Manipulating transient mass diffusion with transformation theory

Zeren Zhang [1], Liujun Xu [2], and Jiping Huang [1]

[1] *Department of Physics, State Key Laboratory of Surface Physics, and Key Laboratory of Micro and Nano Photonic Structures (MOE), Fudan University, Shanghai 200438, China*

[2] *Graduate School of China Academy of Engineering Physics, Beijing 100193, China*

I. Introduction

Manipulating transient mass diffusion plays a crucial role in physics, chemistry, biology, and other fields. Two typical examples are chemical waves and plasmas, whose concentration profiles have spatiotemporal variations. Based on a general model associated with the advection-diffusion equation, an optimized transformation-mass-transfer theory is proposed to flexibly control chemical waves. As an example application, a class of separators is theoretically designed to achieve the separation of chemical waves. Meanwhile, three typical types of devices for cloaking, concentrating, and rotating chemical waves are proposed successfully, thus offering more versatile control methods. For plasma manipulation, we use a simplified diffusion-migration approach to describe plasma transport. The feasibility of the transformation theory for plasma transport is demonstrated. As potential applications, we design three model devices capable of cloaking, concentrating, and rotating plasmas without disturbing the density profile of plasmas in the background. The proposed methods are well confirmed by the computer simulations. These results provide insights into novel manipulation of chemical waves associated with biochemical reactions and promote potential applications for transient mass diffusion and may help advance plasma technology in practical fields, such as medicine and chemistry.

II. Methods for controlling diffusion

We could apply an advection-diffusion equation to describe chemical waves,

$$\frac{\partial c}{\partial t} = \nabla \cdot \left(\overleftrightarrow{D} \cdot \nabla c - vc \right)$$

where c, t, $\overleftrightarrow{D}$, and v are concentration, time, tensorial diffusivity, and advection velocity, respectively. Then the transformed parameters $\overleftrightarrow{D}'$ and v' could be summarized as,

$$\overleftrightarrow{D}' = J\overleftrightarrow{D}J^{\tau} \qquad v' = Jv$$

J is Jacobian matrix and J^{τ} the transpose of J. Plasma transport can be simplified to a diffusion-migration process,

$$\partial_t n - \nabla \cdot (D \nabla n) \pm \nabla \cdot \left[\left(\frac{D\boldsymbol{E}}{T} \right) n \right] = S$$

where n, E and S are the density of plasma, intensity of electric field, and external plasma source, respectively. T (in units of V) is assumed to be a constant plasma temperature. Then the transformed parameters D'', E', and S' could be summarized as,

$$D'' = JDJ^{\tau} \qquad E'' = J^{-\tau}E \qquad S'' = S$$

In this way, the transformed equation could keep form-invariant.

III. Basic model applications of mass diffusion

To realize the cloak, concentrator, rotator, and separator. The coordinate transformations from a virtual space r_i to the physical space r_i' are set as,

For cloak
$$\begin{cases} r' = \dfrac{r_2 - r_1}{r_2} r + r_1, \\ \theta' = \theta. \end{cases}$$

For rotator
$$\begin{cases} r' = r, \\ \theta' = \theta + \theta_0, \quad r < r_1 \\ \theta' = \theta + \theta_0 \dfrac{r - r_2}{r_1 - r_2}, \quad r_1 < r < r_2 \end{cases}$$

For concentrator
$$\begin{cases} r' = \dfrac{r_1}{r_m} r, \quad r < r_m \\ r' = \dfrac{r_1 - r_m}{r_2 - r_m} r_2 + \dfrac{r_2 - r_1}{r_2 - r_m} r, \quad r_m < r < r_2 \\ \theta' = \theta. \end{cases}$$

For separator
$$\begin{cases} x' = (y/(2y_0) + 1) - x_0 y/(2y_0), \\ y' = y, \end{cases}$$
$$\begin{cases} x' = (y/(2y_0) + 1) + x_0 y/(2y_0) + x_0, \\ y' = y, \end{cases}$$

Then according to transformation rules, we are able to determine the corresponding diffusivities, advection velocities, and intensity of electric fields to achieve cloak, concentrator, rotator, and separator for transient mass diffusion.

