Metathermotics: Nonlinear thermal responses of core-shell metamaterials

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I. Abstract

Thermal metamaterials based on core-shell structures have aroused wide research interest, e.g., in thermal cloaks. However, almost all the relevant studies only discuss linear materials whose thermal conductivities are temperature-independent constants. Nonlinear materials (whose thermal conductivities depend on temperatures) have seldom been touched; however, they are important in practical applications. Here we study **the nonlinear responses of thermal metamaterials with a core-shell structure in two or three dimensions.** By calculating the effective thermal conductivity, we derive the nonlinear modulation of a nonlinear core. **Furthermore, we reveal two thermal coupling conditions, under which this nonlinear modulation can be efficiently manipulated.** In particular, we reveal the phenomenon of nonlinearity enhancement. Then this theory helps us to design a kind of intelligent thermal transparency devices, which can respond to the direction of thermal fields.

II. Method

We first consider the two dimensional case: The core with radius r1 has a temperature-dependent (i.e., nonlinear) thermal conductivity given by

$$\kappa_c(T) = \kappa_c^{(0)} + \chi_c T^\alpha$$

Then, the effective thermal conductivity of the core-shell structure reaches

$$\kappa_{e1}(T) = u\kappa_{rr}\frac{\kappa_c(T) + u\kappa_{rr} + [\kappa_c(T) - u\kappa_{rr}]p_1^u}{\kappa_c(T) + u\kappa_{rr} - [\kappa_c(T) - u\kappa_{rr}]p_1^u},$$

and that of the core-shell structure plus the matrix turns to be

$$\kappa_{e2}(T) = \kappa_m\frac{\kappa_{e1}(T) + \kappa_m + [\kappa_{e1}(T) - \kappa_m]p_2}{\kappa_{e1}(T) + \kappa_m - [\kappa_{e1}(T) - \kappa_m]p_2},$$

where $p_1 = r_1^2/r_2^2$, $p_2 = \pi r_2^2/a^2$, and $u = \sqrt{\kappa_{\theta\theta}/\kappa_{rr}}$.

we define the nonlinear modulation $\eta = \chi_e/\chi_c$, which is given by

$$\eta = \frac{16u^2\kappa_{rr}^2\kappa_m^2 p_2 p_1^u}{\left\{u\kappa_{rr}(p_2-1)\left[\kappa_c^{(0)} + u\kappa_{rr} + \left(\kappa_c^{(0)} - u\kappa_{rr}\right)p_1{}^u\right] + \kappa_m(p_2+1)\left[\kappa_c^{(0)} + u\kappa_{rr} - \left(\kappa_c^{(0)} - u\kappa_{rr}\right)p_1{}^u\right]\right\}^2}.$$

