Anyon Exclusion Statistics

On surface with gapped boundary

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abstract

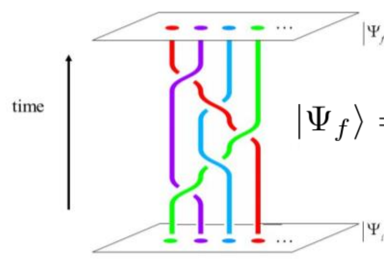
An anyonic exclusion statistics, which generalizes the Bose-Einstein and Fermi-Dirac statistics of bosons and fermions, was proposed by Haldane [1]. When fusion of anyons is involved, certain `pseudo-species' anyons appear in the exotic statistical weights of non-Abelian anyon systems [2]; however, the meaning and signicance of pseudo-species remains an open problem. The relevant past studies had considered only anyon systems without any physical boundary but boundaries often appear in real-life materials.

In this paper, we propose an extended anyonic exclusion statistics on surfaces with gapped boundaries, introducing mutual exclusion statistics between anyons as well as the boundary components. Motivated by refs. [3, 4], we present a formula for the statistical weight of many-anyon states obeying the proposed statistics. Taking the (doubled) Fibonacci topological order as an example, we develop a systematic basis construction for non-Abelian anyons on any Riemann surfaces with gapped boundaries. The basis construction offers a standard way to read off a canonical set of statistics parameters and hence write down the extended statistical weight of the anyon system being studied. The basis construction reveals the meaning of pseudo-species. A pseudo-species has dierent `excitation' modes, each corresponding to an anyon species. The `excitation' modes of pseudo-species corresponds to good quantum numbers of subsystems of a non-Abelian anyon system. This is important because often (e.g., in topological quantum computing) we may be concerned about only the entanglement between such subsystems.

Motivation

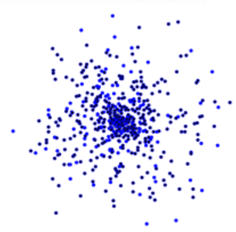
Topological quantum computing

Non-Abelian anyons



$$|\Psi_f\rangle = U|\Psi_i\rangle$$

Thermodynamics of many-body anyon system



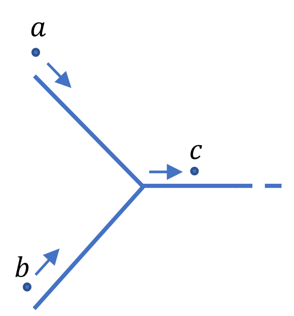
Exclusion statistics!

A direct generalization of Pauli exclusion principle proposed by Haldane and Wu reads

$$W_{\{G_i,N_i\}} = \prod_i \binom{G_i - \alpha_{ij}(N_j - \delta_{ij}) + (N_i - 1)}{N_i}.$$

We go back to boson and fermion when $\alpha_{ij} = 0$ and δ_{ij}, respectively.

Anyon fusion



Anyons can fuse. The rule of fusion can be written as

$$a \times b = \sum_c N_{ab}^c c.$$

There always exists a trivial anyon, s.t.

$$1 \times a = a.$$

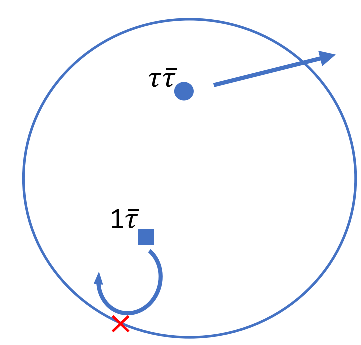
Fibonacci system on surface with gapped boundary

A specific example: the doubled Fibonacci system, with four anyon types: $\{1, \tau 1, 1\bar{\tau}, \tau\bar{\tau}\}$. The fusion rule reads:

$$\tau \times \tau = 1 + \tau,$$

which applies to τ and $\bar{\tau}$ respectively.

In the presence of gapped boundary, anyon 1 and $\tau\bar{\tau}$ can freely move through the boundary, while $\tau 1$ and $1\bar{\tau}$ are confined in the bulk.