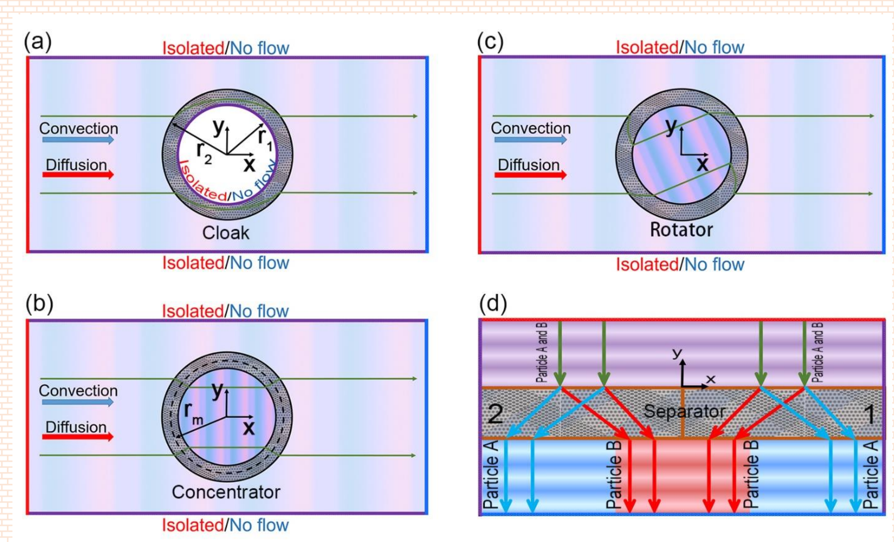

Fig. 1. Schematic diagrams of cloaking, concentrating, rotating, and separating in two dimensions for chemical waves.

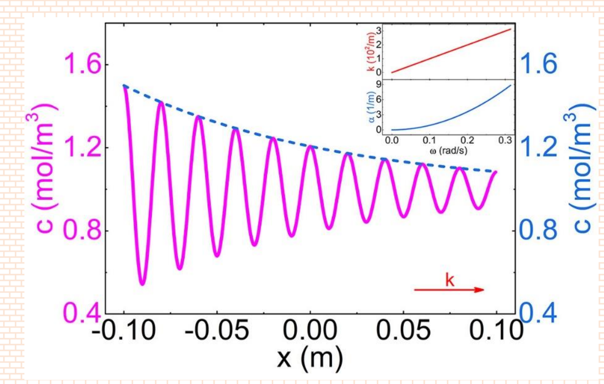

Fig. 2. Particle concentration distribution along x-axis. Inset shows the real and imaginary part of wave number, k and α, as a function of ω, respectively.

