Breaking efficiency limit of thermal concentrators by conductivity couplings

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Abstract Thermal concentrators[1] have drawn broad attention due to their practical applications for heat collection and storage. Till now, many schemes [2,3] have been proposed to design thermal concentrators. Nevertheless, there is an upper limit for the concentrating efficiency of existing thermal concentrators, commonly defined as the ratio between the temperature gradient in the core and that in the background. Here, we manage to break the upper limit by considering coupling conditions of thermal conductivities. For this purpose, we investigate a monolayer scheme and two extended schemes under coupling conditions of thermal conductivities. Moreover, finite-element simulations are performed to confirm theoretical predictions, and experimental suggestions are provided to ensure feasibility.

Theory.

We firstly discuss a two-dimensional circular structure [see Fig. 1(c)]. A thermal-conduction process can be written with the cylindrical coordinates (r, θ) as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\kappa_{srr}\frac{\partial T}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(\kappa_{s\theta\theta}\frac{\partial T}{r\partial \theta}\right)=0.$$

By solving this equation and boundary conditions, we then define concentrating efficiency (the ratio of its interior and exterior temperature gradients),

$$\eta = f^{-(1-\kappa_c/\kappa_{srr})/2} = (r_s/r_c)^{1-\kappa_c/\kappa_{srr}},$$

under coupling relation,$\kappa_{srr} = \kappa_b$ and $\kappa_{srr}\kappa_{s\theta\theta} = \kappa_c^2$.

Obviously, the minimum value $\eta \longrightarrow 0$ appears when $\kappa_{srr}/\kappa_c \longrightarrow 0^+$ and the maximum value $\eta \longrightarrow \infty$ appears when $\kappa_{srr}/\kappa_c \longrightarrow 0^-$. Moreover, we can observe $\eta \longrightarrow r_s/r_c$ when $\kappa_{srr}/\kappa_c \longrightarrow \pm\infty$, which is just the upper limit of existing concentrating efficiency. [see the solid line in Fig. 1(d)]

