Physical Probabilistic Computing

Yunwen Liu[1], Jiang Xiao[1, 2, 3]

1 Department of Physics, Fudan University, Shanghai 200433, China

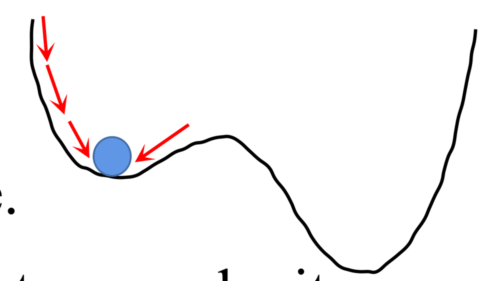
2 Institute for Nanoelectronics Devices and Quantum Computing, Fudan University, Shanghai 200433, China

3 Shanghai Research Center for Quantum Sciences, Shanghai 201315, China

Introduction

Classical deterministic computation methods have encountered great difficulties when it comes to optimization (to find the ground state of a cost function E).

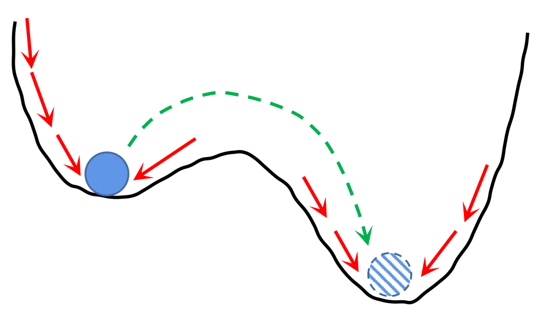
1. Continuous cases: gradient descent is frequectly used, which may be stuck by local minima. Noise may help to escape.

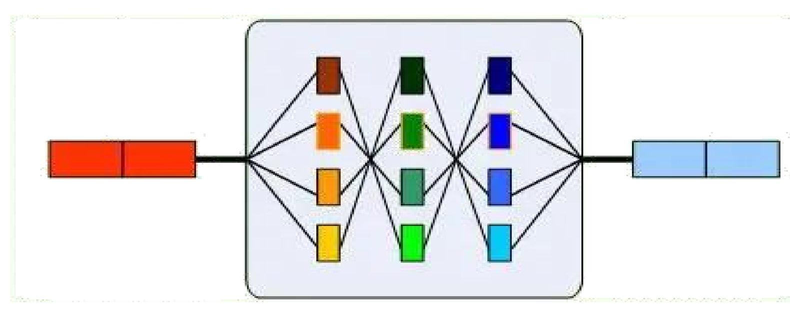


2. Discrete cases: sheer enumeration leads to complexity.

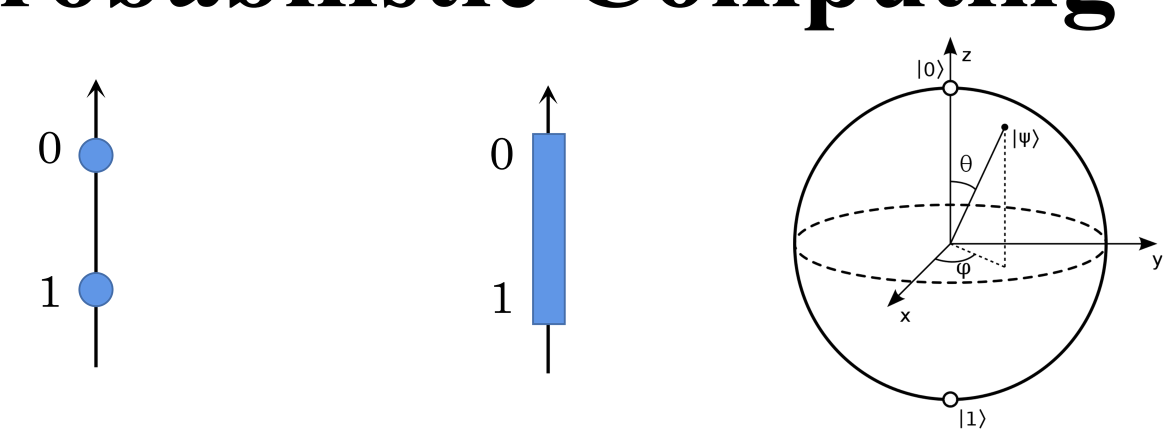
Various physical systems have been investigated to accelerate computation from two aspects:

