Multiple control of thermoelectric dual-function metamaterials

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Introduction

Thermal metamaterials [1,2] based on transformation theory have been developed to control heat flow by manipulating material parameters. However, existing designs [3] are limited to single target functions within a specific domain. In this study [4], we prove the form invariance of thermoelectric (TE) governing equations, allowing precise control of thermal flux and electric current. We propose a dual-function metamaterial that can simultaneously concentrate (or cloak) and rotate the TE field. We introduce two practical control methods: a temperature-switching TE rotating concentrator-cloak and an electrically controlled TE rotating concentrator. Theoretical predictions and simulations confirm the effectiveness of our approach. This work establishes a unified framework for manipulating the direction and density of the TE field, contributing to thermal management applications such as rectification and diode functionality.

Transformation thermoelectric (TE) theory

- **TE domain equations:**

$$0 = \nabla \cdot (\boldsymbol{\sigma}\nabla\mu + \boldsymbol{\sigma S}\nabla T)$$

$$0 = \nabla \cdot [\boldsymbol{\kappa}\nabla T + T\boldsymbol{S}^{\tau}\boldsymbol{\sigma S}\nabla T + T\boldsymbol{S}^{\tau}\boldsymbol{\sigma}\nabla\mu] + \nabla\mu \cdot [\boldsymbol{\sigma}\nabla\mu + \boldsymbol{\sigma S}\nabla T]$$

- **Transformation rules:**

$$\begin{cases} \tilde{\boldsymbol{\kappa}} = \dfrac{A\boldsymbol{\kappa}' A^{\tau}}{det\ (A)} \\ \tilde{\boldsymbol{\sigma}} = \dfrac{A\boldsymbol{\sigma}' A^{\tau}}{det(A)} \\ \tilde{\boldsymbol{S}} = A^{-\tau}\boldsymbol{S}' A^{\tau} \end{cases}$$

These rules guarantee the invariance of the TE coupling equation after coordinate transformation and guides us to design a series of thermal metamaterials.

