Macrospin-based Ising Computer

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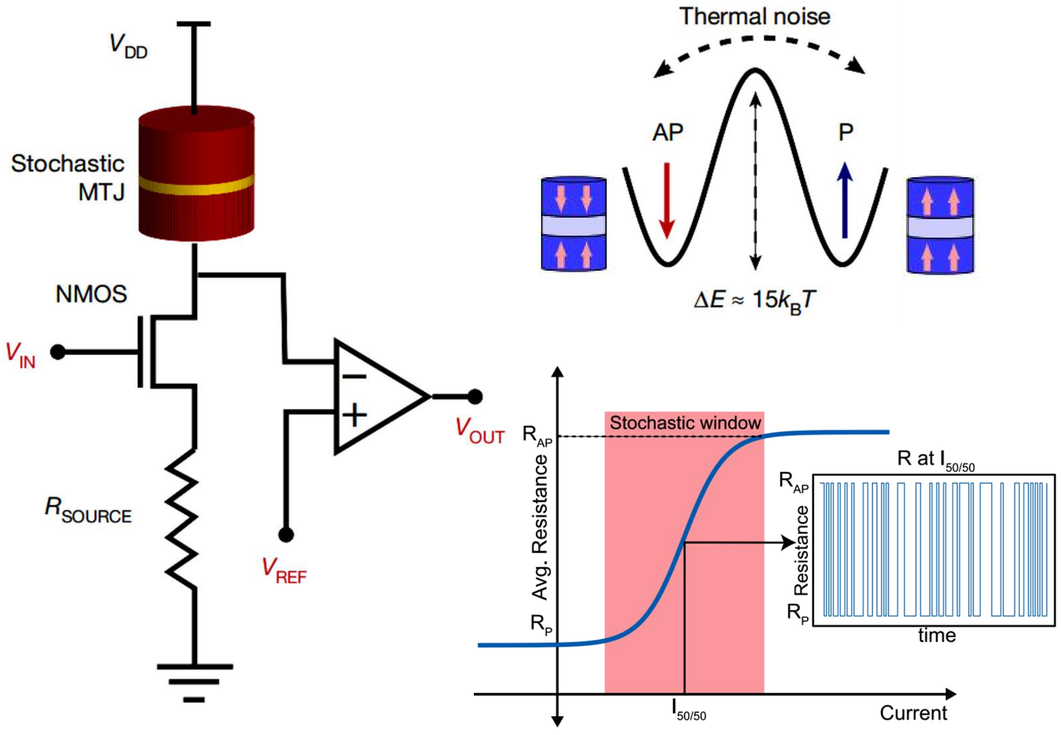
Introduction

Superparamagnetism have been popular in generating true random number and stochastic computation (p-bits)[1][2].



Physical Ising Computer

- Integer factorization $F = XY$ is to minimize the **cost function**

$$E = A\left(\sum_{i=0}^{P} 2^i x_i \sum_{j=0}^{Q} 2^j y_j - F\right)^2 \qquad x_i, y_j \in \{0,1\}$$

Incorporating into micromagnetic dynamics (LLG equation) :

$$\dot{\vec{m}} = -\gamma \vec{m} \times \vec{H}_{\text{eff}} + \alpha \vec{m} \times \dot{\vec{m}} \qquad x, y = \frac{1+m_z}{2} \qquad \vec{H}_{\text{eff}} = -\frac{\partial E}{\partial \vec{m}} + K\vec{m} \cdot \hat{z} + \vec{h}$$

- Thermal noise ***h*** results in Boltzmann distribution. Correct solutions are ground states and emerge most frequently.

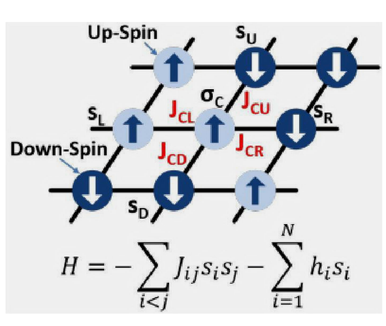
$$\left\langle h_{x,y,z}^2 \right\rangle = \frac{2\alpha k_B T}{\gamma \mu_0 M_s \Delta V \Delta t} \qquad \alpha \sim 0.2$$