Stochasticity **Parallelization**





Probabilistic Computing

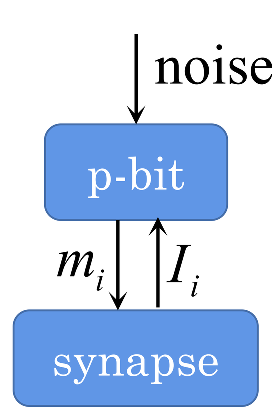


traditional (d-bit) | **probabilistic (p-bit)** | quantum (qubit)

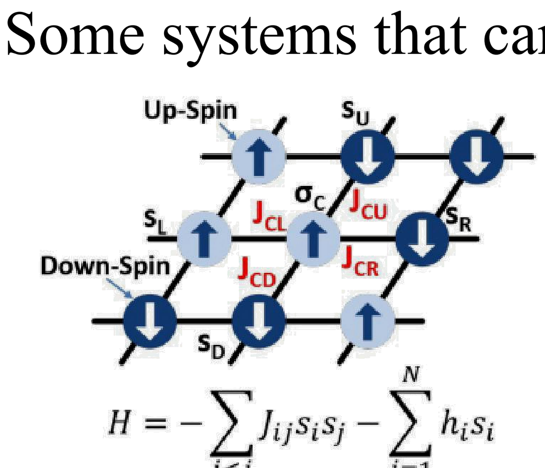
$$E = m^{\mathrm{T}} J m + m^{\mathrm{T}} b$$

$$\text{Input}: \quad I_i = -\frac{\partial E}{\partial m_i} = -2\sum_j J_{ij} m_j - b_i$$

$$\text{Output}^{[1]}: \quad m_i = \mathrm{sgn}[f(I_i) + \mathrm{rand}(-1,1)]$$

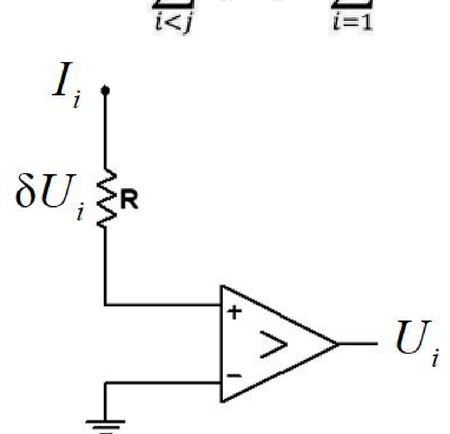


Some systems that can make p-bits:



1. Macrospin
Synapse: Ising interaction
Fluctuation: thermal fields[2]

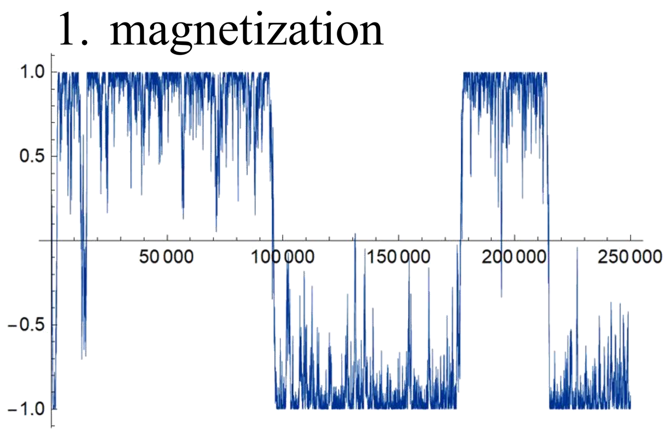
$$\left\langle \delta h^2_{x,y,z} \right\rangle = \frac{2\alpha k_B T}{\gamma \mu_0 M_s} \delta(\vec{r})\delta(t)$$



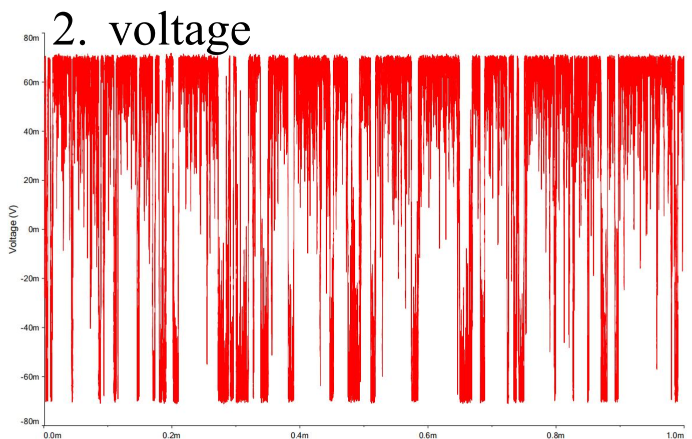
2. Noisy voltage comparator
Synapse: resistor network
Fluctuation: Johnson-Nyquist noise[3]

$$\left\langle \delta U_i^2 \right\rangle = 2Rk_B T\delta(t)$$

1. magnetization



2. voltage



Telegraph noise can be simulated in these systems.