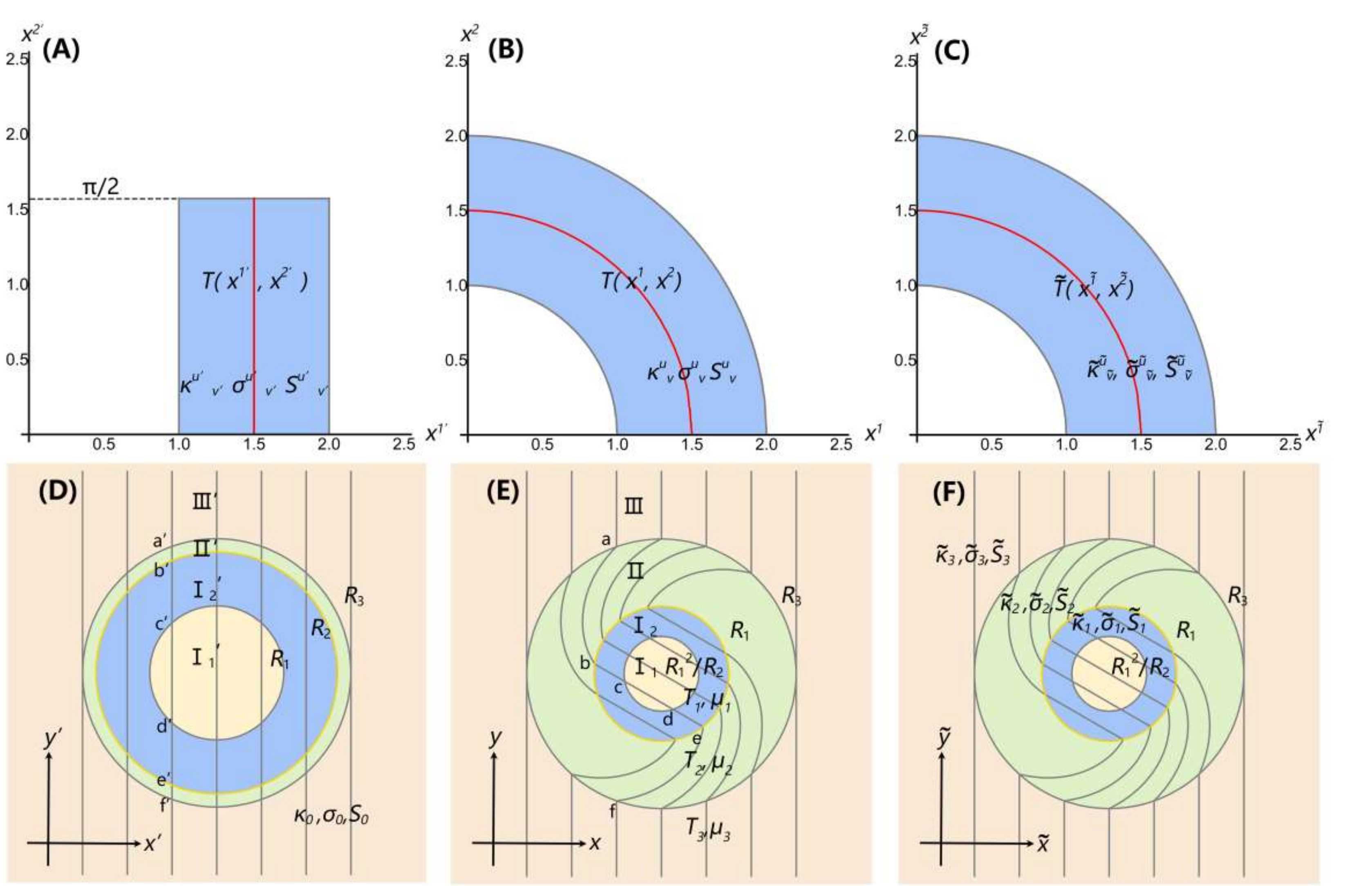

Figure 1. Schematic diagram of the coordinate transformation process in cylinder structure. (A)(D)(C)(F) Cartesian coordinates. (B)(E) Curvilinear coordinates. The isotherm shape in (C)(F) is the same as that in (B)(E), indicating that the effect of parameters' transformation is equivalent to coordinate transformation.