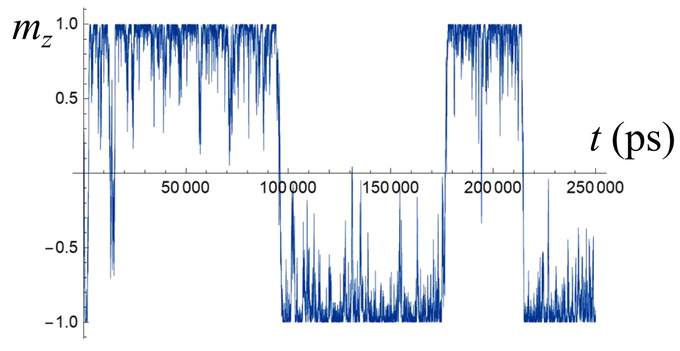
- Effective all-to-all Ising model with ancillary bits[3][4]

$$x_i^2 = x_i, \quad y_i^2 = y_i \qquad A_{ij} \equiv x_i \wedge x_j \qquad B_{ij} \equiv y_i \wedge y_j$$

$$E' = E + \lambda E_{AND} \qquad E_{AND} = \sum_{ij} 3A_{ij}^2 + x_i x_j - 2A_{ij}(x_i + x_j) + [(x, A) \to (y, B)]$$

Numeric simulation is performed by Wolfram Mathematica 12.1.

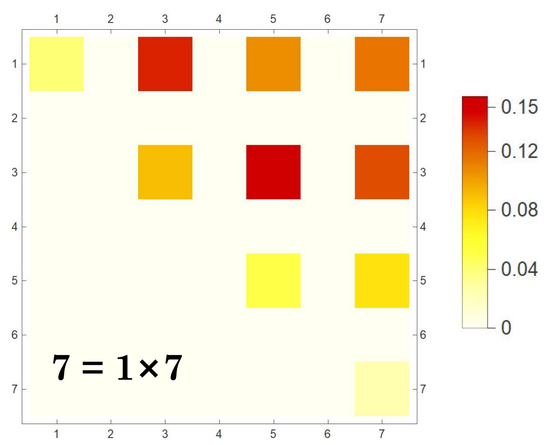




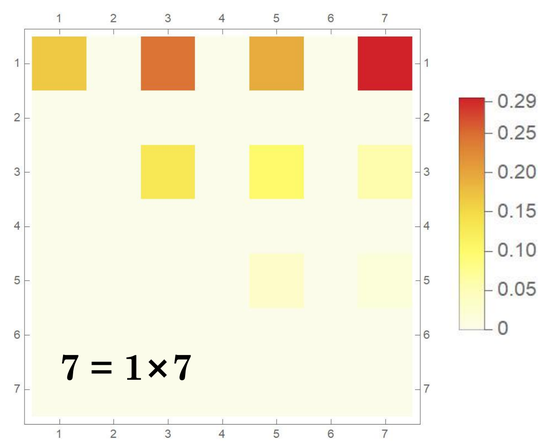
Bit-like random telegraph noise (**RTN**) has been realized, calling for large Gilbert damping and moderate temperature.

Results

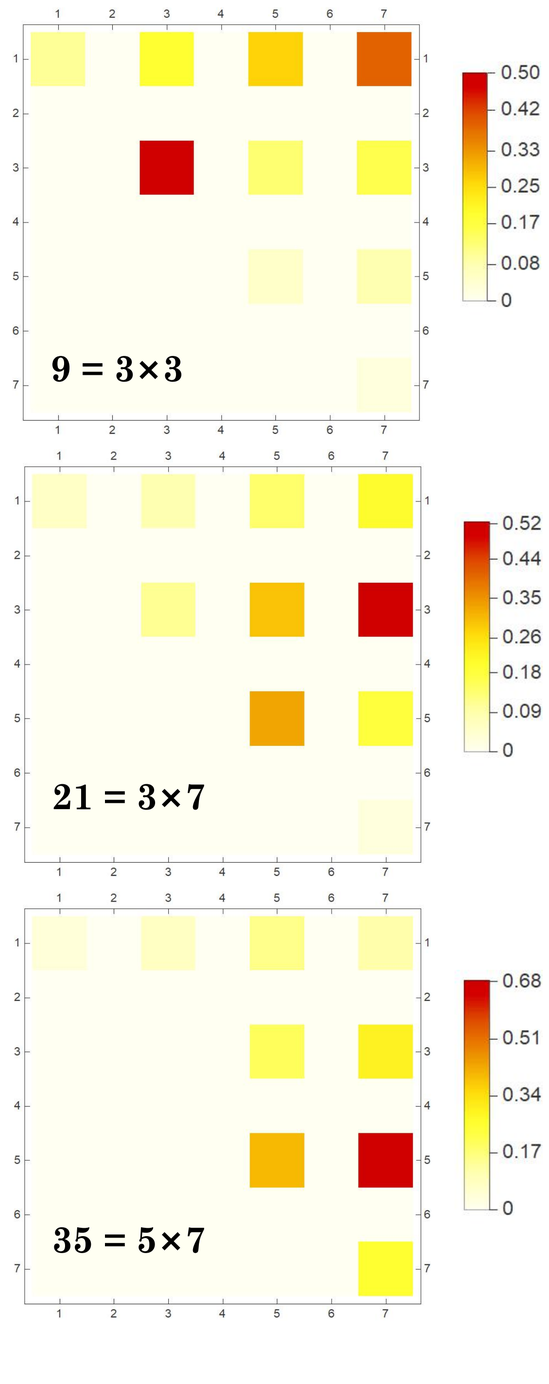
- Constraint for **odd** number factorization : $x_0 = y_0 = 1$
- Simple cases involving **6** p-bits : $x_1, x_2, y_1, y_2, A_{12}, B_{12}$
- **Anneal** to make ground states stand out
- Histogram : probability of different (X, Y) states