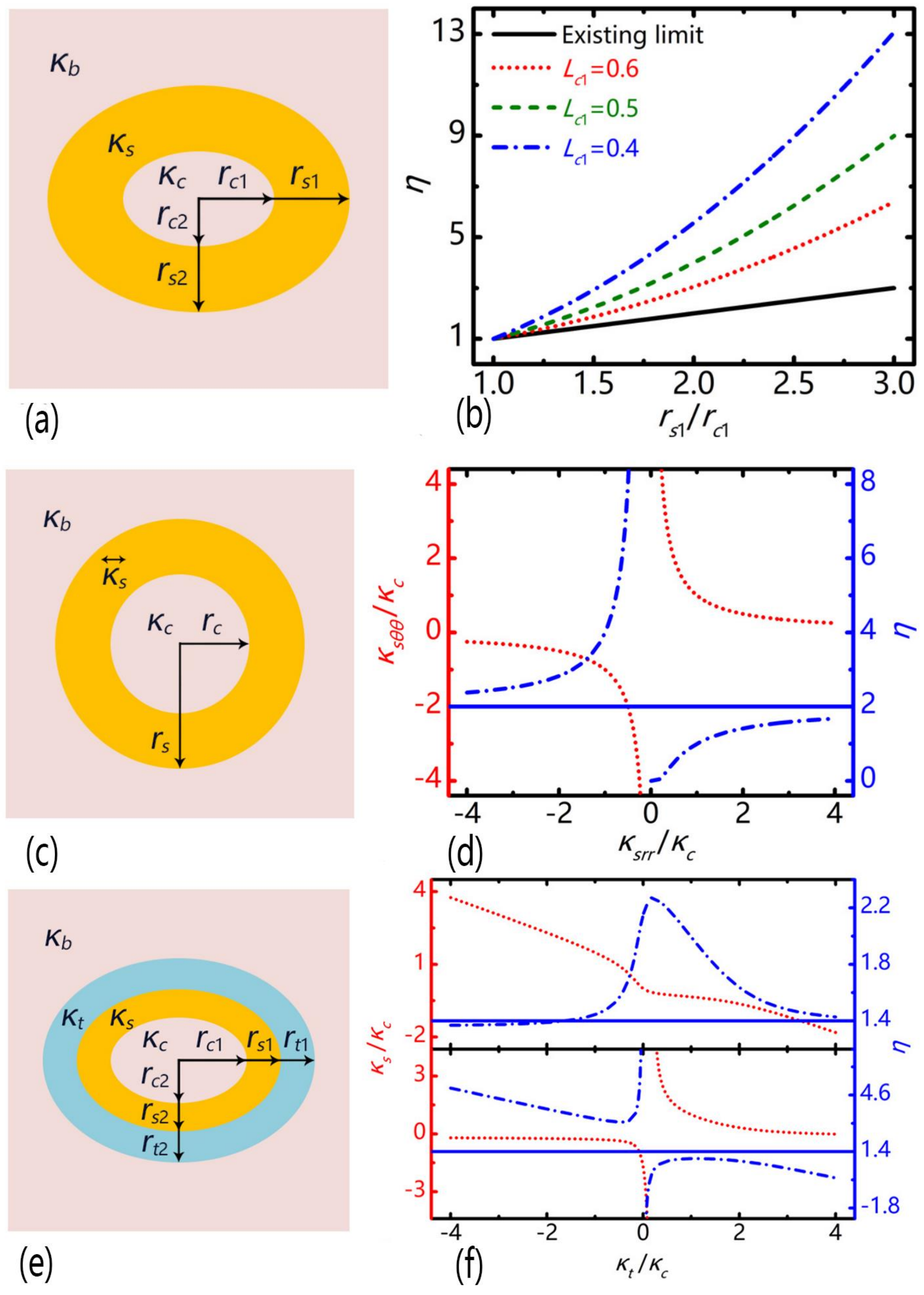

FIG. 1. (a) Monolayer scheme with an isotropic thermal conductivity. (b) Concentrating efficiency η as a function of the geometric configuration r_{s1}/r_{c1}. (c) Monolayer scheme with an anisotropic thermal conductivities. (d) $\kappa_{s\theta\theta}/\kappa_c$ and η as a function of κ_{srr}/κ_c with $r_s/r_c = 2$. (e) Bilayer scheme with isotropic thermal conductivities. (f) κ_s/κ_c and η as a function of κ_t/κ_c with $r_{s1}/r_{c1} = 1.2$, $r_{t1}/r_{c1} = 1.4$, and $L_{c1} = 1/3$.

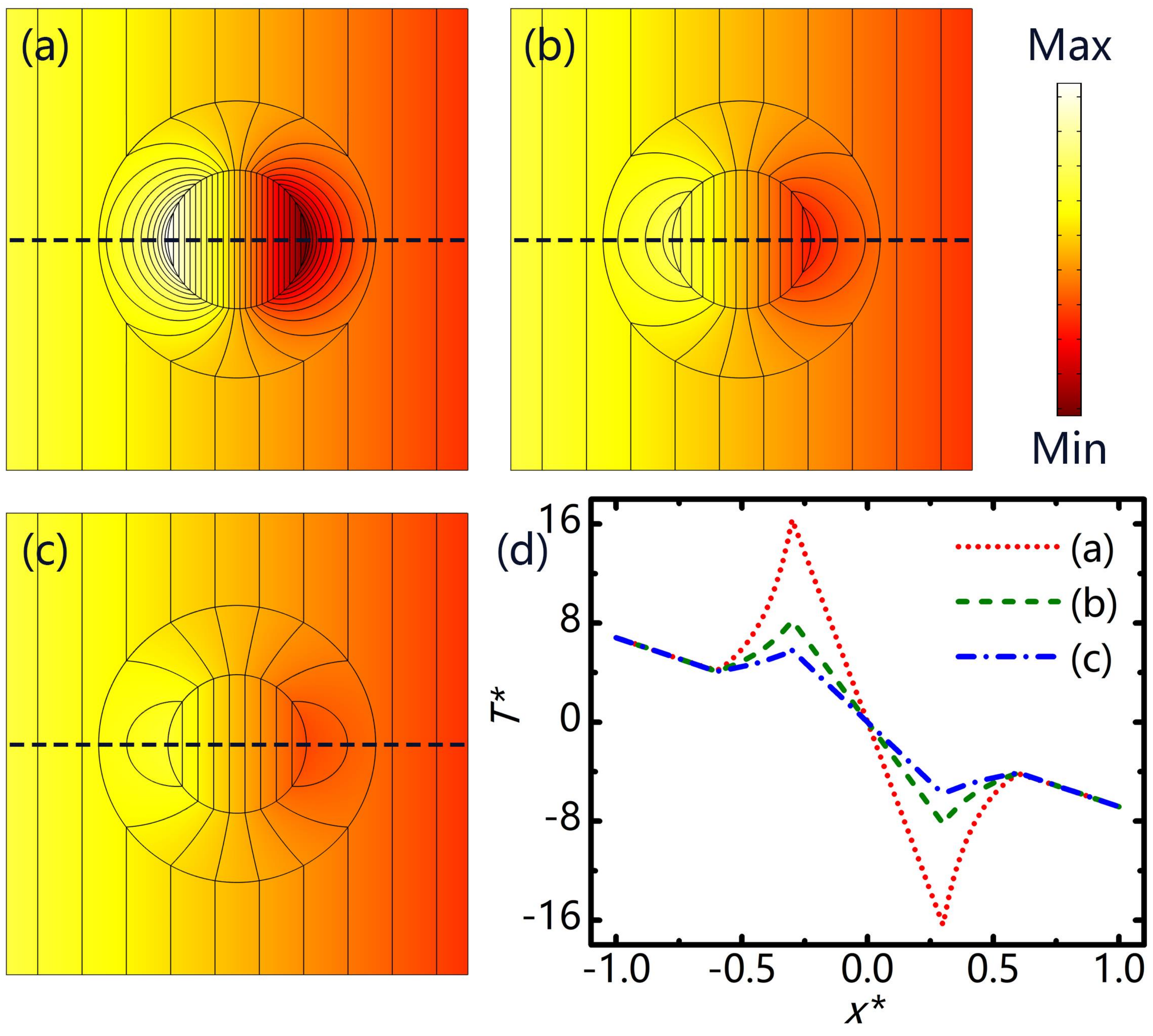

FIG. 2. (a-c) Simulations of the monolayer scheme with an anisotropic thermal conductivity and (d) T^*as a function of x^*.

