Bayesian optimization for state preparation via Markovian feedback control

Yadong Wu[1,2*], Juan Yao[3], Pengfei Zhang[4+]

([1]State Key Laboratory of Surface Physics, Institute of Nanoelectronics and Quantum Computing, and Department of Physics, Fudan University, Shanghai 200438, China)

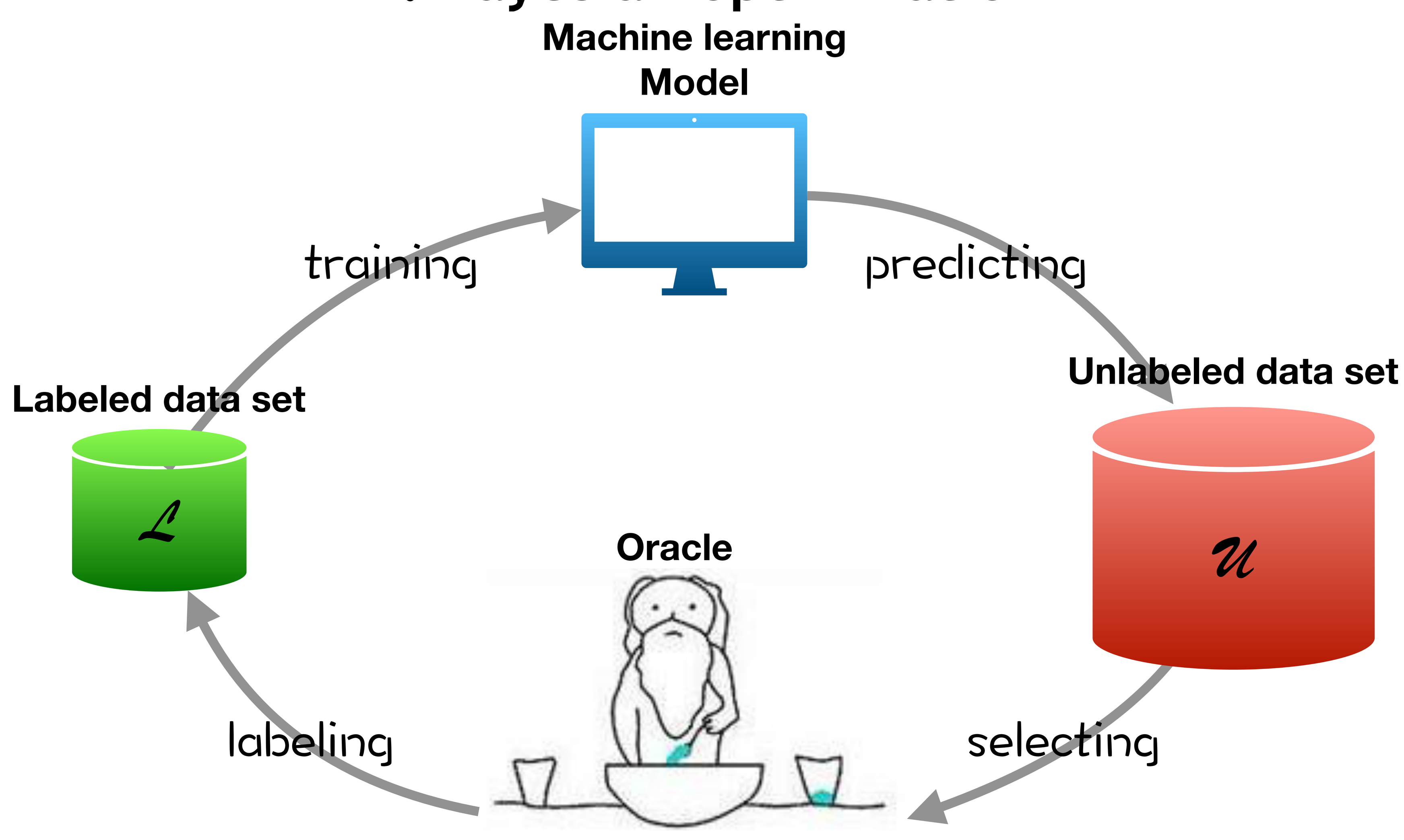
([2]Shanghai Qi Zhi Institute, Shanghai 200030, China)

([3]Guangdong Provincial Key Laboratory of Quantum Science and Engineering, Shenzhen Institute for Quantum Science and Engineering, Southern University of Science and Technology, Shenzhen 518055, Guangdong, China)

([4]Institute for Quantum Information and Matter and Walter Burke Institute for Theoretical Physics, California Institute of Technology, Pasadena, California 91125, USA)

*yadongwu_phy@163.com, +PengfeiZhang.physics@gmail.com

1. Bayesian optimization



Algorithm:

