finite gauge groups on a lattice which describes and classifies a large class of three-dimensional topological orders. This model however cannot handle the situation where the 3-manifolds have boundaries, whereas realistic materials — in particular in three dimensions — mostly do have boundaries. We extend the twisted gauge theory model of topological orders in three spatial dimensions to the case where the three spaces have two dimensional boundaries. We achieve this by systematically constructing the boundary Hamiltonians that are compatible with the bulk Hamiltonian.

TGT model on closed 3-manifold



A portion of a graph that represent the basis vectors in the Hilbert space. Each edge carries an arrow and is assigned a group element denoted by [ab] with a < b.

Hibert space: $\mathcal{H}_{\Gamma,G} = \{[ij] \in G | i,j \in V(\Gamma)\}$

Input data:

1. finite group G
2. a normalized 4-cocycle ω in $H^4(G, U(1))$

Hamiltonian:

$$H_{G,\omega} = -\sum_v A_v - \sum_f B_f$$

Output data:
3-d topological order

A_v and B_f are mutual commuting projectors

Ground states:

$A_v|\Phi_0\rangle = |\Phi_0\rangle$: Gauge invariance at v

$B_f|\Phi_0\rangle = |\Phi_0\rangle$: Gauss law, no flux at face f

TGT model with boundaries

Additional input data: $K \subset G$, $\alpha \in C^3(K, U(1))$

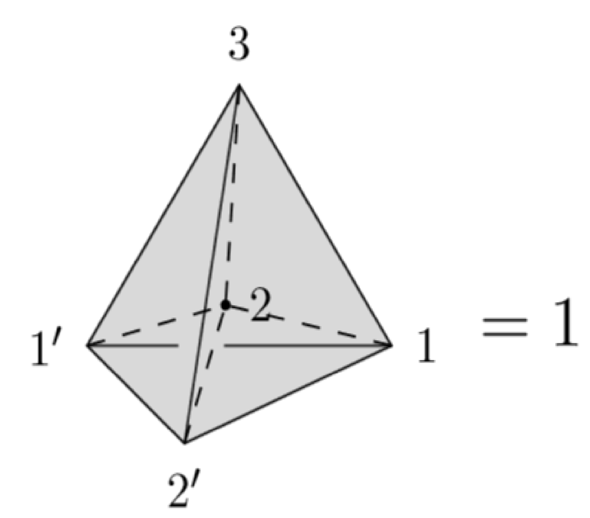
Boundary Hamiltonian:

$$H^{K,\alpha}_{G,\omega} = H_{G,\omega} - \sum_{i=1}^{M}\left(\sum_{v\in\partial_i\Gamma} A_v^{K_i} + \sum_{f\in\partial_i\Gamma} B_f^{K_i} + \sum_{f\in\partial_i\Gamma} C_l^{K_i}\right)$$

Generalized Frobenius condition

The consistency between bulk and boundary gives out:

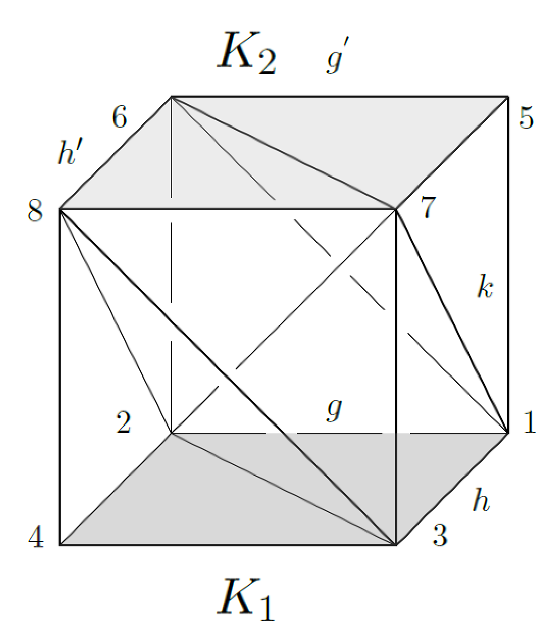
$$\omega(g,h,k,l)d\alpha(g,h,k,l) = 1,\ g,h,k,l \in K$$