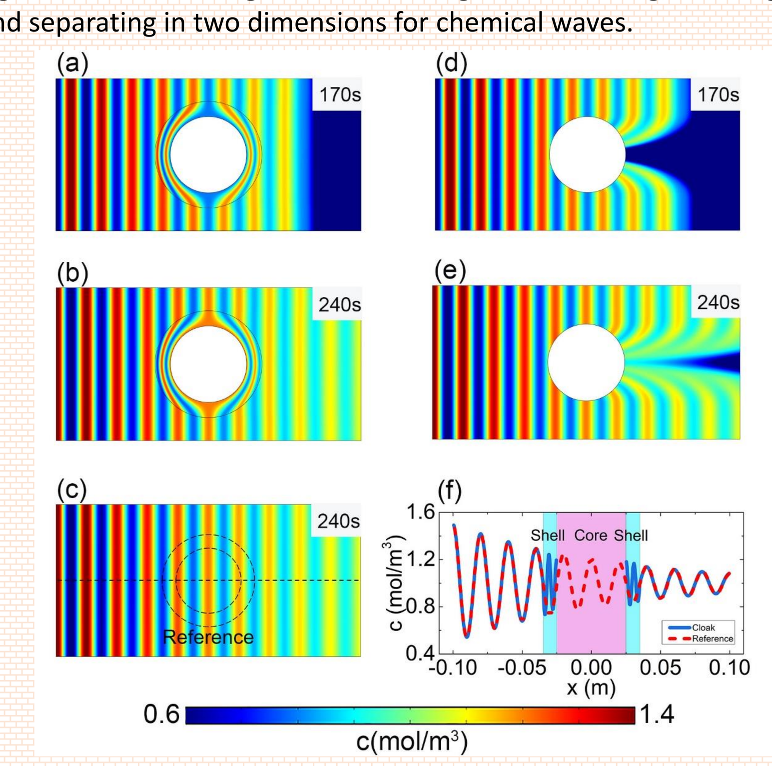

Fig. 3. Simulation results of cloaking chemical waves.

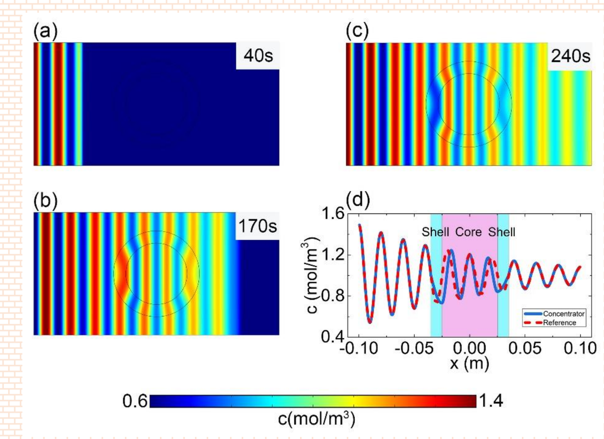

Fig. 4. Simulation results of concentrating chemical waves.

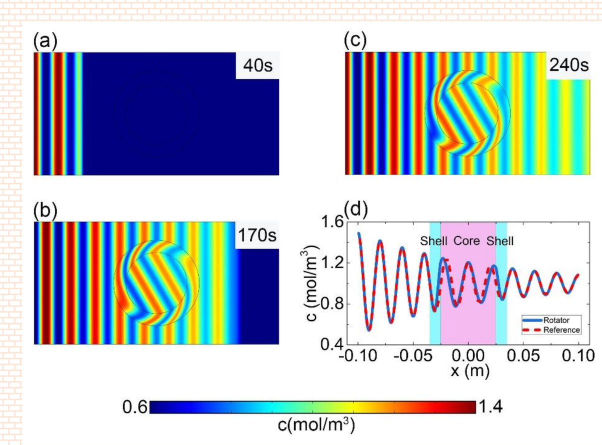

Fig. 5. Simulation results of rotating chemical waves.