When the core-shell structure satisfies

$$\kappa_c^{(0)} + u\kappa_{rr} = 0,$$
$$\kappa_m = \kappa_c^{(0)}$$

the nonlinear modulation is simplified as

$$\eta = p_1^{-u}p_2$$

On the other hand, when the core-shell structure satisfies

$$\kappa_c^{(0)} - u\kappa_{rr} = 0,$$
$$\kappa_m = \kappa_c^{(0)}$$

the nonlinear modulation becomes

$$\eta = p_1^u p_2.$$

III. Nonlinear modulation

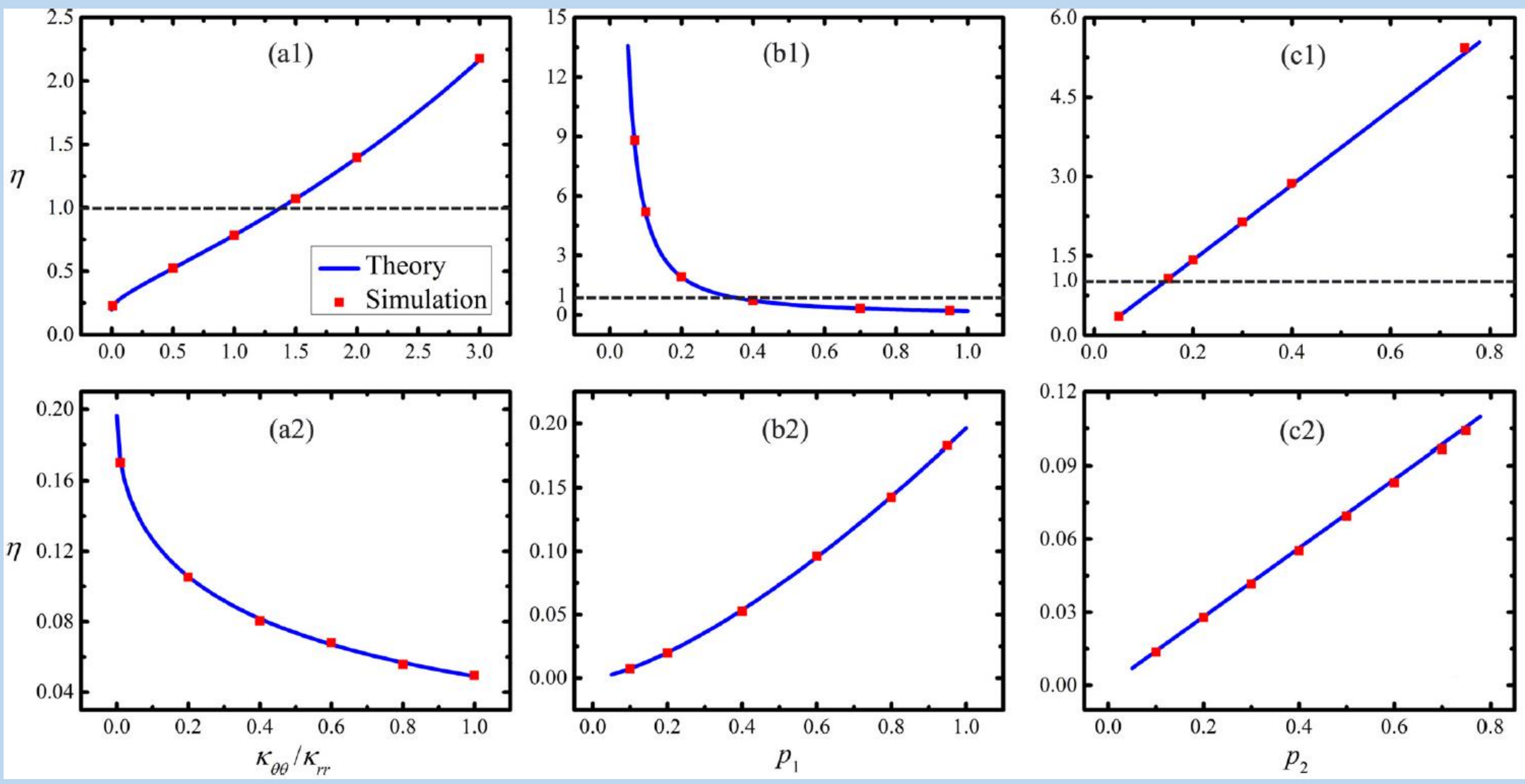

Fig. 1 Two dimensional results.

