Transformation theory for spatiotemporal metamaterials

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I Introduction

The transformation theory provides a distinct method for designing parameters in spatial dimensions[1-8]. However, with the introduction of time dimension, the transformation theory becomes particularly elusive because coordinate transformations apply only to static parameters. Here, we develop transformation theory for designing spatiotemporal metamaterials. In contrast to conventional static parameters, dynamic parameters may provide unique opportunities for achieving thermal functions with the additional asymmetric feature. Our spatiotemporal scheme[9] provides insights into particle or plasma diffusion and wave propagation.

II Theoretical analysis

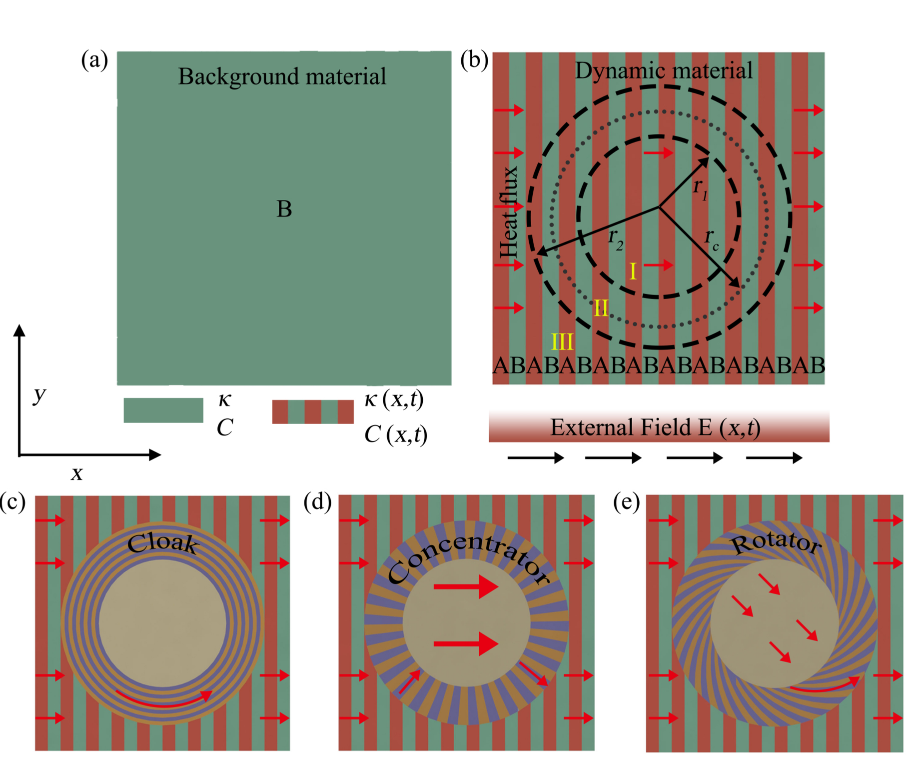

Fig. 1 Schematic diagram of spatiotempoal metamaterials.

As shown in Fig. 1 (a,b), material B (homogeneous background) with static thermal parameters can be converted into material A by an external field. Thus, we obtain a material with dynamic thermal parameters by driving the external field to move.

The Fourier law with spatiotemporally modulated parameters is

$$\frac{\partial\left(C\left(r,t\right)T\right)}{\partial t}+\nabla\cdot\left(-\kappa\left(r,t\right)\nabla T\right)=Q\left(r,t\right),$$

where *C(r, t)* is the product of heat capacity and density. We write it in a curvilinear space as

$$\partial_t\left(\sqrt{g}C\left(r,t\right)T\right)+\partial_i\left(-\sqrt{g}\kappa^{ij}\left(r,t\right)\partial_j T\right)=\sqrt{g}Q\left(r,t\right).$$

We rewrite it in the physical space as

$$\partial_t\left(\sqrt{g}C\left(r,t\right)T\right)+\partial_{i'}\left(-\sqrt{g}J_i^{i'}\kappa^{ij}\left(r,t\right)J_j^{j'}\partial_{j'}T\right)=\sqrt{g}Q\left(r,t\right),$$

where *J* is the Jacobian transformation matrix and $\sqrt{g}=\det^{-1}\mathbf{J}$.

