3+1D Transformation Thermotics

J. R. Liu, L. J. Xu and J. P. Huang
Department of Physics , Fudan University, Shanghai 200438, China

Abstract

Resent researches consider the continuity equation[1] and time modulation[2] in thermal convection. Here we establish the Transformation Thermotics into 3+1D one and create a new way to gradually change the thermal aims without disturbing the environment.

Theory

$$\vec{\nabla}\cdot\left(-\kappa\vec{\nabla}\mathrm{T}\right)+\rho c\vec{v}\cdot\vec{\nabla}T+\rho c\partial_t T=0$$

$$\boldsymbol{\nabla}\cdot(-\boldsymbol{\kappa}\boldsymbol{\nabla}T)+\rho c\boldsymbol{v}\cdot\boldsymbol{\nabla}T=0$$

$$\text{where } \boldsymbol{\nabla}=\left(\partial_t,\vec{\nabla}\right),\boldsymbol{\kappa}=\mathrm{diag}(0,\kappa),\boldsymbol{v}=(1,\vec{v})$$

$$\boldsymbol{\kappa}'=\frac{\boldsymbol{J}\boldsymbol{\kappa}\boldsymbol{J}^T}{\det\boldsymbol{J}},\rho c\boldsymbol{v}=\frac{\boldsymbol{J}\rho c\boldsymbol{v}}{\det\boldsymbol{J}}$$

Inspired by special relativity and continuity equation, we can change the equation of heat transfer with convection into Galilean invariant 4-form. The 4-form is similar to traditional 3D transformation thermotics and follows the same rules under Galilean transformation. $\boldsymbol{J}=\mathrm{diag}\left(1,\frac{\partial \boldsymbol{r}'}{\partial \boldsymbol{r}}\right)=\mathrm{diag}(1,J)$.

Time-modulated Thermal Concentration

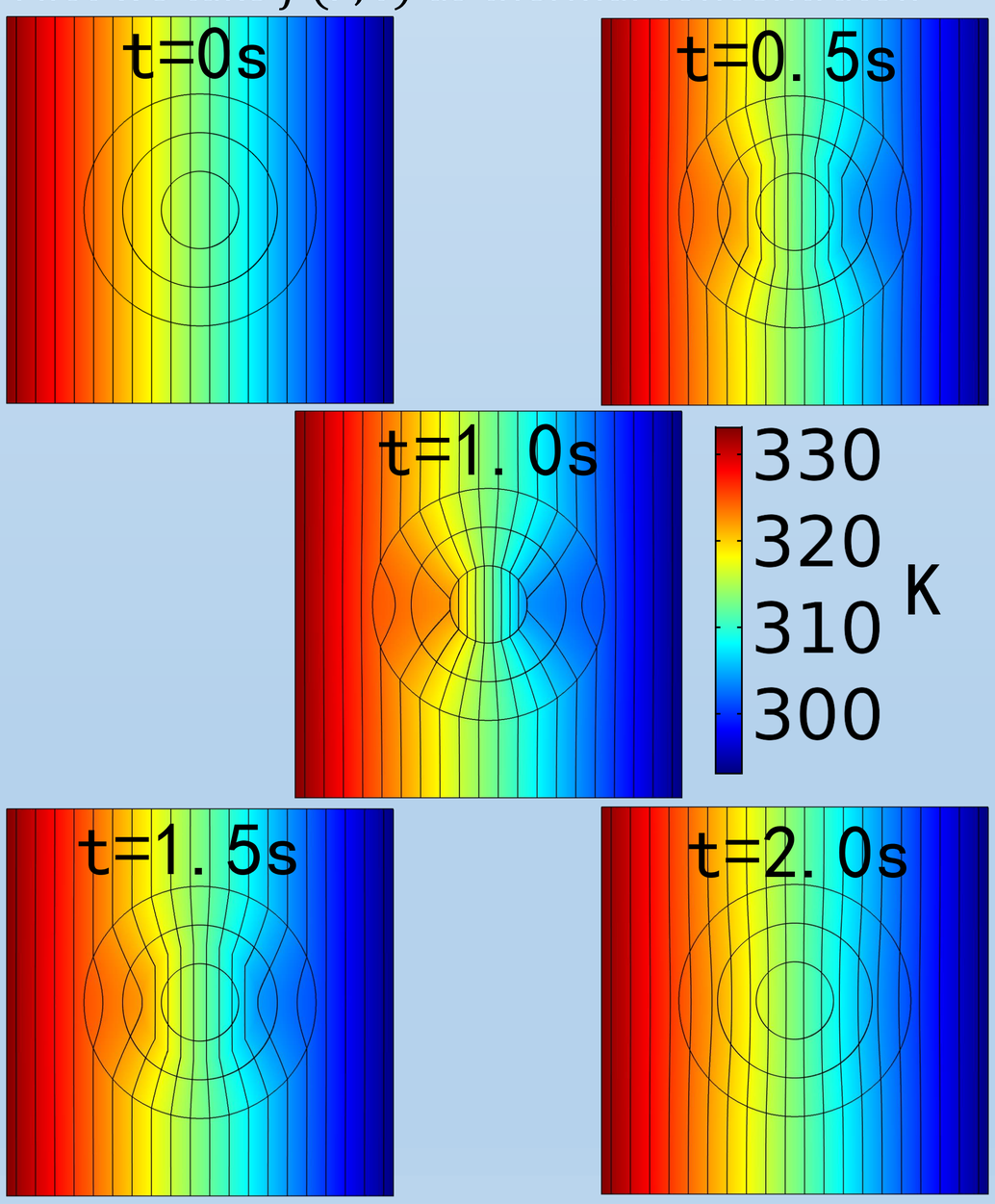
We choose special Galilean transformation that