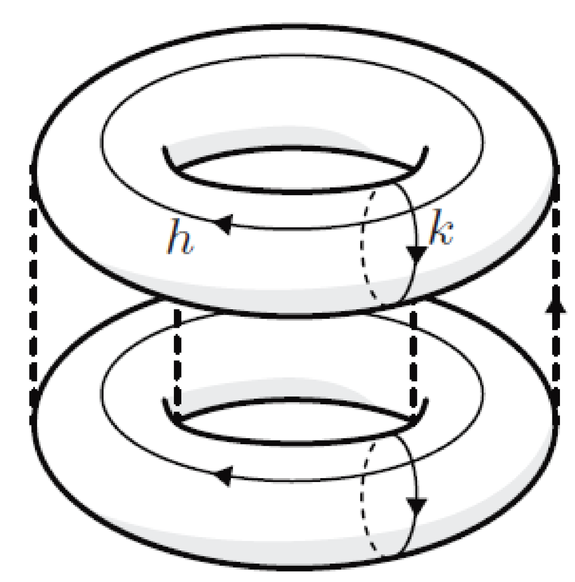


Example:

$G = \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$, $H^4(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2, U(1)) = \mathbb{Z}_2^8$	K	
	$\{1\}$	$\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2$
$\omega^0 \equiv 1$	$\alpha = 1$	$\alpha = 1$
$\omega^{II\ 1st} = -1\ or\ 1$	$\alpha = 1$	×

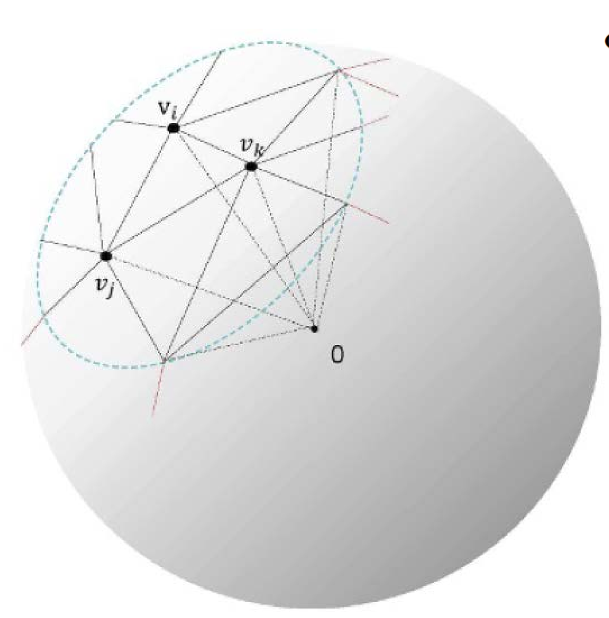
Ground states degeneracy on 3-cylinder





$$GSD_{3-cyl} = \sum_{\substack{x,h,g\in K_1 \\ y,h',g'\in K_2}} \sum_{k\in G} \frac{\delta_{g',\bar{k}gk}\delta_{h',\bar{k}hk}\delta_{g,xg\bar{x}}\delta_{h,xh\bar{x}}\delta_{k,xk\bar{y}}}{|K_1||K_2|} \frac{[\bar{x},k]_{h\bar{g},g}}{[k,\bar{y}]_{h\bar{g},g}} \frac{\eta^g(h\bar{g},x)}{\eta^{\bar{k}gk}(\bar{k}h\bar{g}k,y)}$$

Ground-state wavefunctions on a 3-ball



$$\Phi_0(\Gamma) = \sum_{\substack{x=[0'0]\in G \\ xa_i\in K}} \prod_{\{v_iv_jv_k|2-\text{simplex}\}} [x, a_i, \bar{a}_ia_j, \bar{a}_ja_k]^{-\epsilon(0'0v_iv_jv_k)} \times [xa_i, \bar{a}_ia_j, \bar{a}_ja_k]^{-\epsilon(0'v_iv_jv_k)}$$

When $K = G$ and 4-cocycle is trivial

$$\prod_{a_i,a_j,a_k}[a_i, \bar{a}_ia_j, \bar{a}_ja_k]^{\epsilon(0v_iv_jv_k)}$$

Summary

1. We systematically construct the boundary Hamiltonians of three dimensional TGT with boundary
2. The consistency between the bulk and boundary Hamiltonians is dictated by generalized Frobenius condition
3. We obtain ground states degeneracy on 3-cylinder and ground states wavefunctions on a 3-ball from solely the imput microscopic degrees of freedom

References

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