Reference

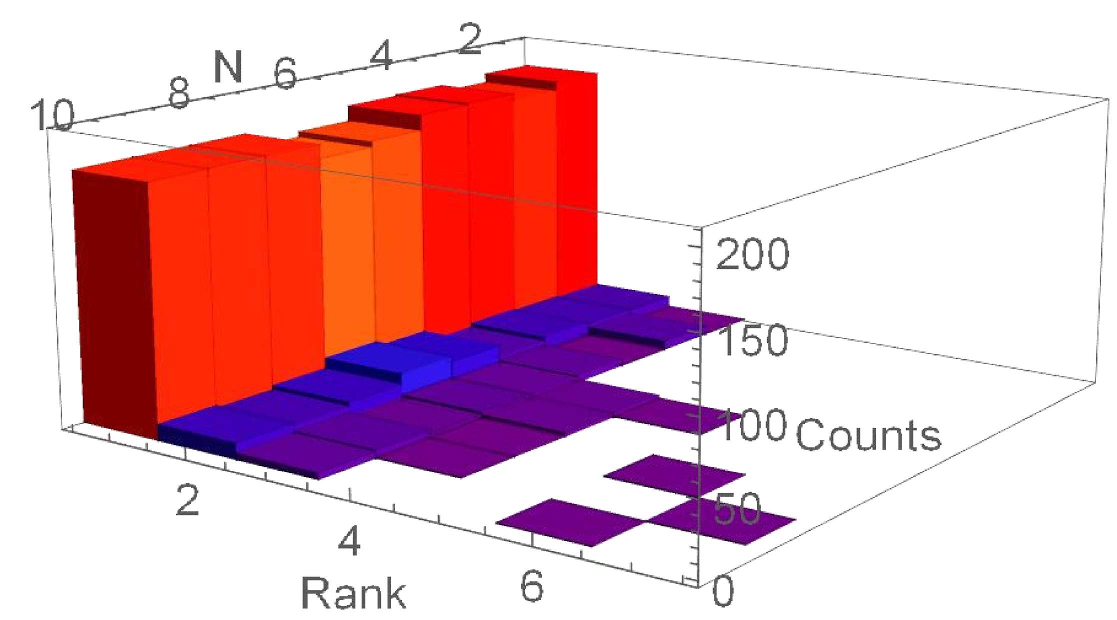
[1]W. Borders et al, Nature, 573
[2]H. Akimoto et el, Jounral of Applied Physics 97, 10N705(2005)
[3]S. Cheemalavagu et al, Proceedings of the IFIP International, 2005
[4]A. Lucas, Frontiers in Physics, 2014
[5]B. Sutton et al, IEEE Access, 2020(8)

Results

1. Random quadratic optimization

$$\min E = m^{\mathrm{T}} J m + m^{\mathrm{T}} b \text{ with random } J \text{ and } b, \quad m \in \{0,1\}^N$$