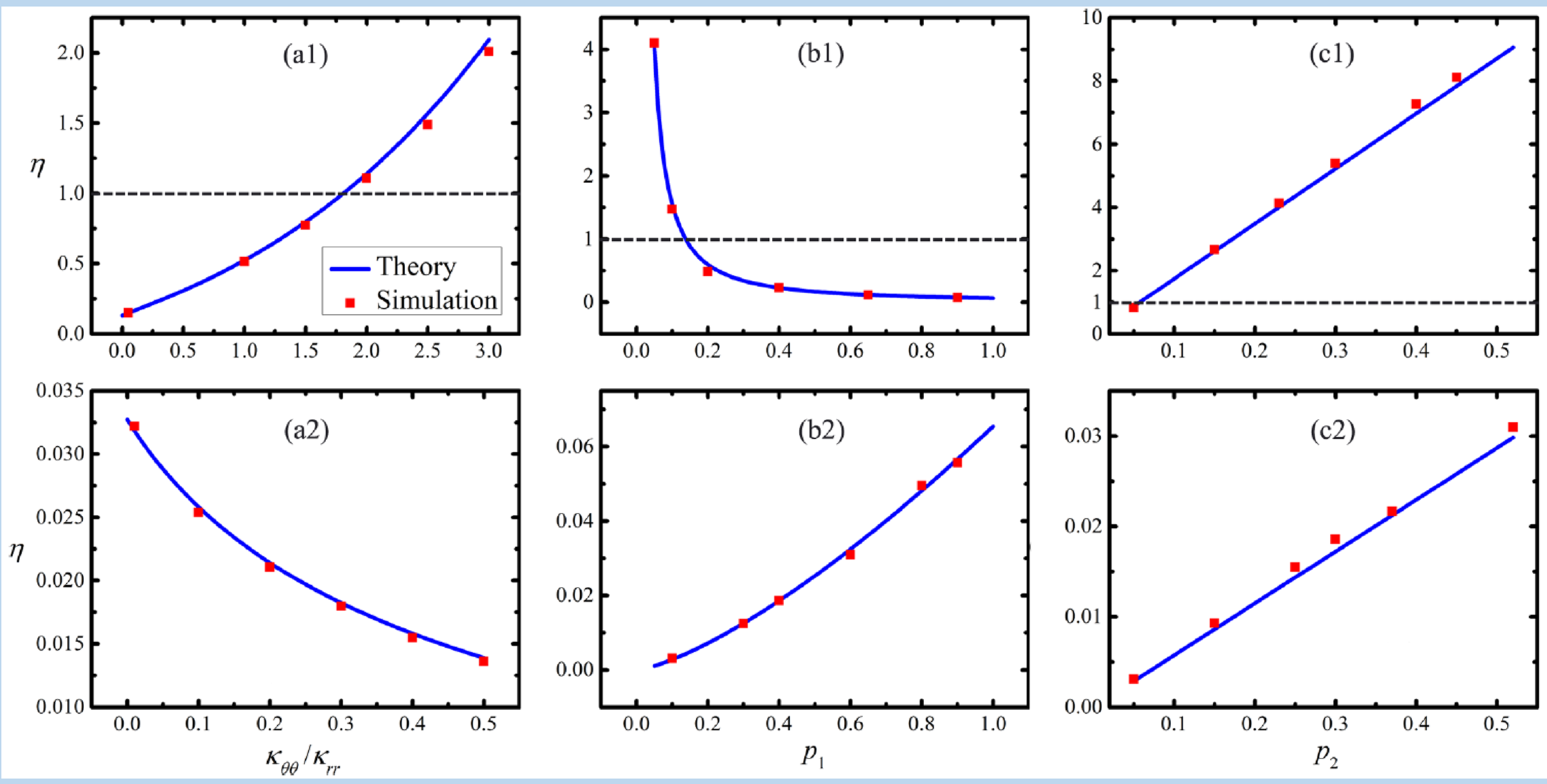

Fig. 2 Three dimensional results.