TE rotating concentrator and cloak

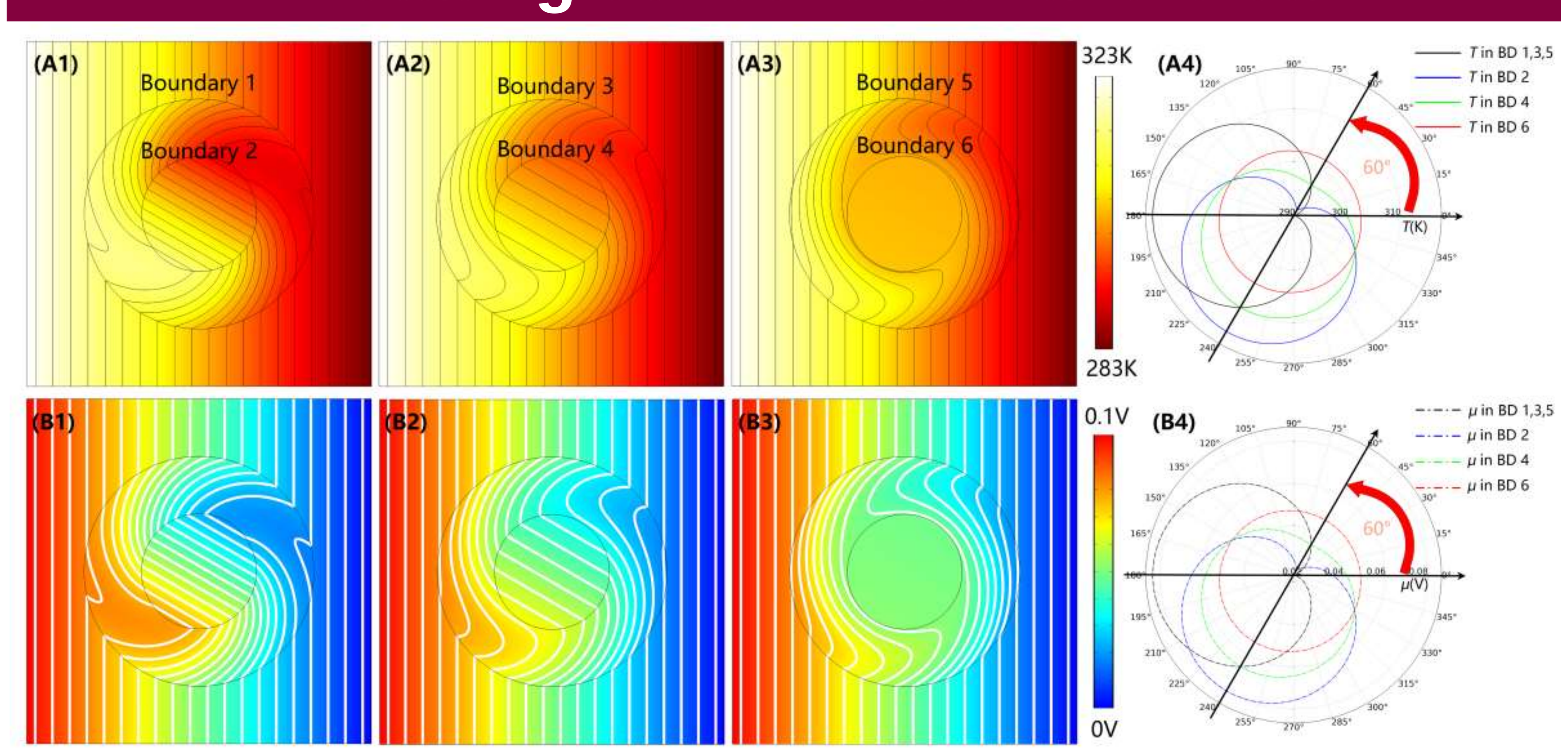

Figure 2. Simulative results of (A1)(B1) rotating cloak, (A2)(B2) rotator, and (A3)(B3) rotating concentrator. (A4) and (B4) respectively denote the temperature and voltage distribution along corresponding boundary (BD).

Temperature-switching TE rotating concentrator cloak

The nonlinearity phenomena are common, and their underlying mechanisms are significant for understanding and designing complex systems. Generally, the thermal conductivities of natural materials are basically dependent on temperature (nonlinear), which provides a hint for function switching at different temperatures. Inspired by this concept, we introduce temperature into geometrical transformation relations,

