Galois conjugates of string-net model

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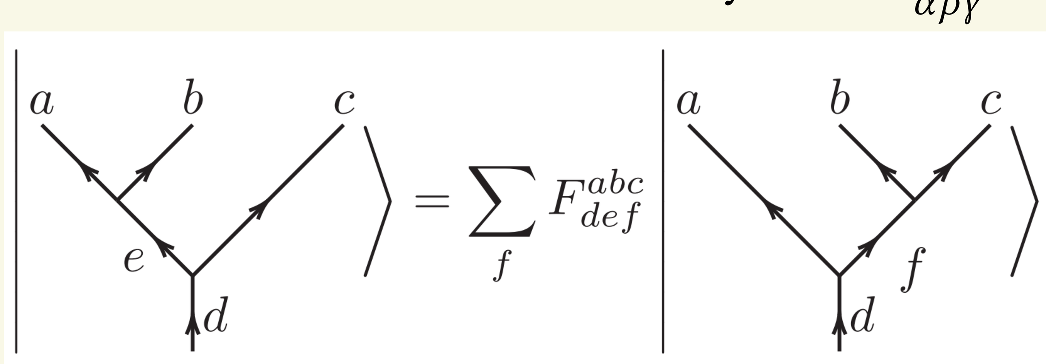
Introduction & background

- **Hermiticity** of a Hamiltonian is one of the key postulates in quantum mechanics.
  - Non-hermitian hamiltonians may have complex eigenvalues and can be used to describe open quantum systems.
  - Non-Hermitian Hamiltonians can also acquire real eigenvalues if they are "parity-time-reversal" (PT) symmetric and describe steady states in open systems.
- The **string-net model** is one important class of lattice models realizing $2+1$ d topological orders.
  - When the input data is a **unitary** fusion category, the resulting hamiltonian is hermitian.
  - A natural question : Can we obtain non-hermtian topological orders if we relax the unitary condition of the input data ?
- We studied a class of **non-hermitian string-net models**, the so-called galois conjugates of hermitian models:

Definition of string-net models

- The basic input data of a string-net model is a fusion category :
  - The objects of the category are called **string types**
  - Each string type i is assigned a quantum weight $v_i = \sqrt{d_i} \in \mathbb{C}$
  - Strings can fuse into new strings: $i \otimes j = \oplus_k N_{ij}^k \, k$
    The fusion rules also determines the F-symbol $F^{abc}_{\alpha\beta\gamma} \in \mathbb{C}$



  - When $(F^{abc}_{\alpha})_{\beta\gamma}$ is a unitary matrix, the corresponding fusion category is **unitary**.
- String-net models can be defined on any trivalent graphs
  - The Hilbert space is spanned by all labelings of graph edges with string types
- The hamiltonian is defined by operators A_v and B_p :

$$H = -\sum_v A_v - \sum_p B_p$$

  - A_v is defined on every vertex v to enforce fusion rules
  - B_p is defined on every plaquette p using F-symbol $F^{abc}_{\alpha\beta\gamma}$
- We **proved** that the **non-hermitian** H has **real spectrum** for a generic non-unitary fusion category.

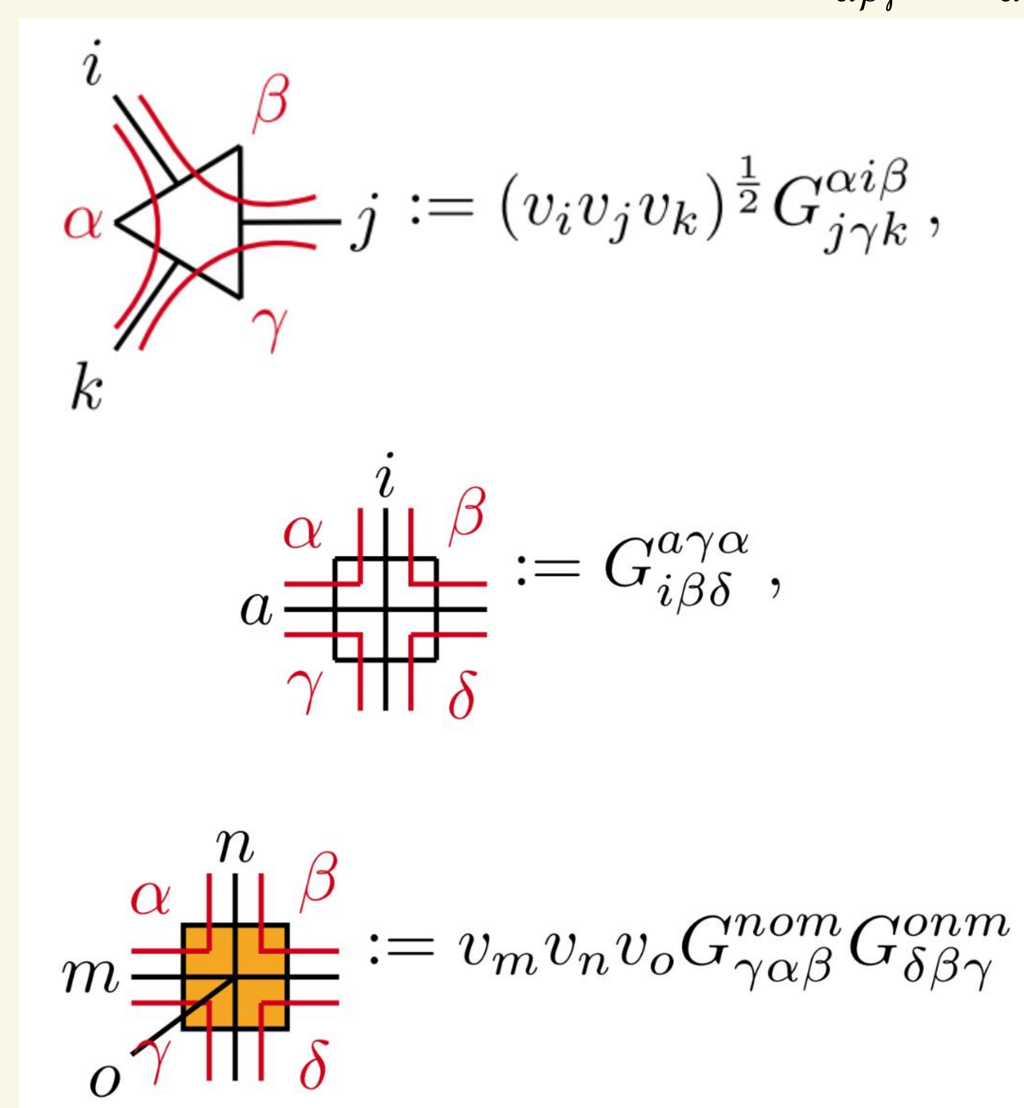
Galois conjugation of unitary string-net model

