Effect of Quantum Statistics on Computational Power of Atomic Quantum Annealers

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Motivation

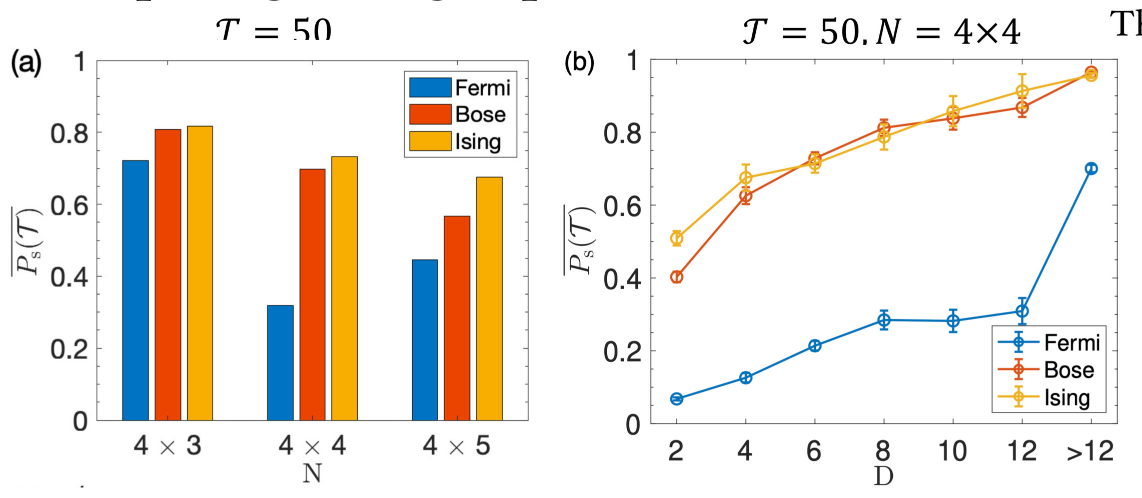
- In the NISQ era, developing alternative quantum annealing protocols and architectures with near-term technology is still in great demand for reaching the quantum computation advantage on combinatorial optimization problems.
- Since the remarkable technological advances in optical lattices provide new opportunities to build a scalable quantum annealer, it is worthy to study **how the quantum statistics affects the computational power based on this atomic platform** from both theoretical interests and practical considerations.

Proposal (demonstrated on the random problem instances of 3-regular graph partitioning)

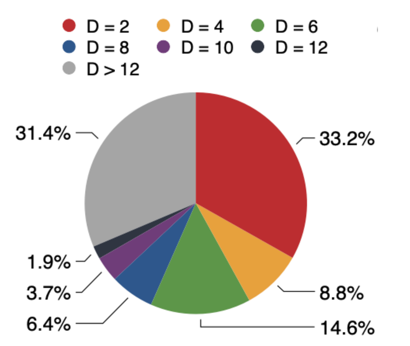
The Atomic Quantum Annealer (spinless fermions vs hard-core bosons)	The Ising Quantum Annealer (as a reference model)
$H^{\text{atomic}}(s) = (1-s)H_V + \lambda s(1-s)H_T + sH_P^{\text{atomic}}$	$H^{\text{Ising}}(s) = (1-s)H_Z + \lambda s(1-s)H_X + sH_P^{\text{Ising}}$
$H_T = \sum_{\langle i,j\rangle, i<j} -(a_i^\dagger a_j + a_j^\dagger a_i)$	$H_X = -\sum_{i=1}^N \sigma_i^x$
$H_V = \sum_{i=1}^N V_i n_i \quad (V_i = -2 \text{ for even } i,\ V_i = 0 \text{ for odd } i)$	$H_Z = \sum_{i=1}^N h_i \sigma_i^z \quad (h_i = -1 \text{ for even } i,\ h_i = 1 \text{ for odd } i)$
$H_P^{\text{atomic}} = \sum_{(v_i,v_j)\in E} \frac{1}{2}[1-(1-2n_i)(1-2n_j)]$	$H_P^{\text{Ising}} = \sum_{(v_i,v_j)\in E} \frac{1}{2}(1-\sigma_i^z\sigma_j^z) + \alpha\left(\sum_{i=1}^N \sigma_i^z\right)^2$