$$R_2 = \frac{R_3}{1 + \mathrm{Exp}[\alpha(T - T_c)]}$$

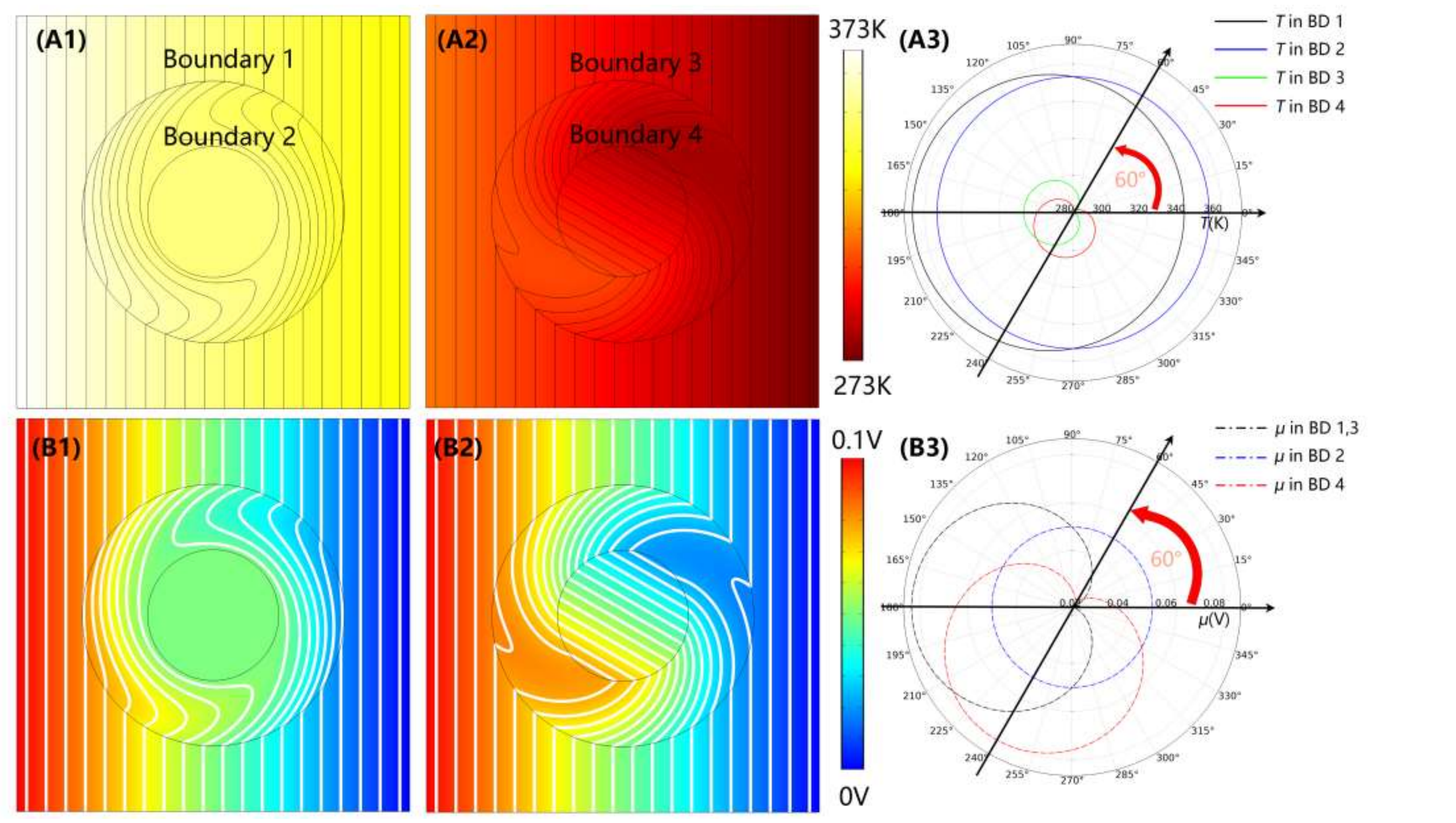

Figure 3. Simulative results of TE rotating cloak concentrator under different external temperatures (T_L and T_R).

Electrically controlled TE rotating concentrator

The external voltage is another manipulating method for temperature distribution. By introducing a generalized auxiliary potential $U = \mu + ST$, the TE domain equations reduce to

$$\begin{cases} \sigma\nabla^2 U = 0, \\ \kappa\nabla^2 T = \sigma\nabla U \cdot \nabla U. \end{cases}$$

The first equation is the Laplace equation with respect to U, which can be easily calculated from

$$U = \frac{U_R - U_L}{L}x' + \frac{U_R + U_L}{2}, \\ = \frac{S(T_R - T_L) + \mu_R - \mu_L}{L}x' \\ + \frac{S(T_R + T_L) + \mu_R + \mu_L}{2}.$$

The latter is a Poisson equation, which has the particular solution,

$$T = \frac{-\sigma U^2}{2\kappa},$$

where the first term is generated by external heat sources and the other term is generated by the TE effect. When the difference of external voltage is low, the TE effect can be ignored, so the temperature distribution is regulated only by transformation theory [Figure 4(A1)]. When the difference increases, the TE effect increases gradually, heating all regions [Figure 4(B1)]. When the difference is relatively high, the TE effect dominates, even leading to maximum temperature higher than that of external sources [Figure 4(C1)].