IV. Application

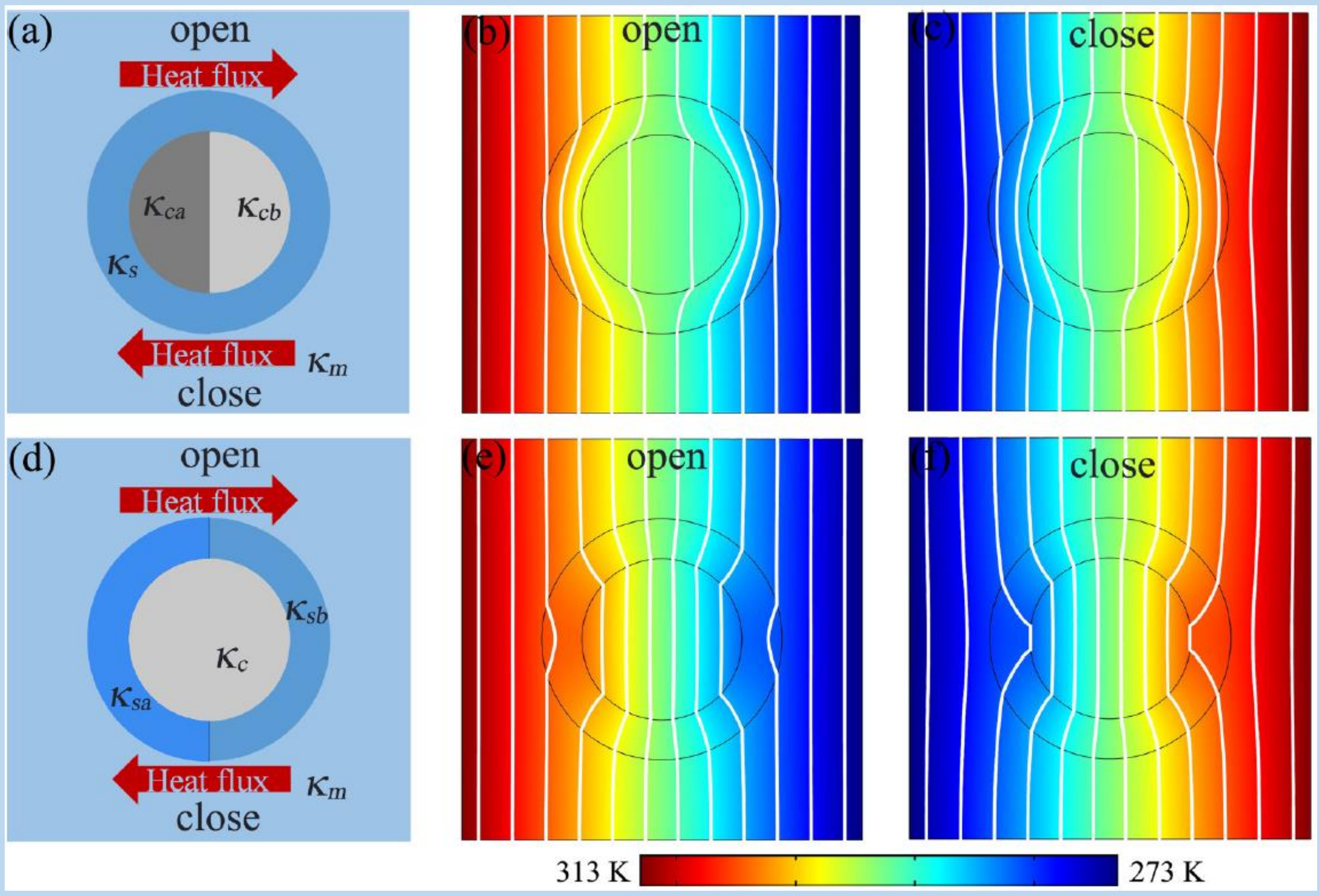

Fig. 3 Switchable thermal transparency device.

V. Conclusion

In this work, we have investigated the nonlinear modulation of a core shell structure embedded in a finite matrix (only the core is nonlinear). **Under two thermal coupling conditions, the nonlinear modulation can be largely simplified, and only depends on three key parameters: the degree of shell anisotropy, the core fraction in the shell, and the core-shell fraction in the matrix. In particular, the nonlinear modulation will be effectively enhanced under the thermal coupling conditions.**

Reference: **Metathermotics: Nonlinear thermal responses of core-shell metamaterials**, S. Yang, L. J. Xu, and J. P. Huang, Physical Review E, volume 99, 042144 (2019).