$$r'=(1-t)r+tf(r,t),t\in[0,1]$$

$$r'=(t-1)r+(2-t)f(r,t),t\in[1,2]$$

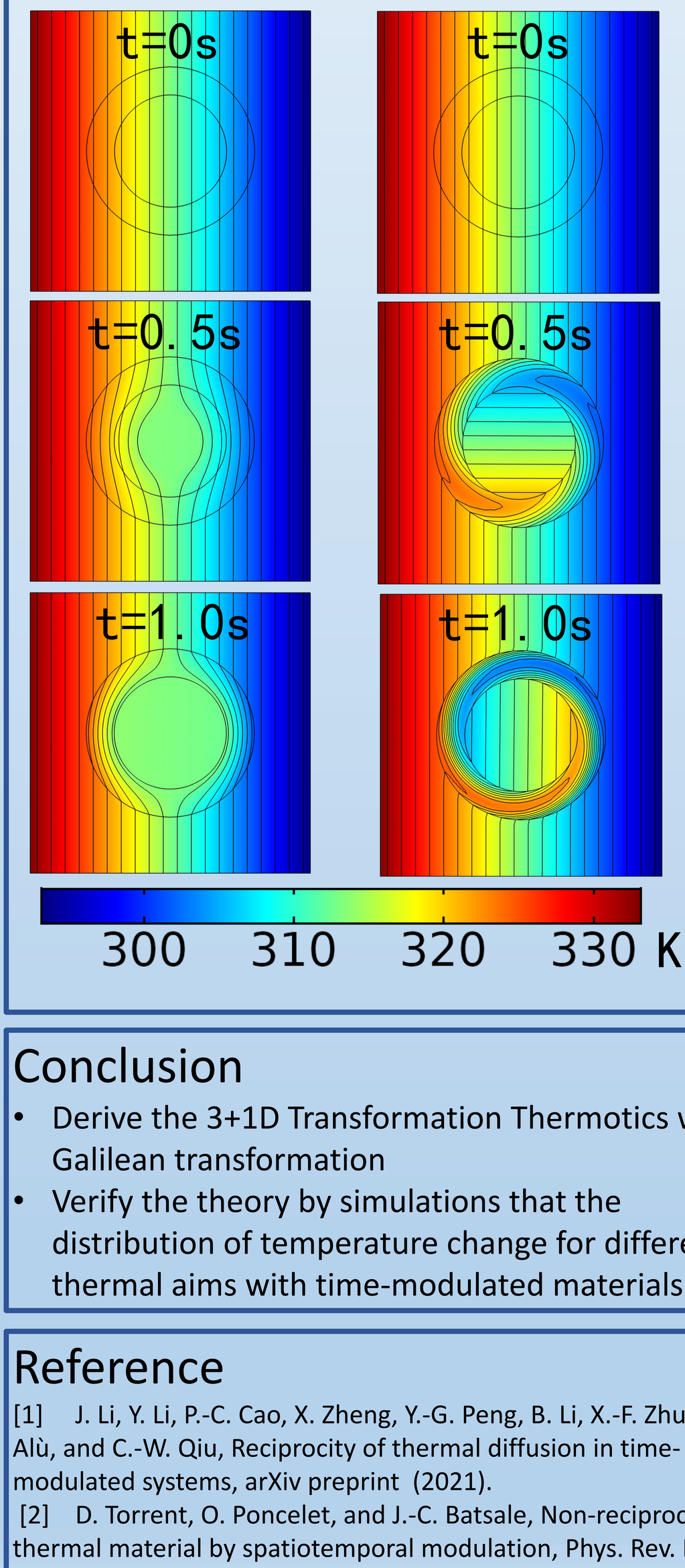
so that the distributions gradually change from original one to our aim $T'(r')$ and return.

First we take $f(r,t)$ as thermal concentrator.



Time-modulated Thermal Cloak and Rotator

Similarly, we take $f(r,t)$ as thermal cloak and rotator. The distribution of temperature transforms under our control.



Conclusion

- Derive the 3+1D Transformation Thermotics with Galilean transformation
- Verify the theory by simulations that the distribution of temperature change for different thermal aims with time-modulated materials

Reference

[1] J. Li, Y. Li, P.-C. Cao, X. Zheng, Y.-G. Peng, B. Li, X.-F. Zhu, A. Alù, and C.-W. Qiu, Reciprocity of thermal diffusion in time-modulated systems, arXiv preprint (2021).

[2] D. Torrent, O. Poncelet, and J.-C. Batsale, Non-reciprocal thermal material by spatiotemporal modulation, Phys. Rev. Lett. **120**, 125501 (2018).