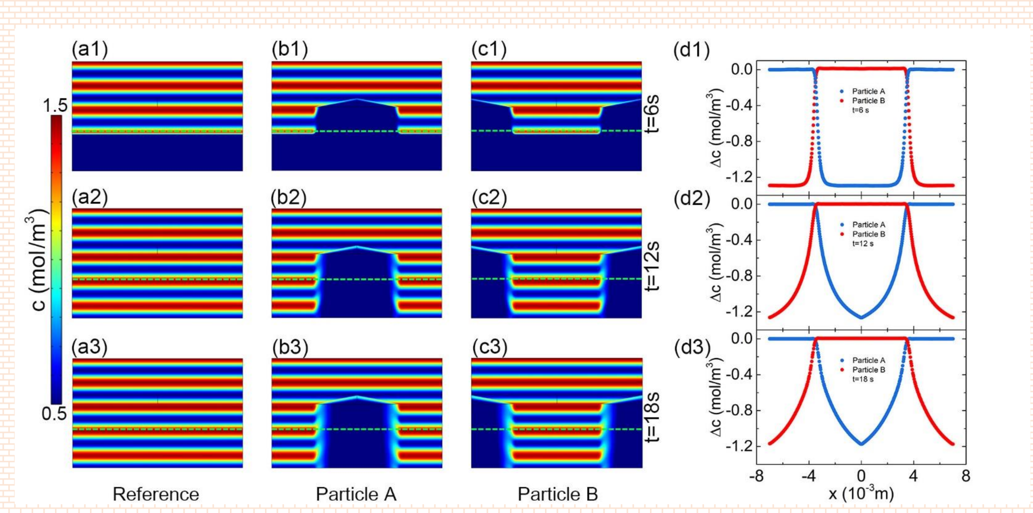

Fig. 6. Simulation results of separating chemical waves composed of particles A and B, respectively. a1–a3) Propagation results of chemical waves in a pure background medium at 6, 12, and 18 s, respectively. b1–b3) Results of particle-A chemical waves in a background medium with separator at 6, 12, and 18 s, respectively. c1–c3) Results of particle-B chemical waves in a background medium with separator at 6, 12, and 18 s, respectively. d1) Particle A's [or Particle B's] concentration difference Δc between (b1) and (a1) [or between (c1) and (a1)] along x axis at 6 s, according to the data extracted from the x-directed green dashed lines in (b1) and (a1) [or in (c1) and (a1)]. Similarly, (d2) and (d3) show Δc at 12 and 18 s, respectively.

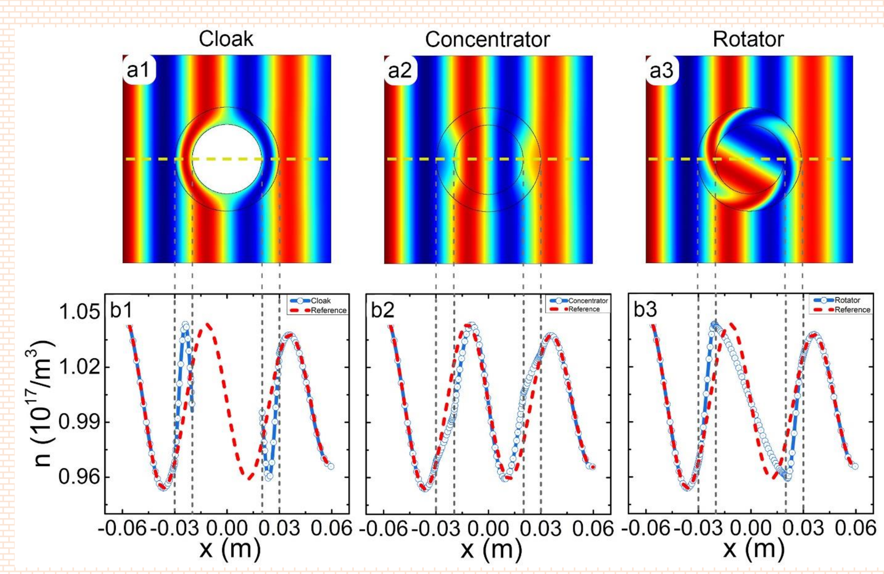

Fig. 7. (a1)–(a3) Color mapping of density profiles at 40 s with a cloak, concentrator, and rotator, respectively. (b1)–(b3) Comparisons of density profiles between pure background (reference) and those with a cloak, concentrator, and rotator, respectively. The gray dashed lines denote the position of the devices. Data are extracted along the yellow dashed line in (a1)–(a3).

IV. Conclusion

1. we have established a model to describe chemical waves and discuss their propagation properties. we proposed an optimized transformation transient-mass transfer theory and further designed four functional devices to flexibly control chemical waves.
2. we have employed a toy model, i.e., the diffusion-migration model, to describe plasma transport in this study. The feasibility of the transformation theory is demonstrated.

Reference

[1] Z. R. Zhang and J. P. Huang, Chin. Phys. Lett. **39**, 075201 (2022).
[2] Z. R. Zhang, L. J. Xu, and J. P. Huang, Adv. Theory Simul. **5**, 2100375 (2022).
[3] Z. R. Zhang, L. J. Xu, X. P. Ouyang, and J. P. Huang, Therm. Sci. Eng. Prog. **23**, 100926 (2021)