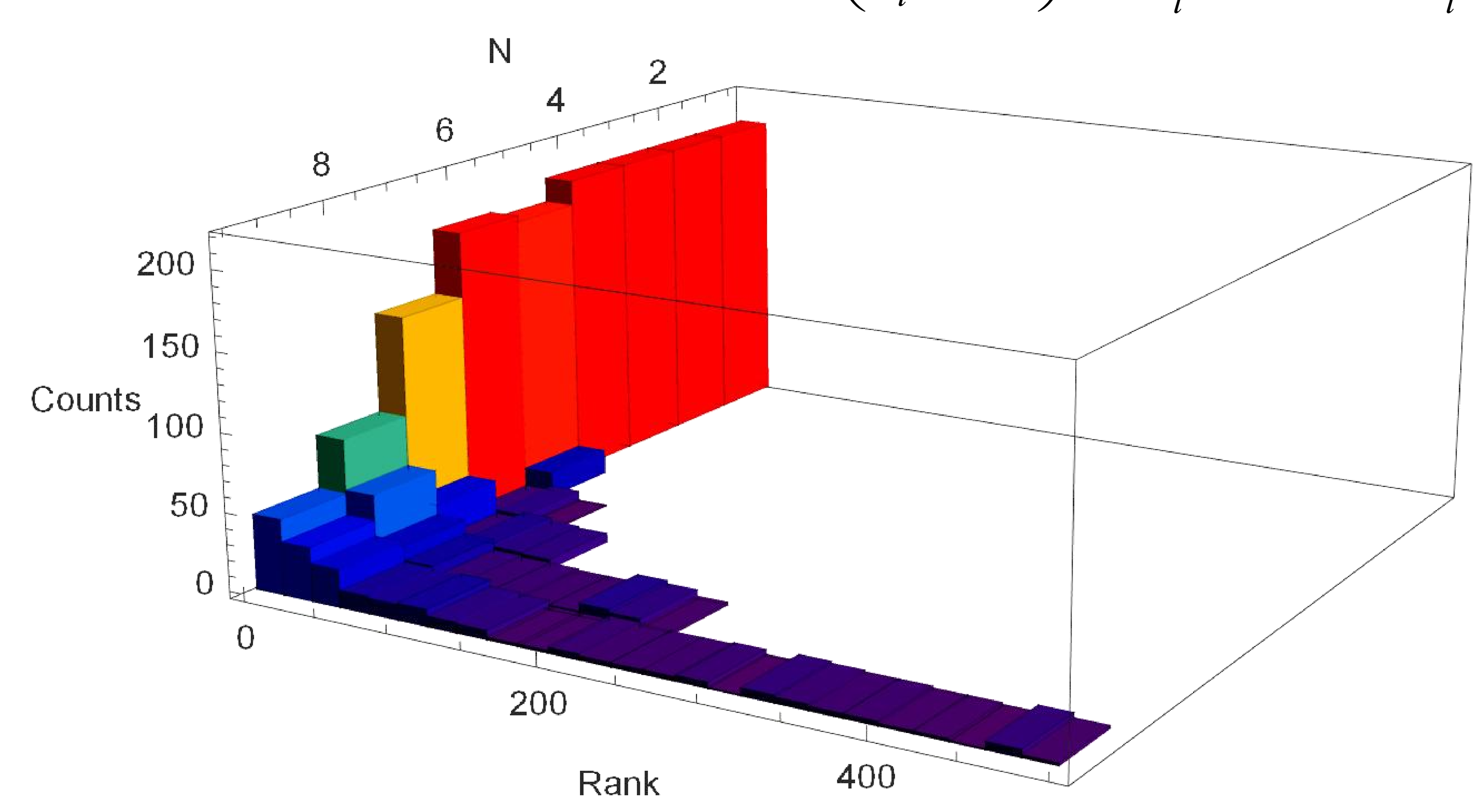
Given p-bit number N, the system computes 200 random problems, and then **the rank of ground states in visit counts** are studied. Most optimization problems of this kind can be correctly solved.



2. Number partitioning[4] (also 200 random problems for each N)

$$S = \{n_1, \dots, n_N\} = \overbrace{\{a_1, a_2, \dots, a_k\}}^{A} \oplus \overbrace{\{b_1, \dots, b_{N-k}\}}^{B}$$

$$\text{Minimize} \,|\, \mathrm{sum}(A) - \mathrm{sum}(B) \,| \quad E = \left(\sum_i n_i s_i\right)^2 \quad \begin{array}{l} s_i = +1 \Leftrightarrow n_i \in A \\ s_i = -1 \Leftrightarrow n_i \in B \end{array}$$



Prospects and Challenges

1. Probabilistic systems have the following advantages:
- (Relative) safety from local minima
- Endurability of noise in the environment
- Scalability close to traditional computers
- Effective parallel operation owing to asynchronized update[5]

→ Less time / power consumption, mild condition, integration

2. Issues remaining to be handled:
- NP character caused by exponential increase of local minima

# of local minima for general quardratic problem ~ 1.05^N
# of local minima for number partitioning ~ 1.5^N

- Misleading of -▿E in discrete problems (outperformed by Monte Carlo simulation that directly involves E)
- How to generate true noise in reality: giant resistors, chaotic circuits, etc.