- Galois conjugation replaces a root of a polynomial by another one with identical algebraic properties :
  - i and $-i$ are galois conjugate of $z^2+1=0$
  - $\phi = (1+\sqrt{5})/2$ and $-1/\phi$ are galois conjugate of $z^2 - z - 1 = 0$
  - **Galois conjugate string-net model** replaces all **F-symbol** with their galois conjugate counterparts.
- E.g. The F-symbol of **Yang-Lee** theory is the galois conjugate of its unitary counterpart–**Fibonacci** theory.

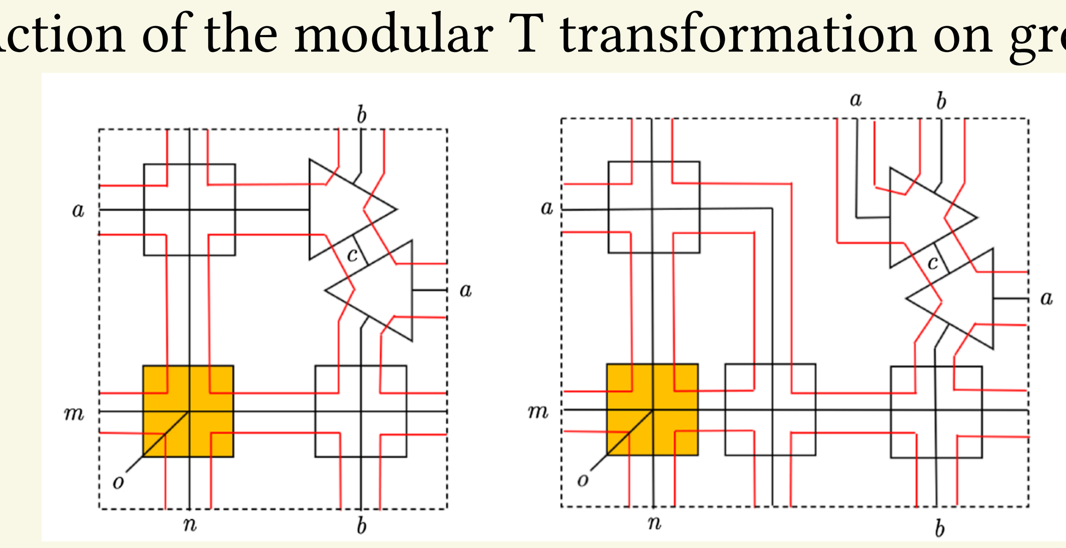
$$(F^{\tau\tau\tau}_{\tau})_{\text{Fib}} = \begin{pmatrix} \phi^{-1} & \phi^{2} \\ \phi^{-2} & -\phi^{-1} \end{pmatrix} \leftrightarrow (F^{\tau\tau\tau}_{\tau})_{\text{YL}} = \begin{pmatrix} -\phi^{1} & \phi^{-2} \\ \phi^{2} & \phi^{1} \end{pmatrix}$$

Topological data: Modular S, T Matrix

- We use tensor network (**TN**) methods to construct **biorthogonal basis** explicitly and futher extract the modular matrices.
  - The basic component tensors are : (where $G^{abc}_{\alpha\beta\gamma} = F^{abc}_{\alpha\beta\gamma}/v_\beta v_\gamma$)



  - left : TN representation of right eigen ground state $|T^n_{mom}\rangle$
    right : Action of the modular T transformation on ground states



  - T matrix twists the torus and its eigenvalues encode the topological spins h_i of the anyons.
  - S matrix rotates the torus by 90° and carries information of braiding between anyons.
- For Yang-Lee theory, the results are:

$$h_1 = h_4 = 0, \ h_2 = -\frac{2}{5}, \ h_3 = \frac{2}{5}$$

$$S = \frac{1}{\mathcal{D}} \begin{pmatrix} 1 & -\phi^{-1} \\ -\phi^{-1} & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & -\phi^{-1} \\ -\phi^{-1} & -1 \end{pmatrix}, \quad \mathcal{D} = 1 + d_\tau^2$$

Entanglement entropy

- We compute the entanglement entropy from the **generalized density matrix** constructed from the left/right eigenstates.
- We find that real part of the **topological entanglement entropy** is exactly the same as its hermitian counterpart :

$$Re[S_{\text{top}}] = -\log \mathcal{D}$$

Conclusion & outlook

- We show that modular matrices and entanglement entropies are **topological invariants**, exactly as their Hermitian counterpart
- Much more is needed to understand non-Hermitian topological orders and how to utilize their **topological robustness** in **open system experiments**.

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