We derive the transformation rules:

$$C'\left(r',t\right)=\frac{C\left(r,t\right)}{\det\mathbf{J}},\quad\kappa'\left(r',t\right)=\frac{\mathbf{J}\kappa\left(r,t\right)\mathbf{J}^{\dagger}}{\det\mathbf{J}},\quad Q'\left(r',t\right)=\frac{Q\left(r,t\right)}{\det\mathbf{J}}$$

III Simulation results

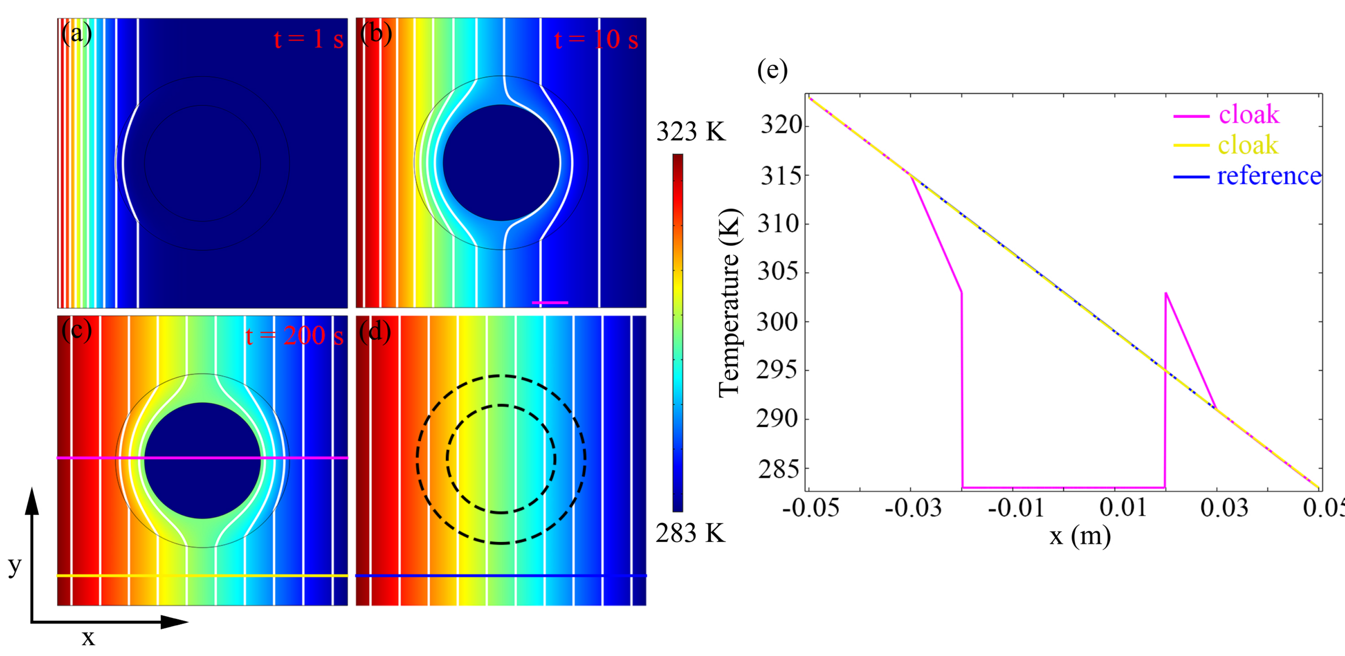

Fig. 2 Simulation results of spatiotemporal thermal cloak.