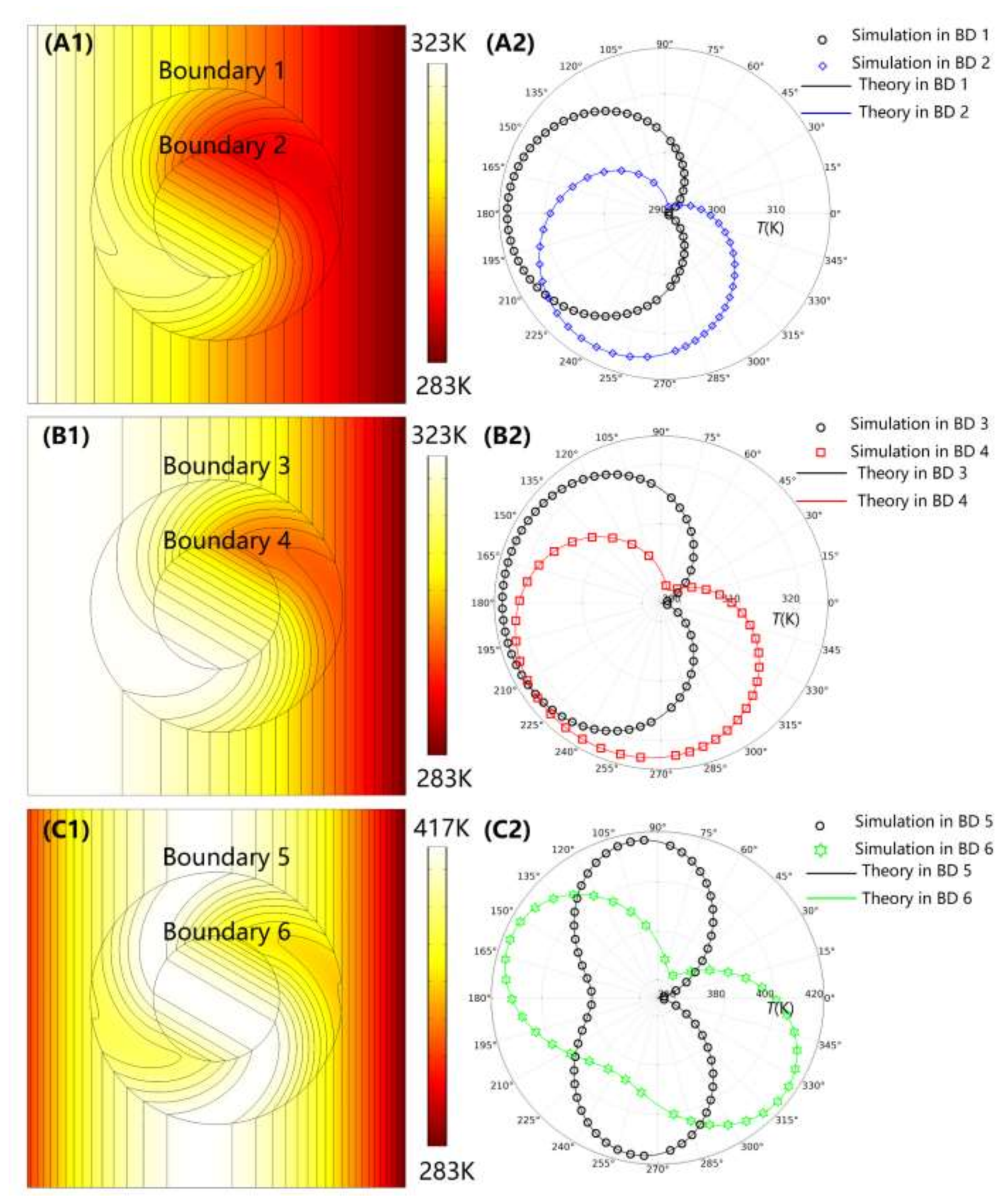

Figure 4. (A1)(B1)(C1) The temperature fields of TE rotating concentrator under different external voltages (μ_L and μ_R). (A2)(B2)(C2) respectively present the temperature distribution along corresponding boundary.

Conclusions

- We systematically demonstrate a unified framework to regulate the direction and density of heat flux (electric current).
- A dual-function metamaterial that can concentrate (or cloak) and rotate the thermoelectric at the same time is designed.
- To further manipulate the thermoelectric field, we introduce additional degrees of freedom into the metamaterial design: one is temperature-dependent thermal conductivity, and the other is the difference of external voltage.

References:

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