Success probability

- Comparing averaged performance



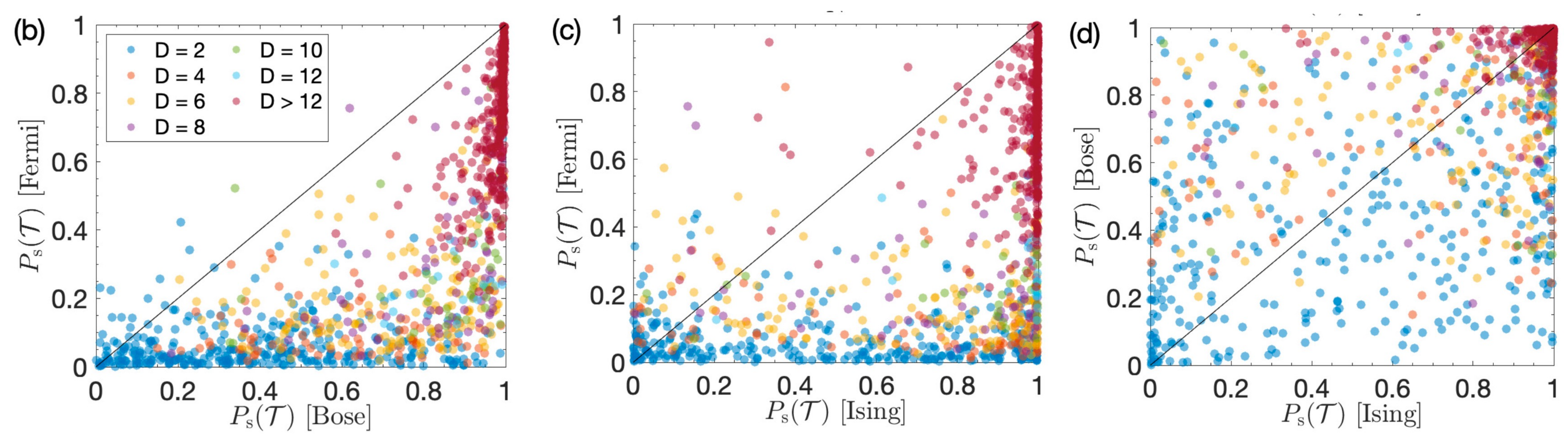
The solution degeneracy distribution of 1000 random instances



$2 \le D \le 96$

- Averaged success probability: Fermi < Bose ≈ Ising.
- For all three quantum annealers, the success probability grows monotonically with D.

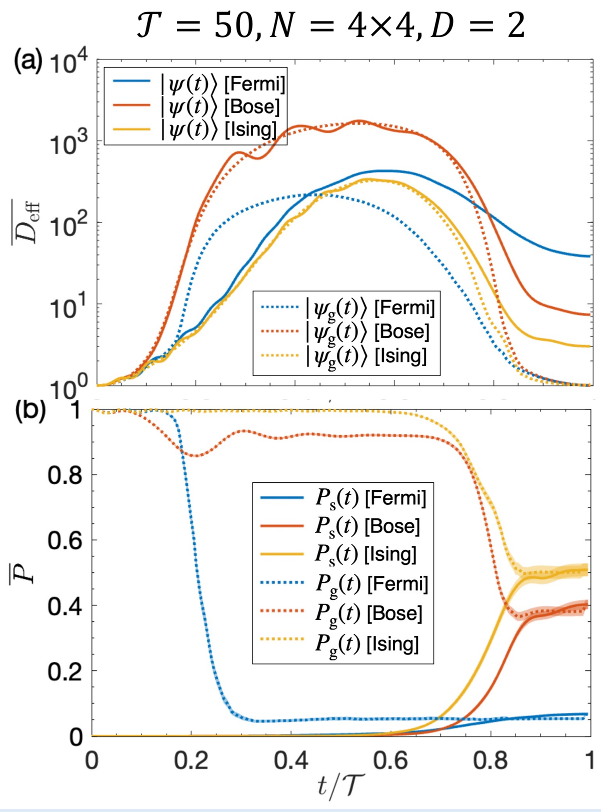
- Comparing performance instance by instance $\mathcal{T} = 50, N = 4\times4$



