

## Introduction

According to quantum mechanics, the electron has wave property, so it can be described by wave function. For electron wavefunction, the intensity values at different positions indicate the probability of the electron appearing here, and the phase of it indicates the direction of the electron. We can get more sample information by detecting the phase.

But the detector can only get the intensity information of the electron wave function, we need extra method to get the phase information. The direction of the electron is reflected by the change of the intensity in different planes, so we can restore the phase by the intensity.

## Theory

In 1983 Teague proposed TIE (the transport of intensity equation), The change of intensity and phase in paraxial approximation is revealed.

$$\nabla_{\perp} \cdot [I(r_{\perp}, z) \nabla_{\perp} \phi(r_{\perp}, z)] = -k \partial_z I(r_{\perp}, z)$$

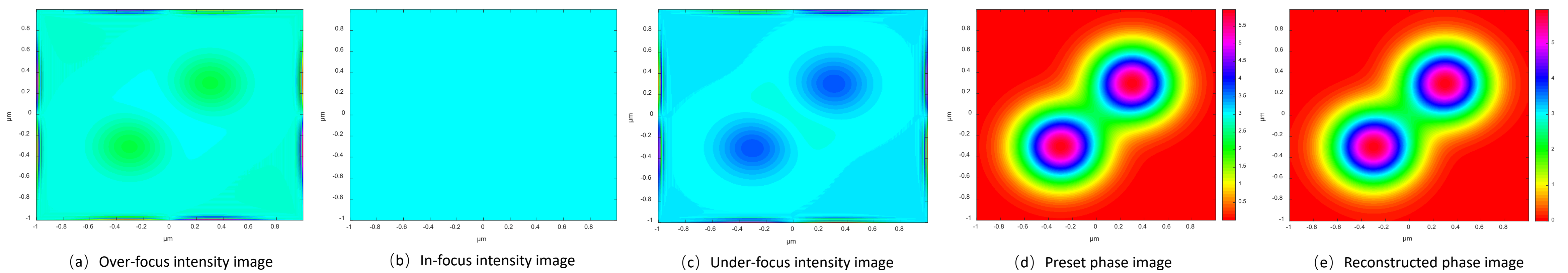
Introducing an auxiliary function

$$\nabla_{\perp}^2 \psi(r_{\perp}, z) = -k \partial_z I(r_{\perp}, z)$$

Solve the Laplacian operator by FFT, we can get the phase through the following function

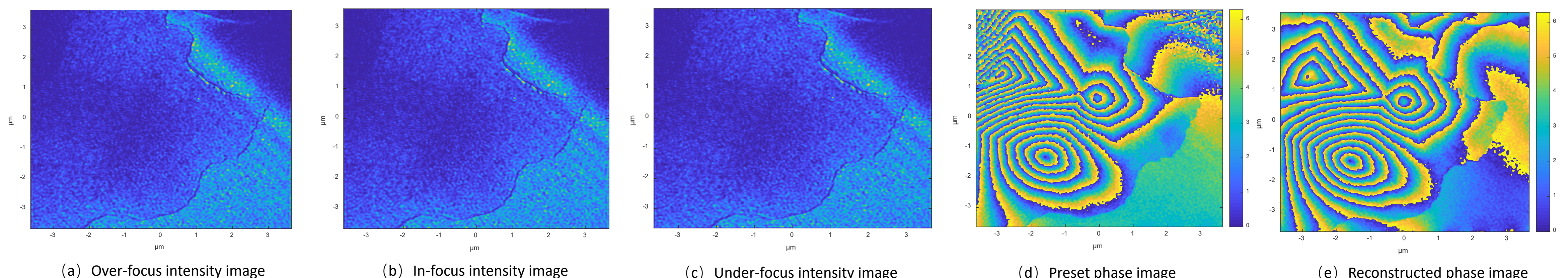
$$\phi(r_{\perp}, z) = \mathcal{F}^{-1} q_{\perp}^{-2} \mathcal{F}\{\nabla_{\perp} \cdot [I^{-1}(r_{\perp}, z) \nabla_{\perp} \psi(r_{\perp}, z)]\}$$

## Simulation I



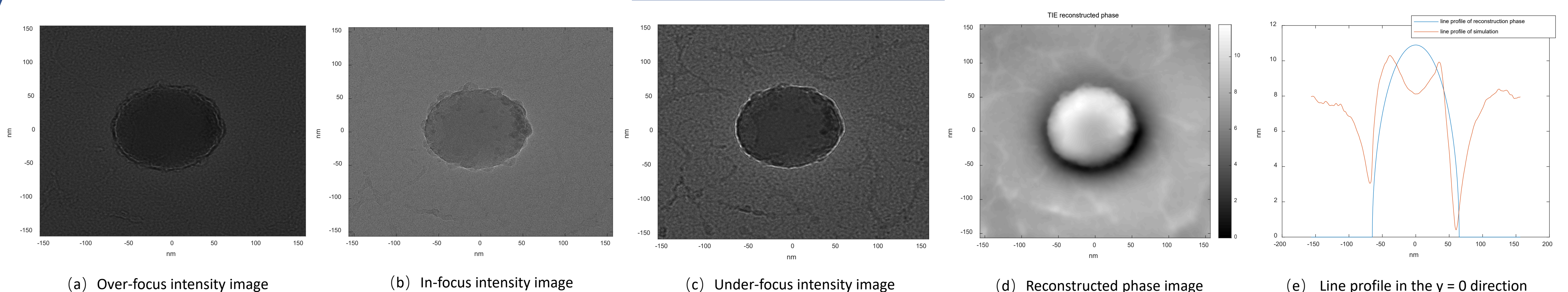
It is simulation experiment of restoring preset wave function phase with TIE. There are three different focus images with 100  $\mu\text{m}$  defocus step. The original image size (1024 $\times$ 1024 pixels) is 2  $\mu\text{m}$  and the simulated acceleration voltage of the electron beam is 200kV. The over- and under-focus intensity images generate from the Fresnel diffraction of the preset in-focus wavefunction. THE RMS between (d) and (e) is 0.007

## Simulation II



It is simulation experiment of restoring preset wave function phase which comes from the hologram of the real material with TIE. The defocus step three of different focus images is 0.1  $\mu\text{m}$ . 0.182  $\mu\text{m}$  original image size (1024 $\times$ 1024 pixels) and 200keV electron beam is used in simulation. THE RMS between (d) and (e) is 1.02

## Experiment



This is the phase reconstruction experiment of SiO<sub>2</sub> Ball by TIE. Here, we show three images selected from the focal series of 33 images taken with 250.57 nm defocus step. 313.17 nm original image size (4096 $\times$ 4096 pixels) and 200keV electron beam is used in experiment. The defocus distance between the under and over-focus images is 4.01  $\mu\text{m}$ . And (e) is the comparison of the line profile of the reconstructed phase image and the simulated phase image of the SiO<sub>2</sub> Ball in the  $y = 0$  direction.

## Conclusion

The feasibility of using TIE method to restore phase is proved by simulation and experiment, but there are still some problems in the quantitative phase reconstruction in experiment.

## Reference

- [1] Allen, L.J. and M.P. Oxley, Phase retrieval from series of images obtained by defocus variation. Optics Communications, 2001. 199(1-4): p. 65-75
- [2] Teague, M.R., DETERMINISTIC PHASE RETRIEVAL - A GREEN-FUNCTION SOLUTION. Journal of the Optical Society of America, 1983. 73(11): p. 1434-1441.