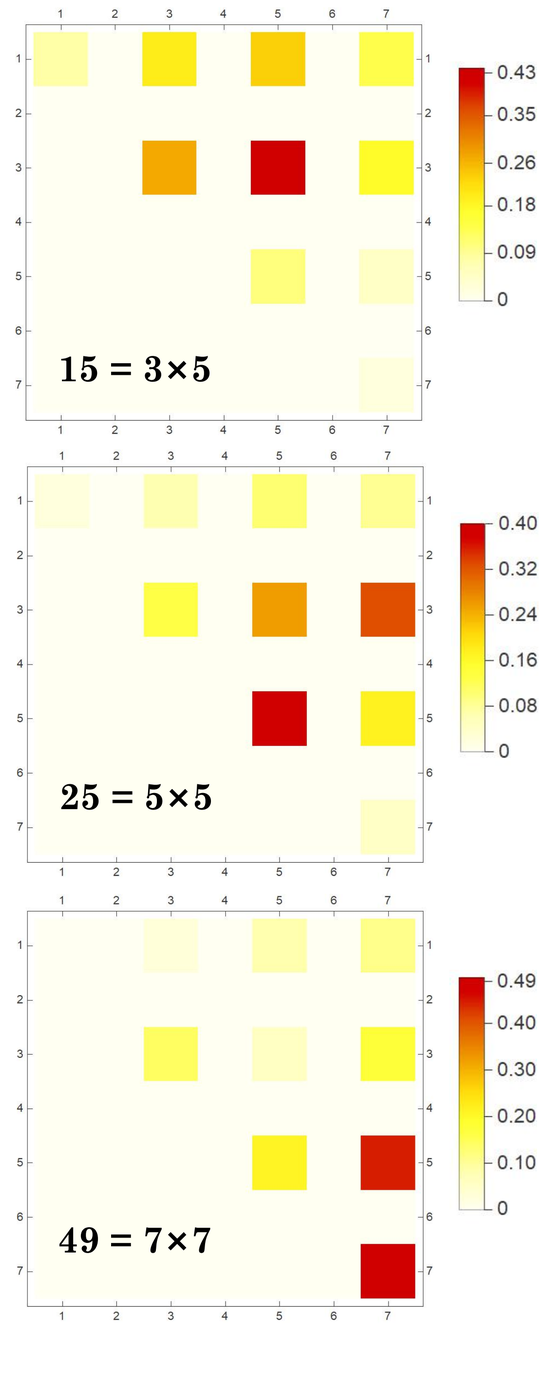


Constant temperature



After annealing





Conslusion and Outlook

Macrospin-based Ising computing has turned out feasible for small problem scales, but remains to be optimized and tested for increasing number of p-bits.

- Better annealing rate to help escape local minima
- Better statistical methods to eliminate intermediate states
- Better design of cost function to prevent high barriers and facilitate travelling in phase space

Reference

[1]W. Borders et al, Nature, 573, 390-393
[2]N. Maciel et al, Sensors (Basel, Switzerland), 2020, 20(1)
[3]Biamonte J. D., Physical Review A, 2008, 77(5): 52331
[4]A. Agrawal et al, IEEE Transaction on Very Large Scale Integration (VLSI) Systems, Vol. 28, No. 12