Time-Optimal Quantum Driving by Variational Circuit Learning

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Introduction

In this context, using the hybrid quantum-classical algorithm, combining classical optimizers and quantum computers, is a competitive strategy for solving specific problems. We put forward its use for optimal quantum control. We simulate the wave-packet expansion of a trapped quantum particle on a quantum device with a finite number of qubits. We then use circuit learning based on gradient descent to work out the intrinsic connection between the control phase transition and the quantum speed limit imposed by unitary dynamics. We further discuss the robustness of our method against errors and demonstrate the absence of barren plateaus in the circuit. The combination of digital quantum simulation and hybrid circuit learning opens up new prospects for quantum optimal control.

Reference

[1] Tangyou Huang, et al, arxiv: 2211.00405(2022).

[2] X. Chen, A. Ruschhaupt, S. Schmidt, A. del Campo, D. Guéry- Odelin, and J. G. Muga, Phys. Rev. Lett. 104, 063002 (2010).

[3] G. S. Giuliano Benenti, arXiv , 0709.1704 (2007).

[4] P. J. Ollitrault, G. Mazzola, and I. Tavernelli, Phys. Rev. Lett. 125, 260511 (2020).

Motivation

arXiv: 2211.00405 (2022)

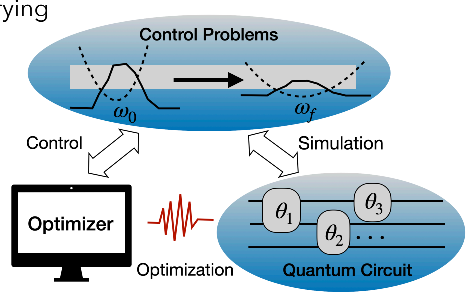
Using a digital quantum computer to solve optimal control problem.

We consider a quantum particle trapped in a time-varying parabolic potential:

$$H(t) = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega^2(t)[\hat{x} - x_0(t)]^2$$

The dynamic is engineered by trap frequency : $\omega_0 \rightarrow \omega_f$.

The time-optimal control of wave-packet expansion is a 'bang-bang' control, see ref [1].



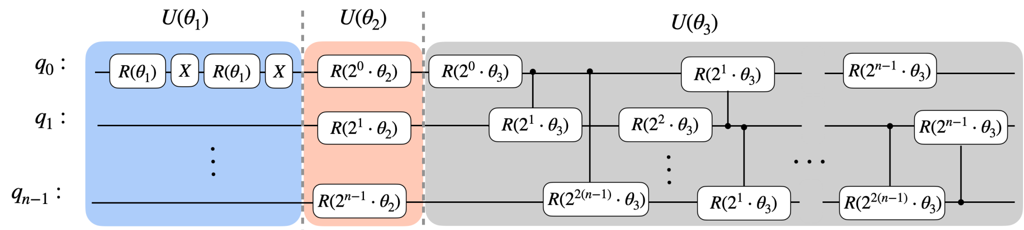
Methods

- Encoding the wave-packet into qubit states :

$$|\Phi\rangle = \sum_{i=0}^{2^n-1} \Psi(x_i)|i\rangle = \Psi(x_0)|0\cdots0\rangle + \ldots + \Psi(x_{2^n-1})|1\cdots1\rangle$$

- Reproducing the dynamic as: $|\tilde{\Phi}(t+dt)\rangle = \mathscr{V}(t)_{dt/2}\text{QFT}\mathscr{T}(t)_{dt}\text{QFT}^\dagger\mathscr{V}(t)_{dt/2}|\tilde{\Phi}(t)\rangle,$

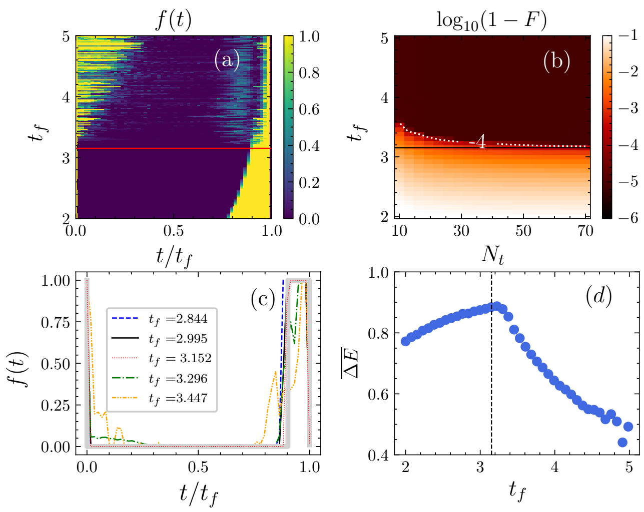
- The circuit realization for the time-operator of quadratic Hamiltonian [2,3]:



$$\mathscr{A}_j = \exp\left\{-i\,dt[h(jdx+x_0)^2+\sigma]\right\},$$

$$\theta_1 = -(hx_0^2+\sigma)\cdot dt,\ \theta_2 = -2hx_0\cdot dx\cdot dt,\ \theta_3 = -h\cdot dx^2\cdot dt$$

Results



(a). The control phase diagram, where the fidelity-optimal control sequence $f(t) = \omega^2(t)$ as functions of t/t_f for different $t_f \in [2,5]$. (b).The logarithm of infidelity $\log_{10}(1-F)$ as a function of $\{t_f, N_t\}$ in (b) and (c). Five selected control sequence for different total time $t_f$are compared with the bang-bang optimal control (thick gray line). The solid line, as a reference, stands for the case of $t_f^{opt} = 3.152$ in (a) and (b) .

Discussion

- The absence of Barren Plateau

$$\overline{|\partial_{\theta_k} J|} = \sum_{i=1}^{N_r} \frac{1}{2N_r}\left|J_i(\theta_k + \frac{\pi}{2}) - J_i(\theta_k - \frac{\pi}{2})\right|$$

- Noise resilience (bit-flip errors)

$$\xi_{BF}(\rho) = (1-\beta)\rho + \beta X\rho X$$

- The circuit complexity

The total gate count of the ansatz is: $\sim 5N_t n^2/2$, with exponentially enlarged Hilbert space requires $\sim n^2/2$ control-phase gates.