

## Introduction

According to quantum mechanics, the electron has wave property, so it can be described by wave function. For electron wavefunction, the intensity values at different positions indicate the probability of the electron appearing here, and the phase indicates the direction of the electron. We can get more sample information by detecting the phase.

But the detector can only get the intensity information of the electron wave function, so it is particularly important to detect the phase information. The direction of the electron is reflected by the change of the intensity in different planes, so we can restore the phase by the intensity.

## Theory

In 1983 Teague proposed TIE (the transport of intensity equation), The change of intensity and phase in paraxial approximation is revealed.

$$\nabla_{\perp} \cdot [I(r_{\perp}, z) \nabla_{\perp} \phi(r_{\perp}, z)] = -k \partial_z I(r_{\perp}, z)$$

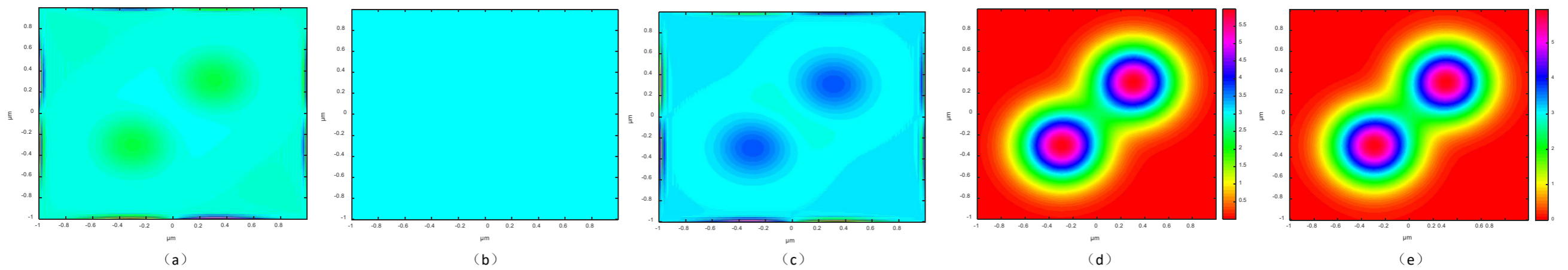
Introducing an auxiliary function

$$\nabla_{\perp}^2 \psi(r_{\perp}, z) = k \partial_z I(r_{\perp}, z)$$

Solve the Laplacian operator by FFT, we can get the phase through the following function

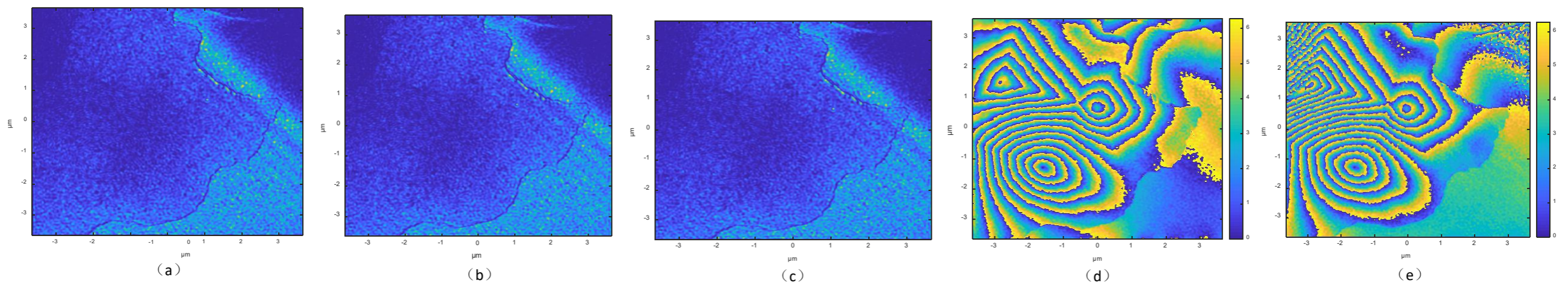
$$\phi(r_{\perp}, z) = \mathfrak{I}^{-1} q_{\perp}^2 \mathfrak{I} \{ \nabla_{\perp} \cdot [I^{-1}(r_{\perp}, z) \nabla_{\perp} \psi(r_{\perp}, z)] \}$$

## Simulation I



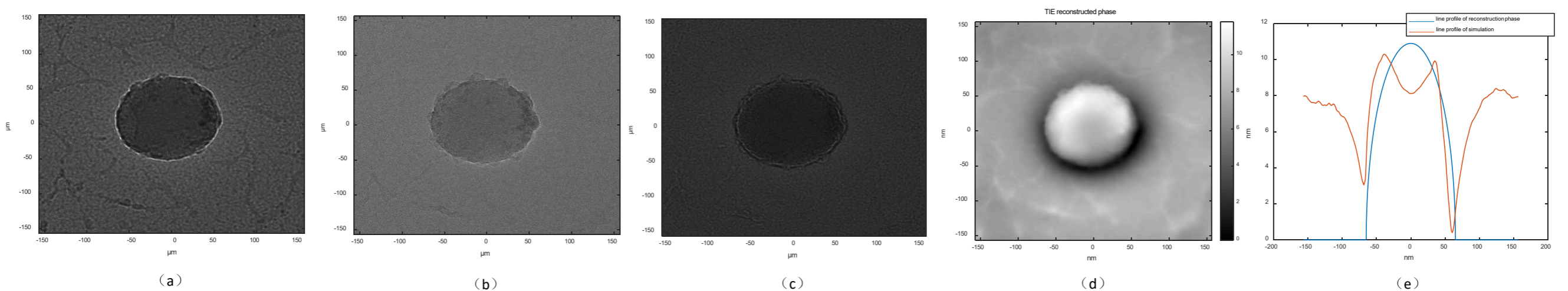
There are three different focus images with 100  $\mu\text{m}$  defocus step. The original image size (1024 $\times$ 1024 pixels) is 2  $\mu\text{m}$ . (a) The in-focus intensity image and (b) and (c), over- and under-focus intensity images from the Fresnel diffraction propagation of the in-focus wavefunction which set in advance. (d) is the preset phase image. (e) is the reconstructed phase by TIE. THE RMS between (d) and (f) is 0.007

## Simulation II



There are three different focus images with 0.1  $\mu\text{m}$  defocus step. The original image size (1024 $\times$ 1024 pixels) is 0.182  $\mu\text{m}$ . (a) The in-focus intensity image and (b) and (c), over- and under-focus intensity images. (d) is the preset phase image. (e) is the reconstructed phase by TIE. THE RMS between (d) and (f) is 1.02. The preset intensity and phase data come from the hologram of the real material.

## Experiment



Here, we show three images selected from the focal series of 33 images taken with 250.57 nm defocus step. The original image size (4096 $\times$ 4096 pixels) is 313.17 nm. (a) A in-focus image, and (b) and (c), over- and under-focus images. The defocus of the center image is 0 nm under-focus, and the defocus distance between the under and over-focus images is 4.01  $\mu\text{m}$ . (d) is the reconstructed phase by TIE (e) is the comparison of the line profile of the reconstructed phase and the simulated phase in the  $y = 0$  direction

## Reference

- [1] Allen, L.J. and M.P. Oxley, Phase retrieval from series of images obtained by defocus variation. Optics Communications, 2001. 199(1-4): p. 65-75
- [2] Teague, M.R., DETERMINISTIC PHASE RETRIEVAL - A GREEN-FUNCTION SOLUTION. Journal of the Optical Society of America, 1983. 73(11): p. 1434-1441.