1. $\mathcal{U}$ = a pool of unlabeled instances
2. $\mathcal{L}$ = set of initial labeled instances
3. **for** $t = 1, 2\cdots$ **do**
4. $\quad\theta$ = **train** ($\mathcal{L}$)
5. $\quad$Select x^* in $\mathcal{U}$ with some selection rule.
6. $\quad$Query oracle to obtain label y^*
7. $\quad$Add (x^*, y^*) to labeled data set $\mathcal{L}$
8. $\quad$Remove (x^*, y^*) from unlabeled data set $\mathcal{U}$
9. **end**

$$P(x|y) = \frac{P(x,y)}{P(y)} = \frac{P(y|x)P(x)}{P(y)}$$

2. Weak measurement in quantum channel with feedback control

First, consider a density matrix $\hat{\rho}$ and a set of measurement operator $\{\hat{\Omega}_m\}$, where $\sum \hat{\Omega}_m^\dagger \hat{\Omega}_m = \hat{I}$.

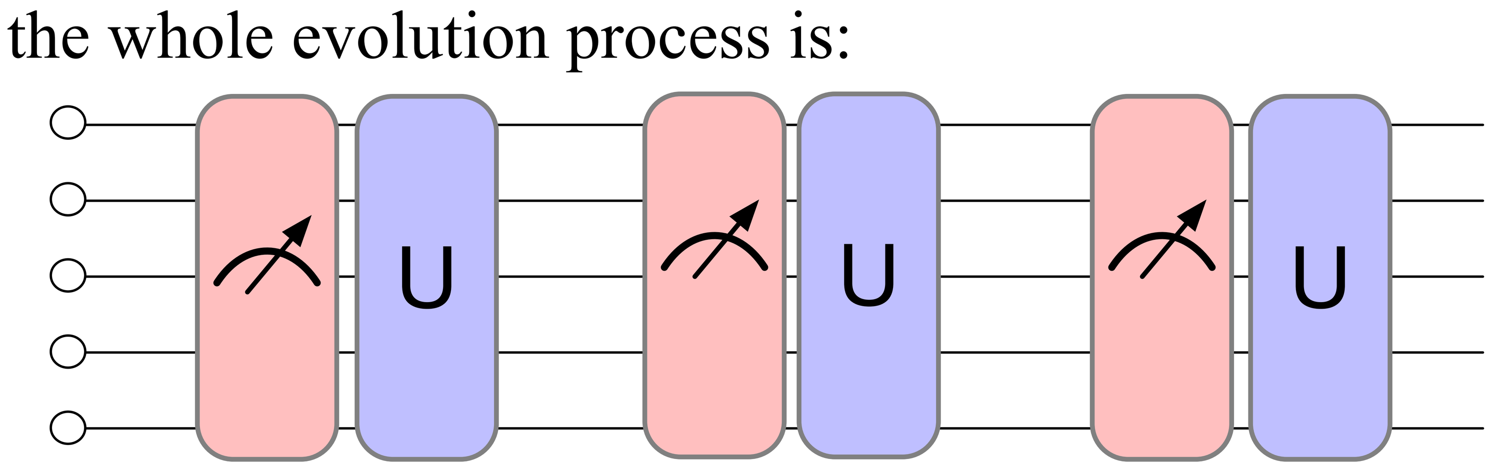
After measurement, $\hat{\rho}_m = \frac{\hat{\Omega}_m \hat{\rho} \hat{\Omega}_m^\dagger}{Tr[\hat{\Omega}_m \hat{\rho} \hat{\Omega}_m^\dagger]}$ with $P_m = Tr[\hat{\Omega}_m \hat{\rho} \hat{\Omega}_m^\dagger]$.

For the weak measurement, measure operator:

$$\hat{A}(\alpha) = \left(\frac{4\gamma\Delta t}{\pi}\right)^{1/4} \int_{-\infty}^{\infty} e^{-2\gamma\Delta t(c-\alpha)^2} |c\rangle\langle c| dc \propto e^{-2\gamma\Delta t(\hat{c}-\alpha)^2}.$$

The weak measurement result: $I_{hom} = Tr[(\hat{c} + \hat{c}^\dagger)\hat{\rho}] + \xi(t)$, $\xi(t) = \Delta W/dt$, $dW \sim \mathcal{N}(0, \sqrt{dt})$.