Pseudo-species generalization

$$W_{G_0,N_0} = \binom{P}{N_0} \sum_{N_k} \binom{\tilde{G}_k - \sum_j \tilde{\alpha}_{kj}(N_j - \delta_{kj}) + (N_k - 1)}{N_k}$$

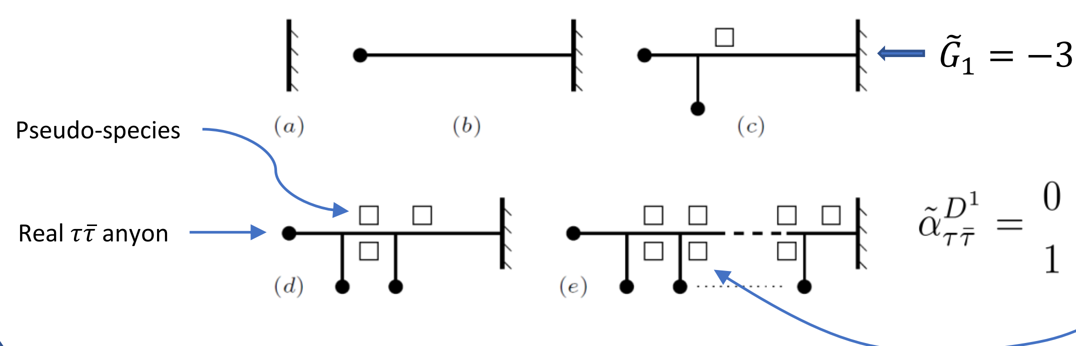
Real $\tau\bar{\tau}$ anyon; Number of Single particle states; Matrix of mutual statistical interaction; Pseudo-species

State counting of Fibonacci anyon on a disk

Table 1. State counting of $\tau\bar{\tau}$ on a disk.

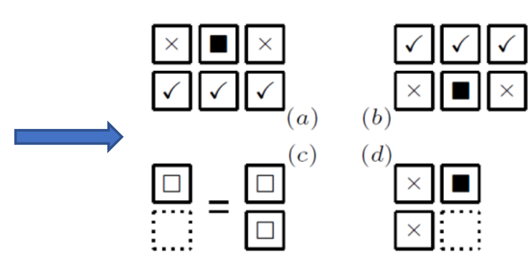
$N_{\tau\bar{\tau}}$	0	1	2	3	4	5	$\cdots$
$w_{P,N,C}$	1	1	2	5	13	34	$\cdots$

Fusion basis of Fibonacci anyon

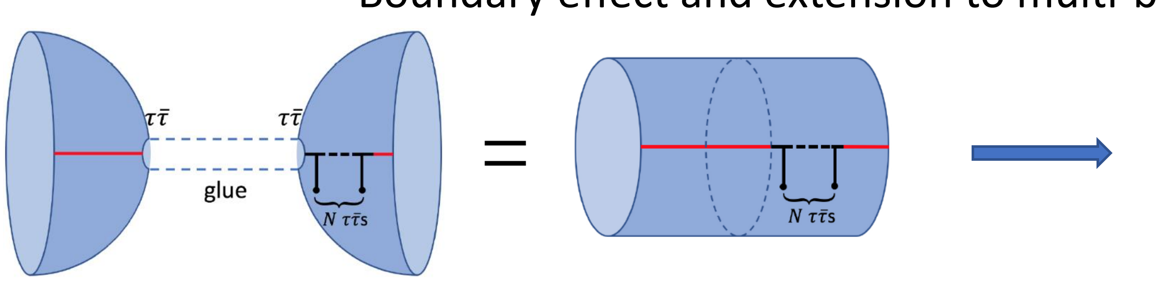


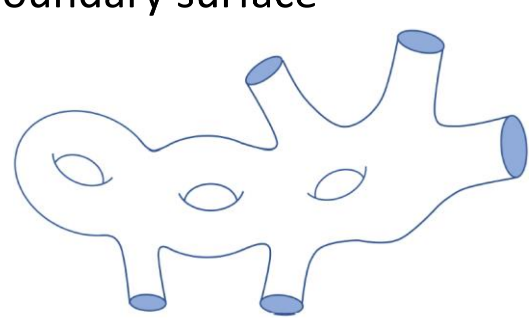
$$\tilde{\alpha}^{D^1}_{\tau\bar{\tau}} = \begin{matrix} & 0 & 1 \\ 0 & 1 & 0 \\ 1 & -2 & 2 \end{matrix}$$

Generalized Pauli exclusion principle



Boundary effect and extension to multi-boundary surface





- Gapped boundaries and bulk anyons are treated on an equal footing!

Reference:
[1] F.D.M. Haldane, Phys. Rev. Lett. 67 (1991) 937
[2] S. Guruswamy and K. Schoutens, Nucl. Phys. B 556 (1999) 530 [cond-mat/9903045]
[3] Y.-S. Wu, Phys. Rev. Lett. 73 (1994) 922
[4] Y. Hu, S.D. Stirling and Y.-S. Wu, Phys. Rev. B 89 (2014) 115133 [arXiv:1303.1586]