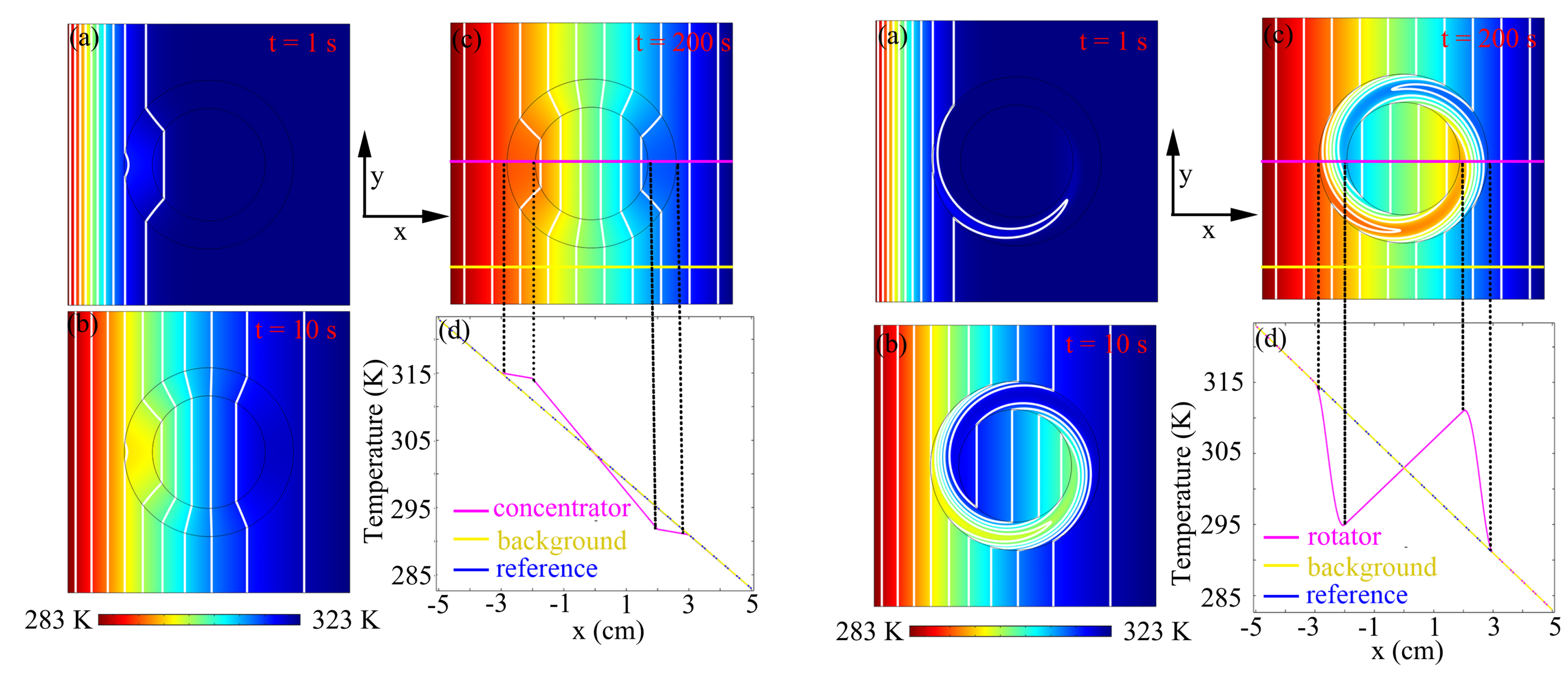

Fig. 3 Simulation results of spatiotemporal thermal concentrator (left) and thermal rotator (right).

Fig. 2-3 are simulation results of designed spatiotemporal thermal cloak, concentrator and rotator. For experimental demonstration, many studies suggest that thermal conductivity and heat capacity can be regulated by external fields, such as electric fields and light fields. Therefore, we can realize spatiotemporal transformation metamaterials by applying appropriate external fields to target regions.

IV Conclusion

- We develop the transformation theory and proposed the concept of spatiotemporal transformation metamaterials.
- The simulated results demonstrate the robustness of the proposed theory.
- The work provides ways to design dynamic parameters, promotes applications of spatiotemporal metamaterials.

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