So the whole evolution process is:



i). Applying measurement hamiltonian to the system $|\psi(t)\rangle$.

$$|\psi(t+dt)\rangle \propto e^{-dt\gamma(\hat{c}-c_m)^2 + \sqrt{\gamma}(\hat{c}-c_m)dW} |\psi(t)\rangle.$$

where $c_m = \langle\psi(t)|\hat{c} + \hat{c}^\dagger|\psi(t)\rangle/2$ and $dW \sim \mathcal{N}(0, \sqrt{dt})$.

ii). According the measurement result $I_{hom} = c_m + dW/dt$. Evolving the state with original hamiltonian and feedback operator: $|\psi(t+dt)\rangle = e^{-i(\hat{H}_0 + I_{hom}\hat{F})dt} |\psi(t)\rangle$.

3. The ground state preparation of BH model

Bose-Hubbard model hamiltonian:

$$\hat{H}_{BH} = -J\sum_i \hat{a}_i^\dagger \hat{a}_{i+1} + h.c. + \frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1).$$

The measurement operate is:

$$\hat{c} = \sum w_l \hat{c}_l, \quad \{\hat{c}_l = \hat{n}_l = \hat{a}_l^\dagger \hat{a}_l\}.$$

The feedback operator is $\hat{F} = q_1\hat{F}_1 + q_2\hat{F}_2$:

$$\{\hat{F}_1 = \textstyle\sum_l \hat{a}_l^\dagger \hat{a}_{l+1} + h.c., \hat{F}_2 = i\sum_l \hat{a}_l^\dagger \hat{a}_{l+1} - h.c.\}.$$

SME: $d\hat{\rho} = -i[\hat{H} + \hat{H}_{fb}, \hat{\rho}]dt + \mathscr{D}[\hat{A}]\hat{\rho}dt + \mathscr{H}[\hat{A}]\hat{\rho}dW$,

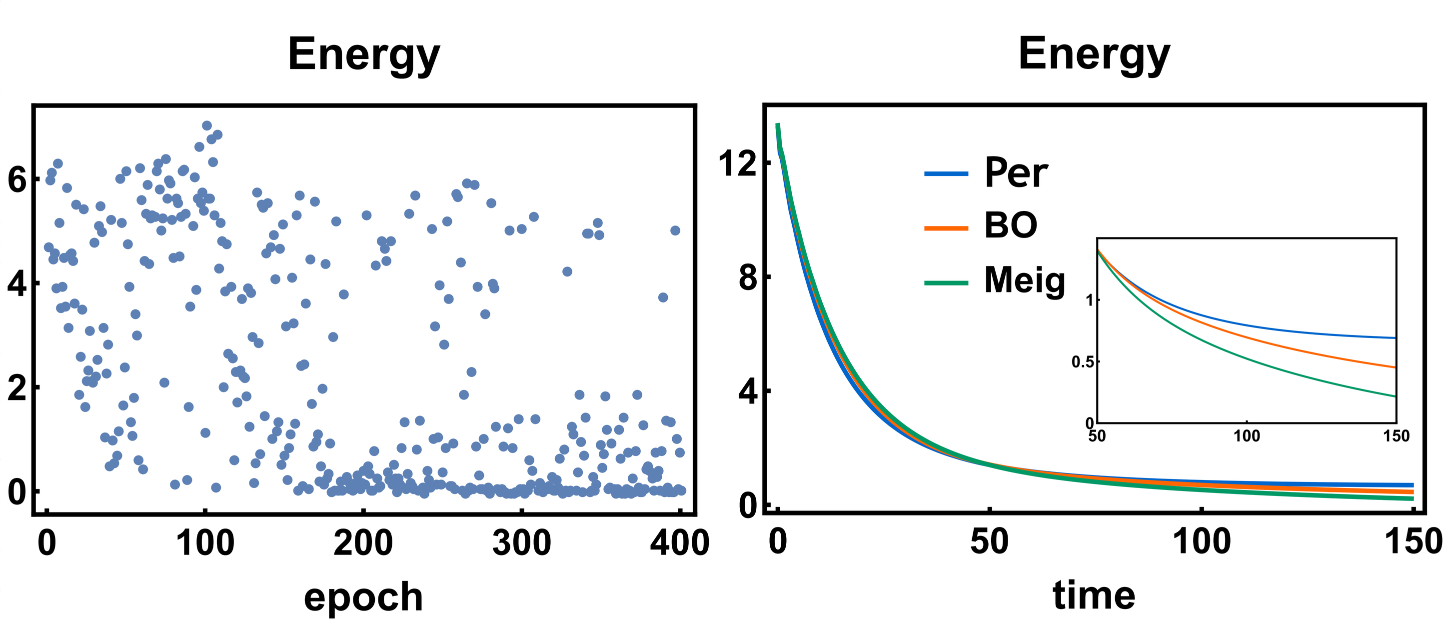
where $\hat{A} = \hat{c} - i\hat{F}$. Steady state $|\psi_s\rangle, \hat{A}|\psi_s\rangle \to 0$.