Bose > Fermi : 97.3%   Fermi > Ising : 0.095%   Bose > Ising : 36.4%

Annealing dynamical behavior

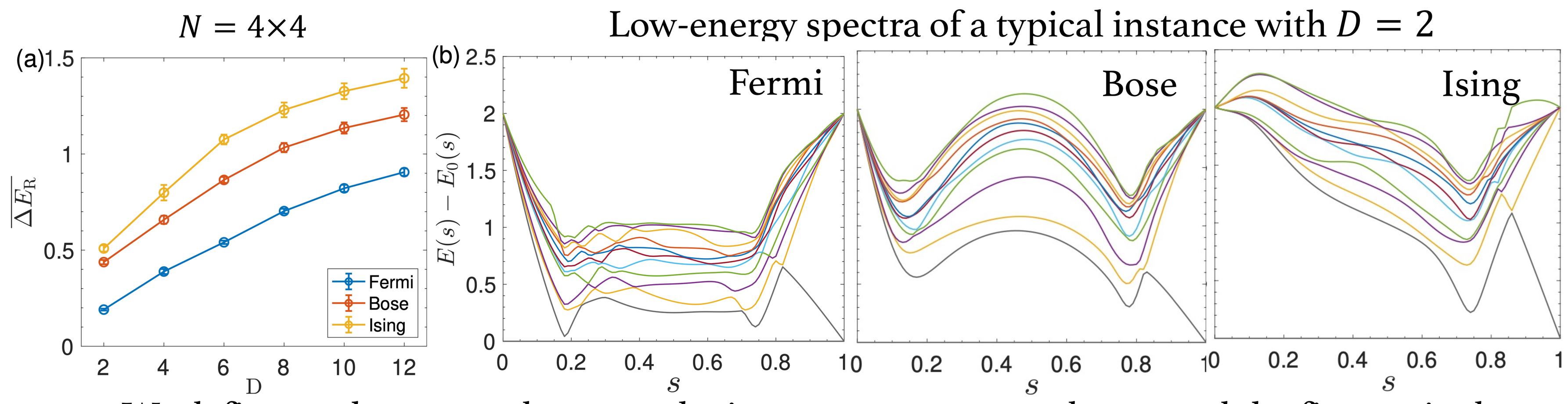
$\mathcal{T} = 50, N = 4\times4, D = 2$



- We investigate the effective dimension, $D_{eff}(|\psi\rangle) = \left(\sum_{i=1}^{\mathcal{D}} |c_i|^4\right)^{-1}$, which characterizes how efficiently the intermediate dynamical state explores the entire Hilbert space.
- The bosonic quantum annealer expands its dynamical state to the large Hilbert space with a faster rate than the fermionic annealer and its resultant peak value of the effective dimension at the intermediate time is also significantly larger.
- The dynamical behaviors of the ground state probability of the three quantum annealers are consistent with our observations on their minimum gaps and the deviations of their effective dimensions.

Relevant gap & Typical low-energy spectra

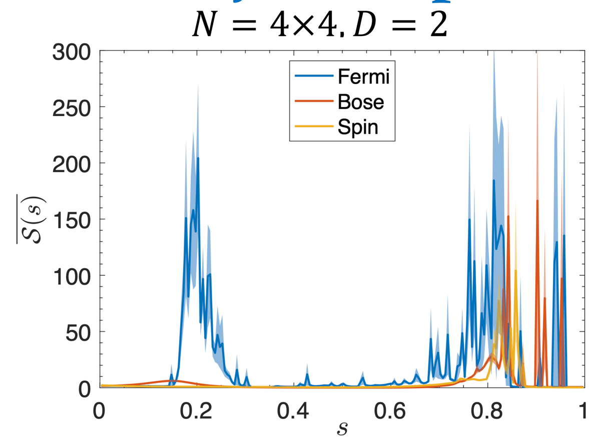
$N = 4\times4$ — Low-energy spectra of a typical instance with $D = 2$