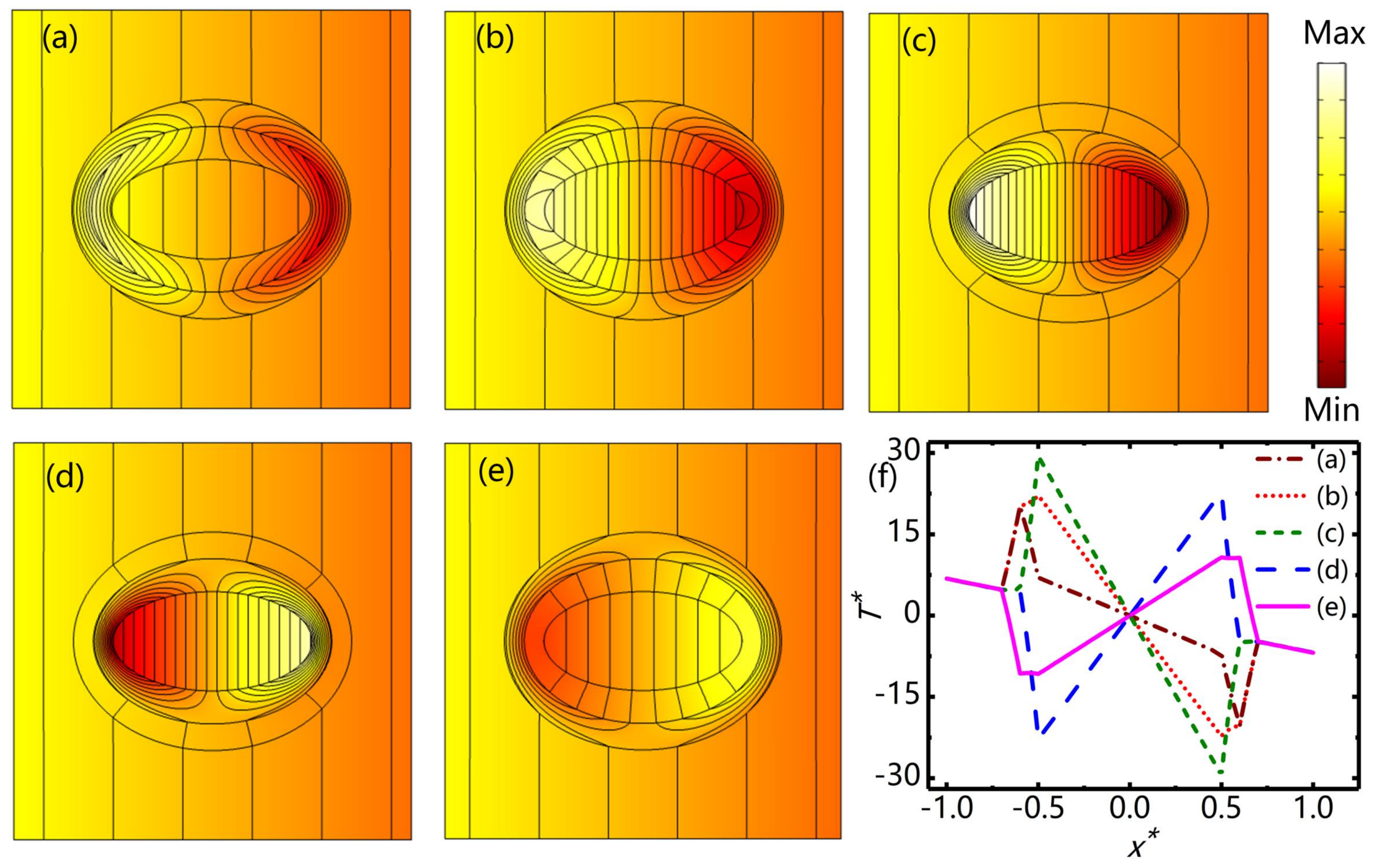

FIG. 3. (a-e) Simulations of the bilayer scheme with isotropic thermal conductivities and (f) T^*as a function of x^*.

Conclusion.

- **We break the efficiency limit of existing thermal concentrators.**
- **A systematic theory is proposed for conductivity couplings.**
- **Three typical schemes are proposed with different features.**

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