$\min_A \langle GS|\hat{A}^\dagger \hat{A}|GS\rangle \to \min_u u^\dagger M u, M_{ij} = \langle GS|\hat{c}_i^\dagger \hat{c}_j|GS\rangle$.

Target: optimizing $\{w_l, q_l\}$ to minimize $\langle\psi(T)|H_{BH}|\psi(T)\rangle$.

I) Gapped system with strong onsite interaction:

N=4, M=4, U/J=5;

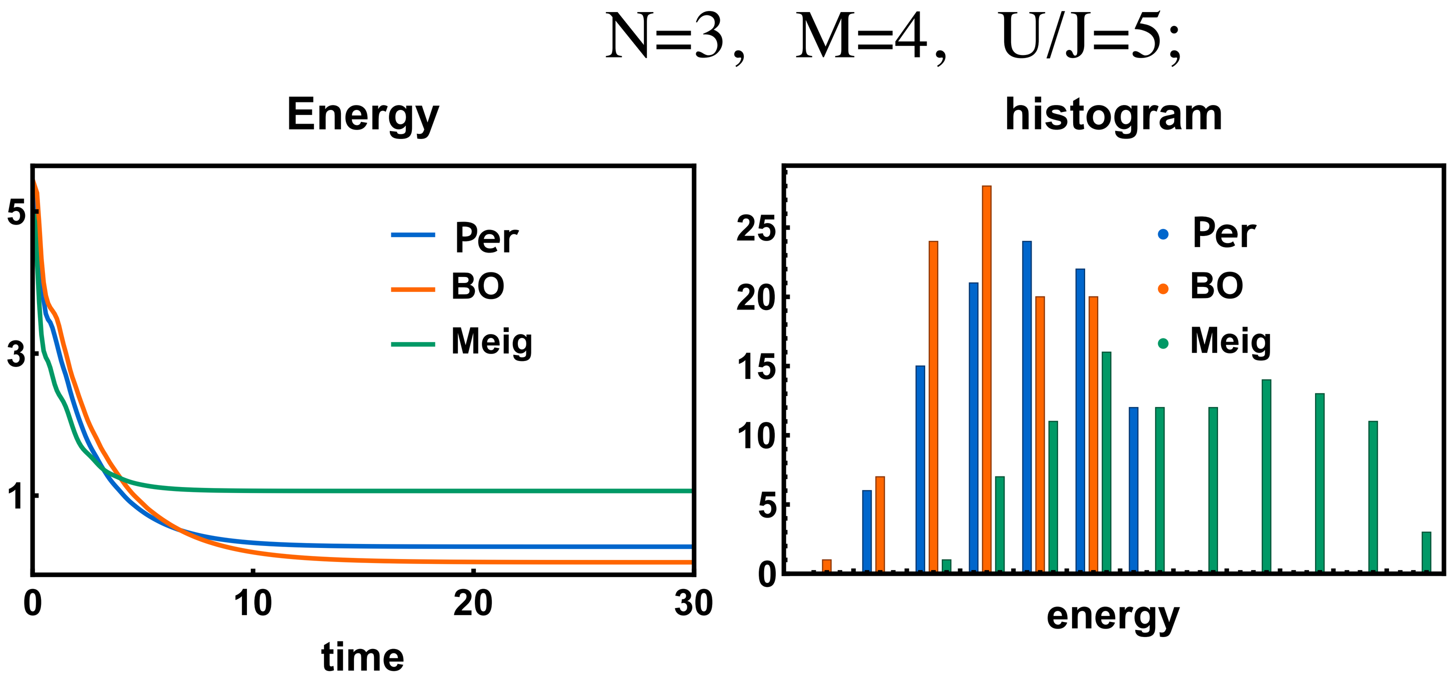


GS expectation

Per:	6.745E-05
BO:	0.00011
Meig	5.4918E-05

II) Gapless system with strong onsite interaction:

N=3, M=4, U/J=5;



GS expectation

Per:	0.0032
BO:	0.0042
Meig	0.0020

4. Measurement induced phase transition

Measurement operator: $\{\hat{c}\}$ local operators;

$d\hat{\rho} = -i[\hat{H}, \hat{\rho}]dt + \mathscr{D}[\hat{A}]\hat{\rho}dt + \mathscr{H}[\hat{A}]\hat{\rho}dW, \quad \hat{A} = \gamma(\hat{c} - i\hat{F})$

Weak measurement strength: $\gamma/J \leq 1$, volume law, $S_{sub} \propto L_{sub}$

Strong measurement strength: $\gamma/J \geq 1$, area law, S_{sub} = constant

With feedback hamiltonian, $\hat{F}$ is the hopping of the nearest neighbor sites. Thus it will re-entangle the system again.