- We define a relevant gap between the instantaneous ground state and the first excited state outside the degenerate ground state subspace, $\Delta E_R = \min_{s\in[0,1]}[E_D(s) - E_0(s)]$.
- The averaged relevant gap increases systematically with the solution degeneracy, which is consistent with the behavior of the success probability.
- The smallest relevant gaps of the fermionic quantum annealer is expected to be the main reason for its computation performance being the worst as compared to the bosonic and the Ising models.

Fidelity susceptibility as a bottleneck indicator

$N = 4\times4, D = 2$

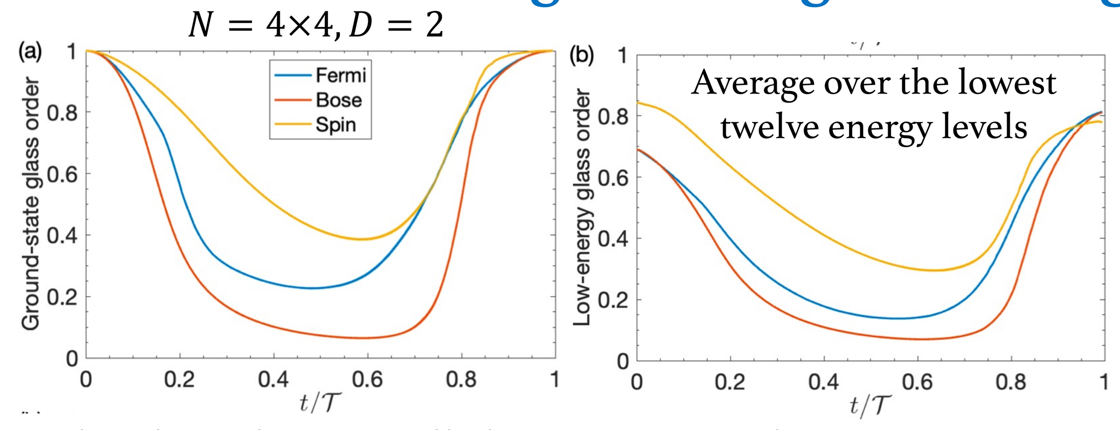


$$\mathcal{S}(s) = \lim_{\delta s\to 0} -2\ln \mathcal{F}(s, s+\delta s)/N(\delta s)^2$$

- A peak in this susceptibility signals the ground-state wave function changes dramatically, thus it is more difficult to maintain quantum adiabaticity at high $\mathcal{S}(s)$.
- The peaks of averaged fidelity susceptibility corresponds to the appearance of the minimum gaps.
- The peaks for bosonic quantum annealer, especially the first one, is less prominent than the fermionic one.
- The peak value of the Ising quantum annealer is comparable to that of the bosonic annealer at the same location.

Glass order strength during annealing

$N = 4\times4, D = 2$



The atomic quantum annealer $q_n = \frac{1}{N}\sum_{i=1}^N \langle 2n_i - 1\rangle^2$

The Ising quantum annealer $q_z = \frac{1}{N}\sum_{i=1}^N \langle\sigma_z\rangle^2$

- In the glass phase, small changes in Hamiltonian parameters may lead to a chaotic reordering of associated energy levels, which causes level crossings with exponentially small energy gaps.
- The bosonic quantum annealer suffers from weaker glass order along the whole process than the fermionic one, consistent with our observation on their low-energy spectra.

Conclusion

- **The bosonic quantum annealer outperforms the fermionic one.**
- The superior performance of the bosonic quantum annealer is attributed to **larger excitation gaps** and the consequent **smoother adiabatic transformation** of its instantaneous quantum ground states.
- Along our annealing schedule, the bosonic quantum annealer is **less affected by the glass order** and **explores the Hilbert space more efficiently**.
- Our theoretical finding could shed light on constructing atomic quantum annealers using Rydberg atoms in optical lattices.

• *Reference: arXiv:2209.00201.*

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