# Non-Bloch Theory for Spatiotemporal Photonic Crystals Assisted by Continuum Effective Medium


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## Background and Motivation

1. In lattice models, non-reciprocal hopping amplitudes lead to non-Hermitian skin effects (NHSE). What about non-reciprocal electromagnetic materials?
2. Higher spatial dimensions bring novel non-Hermitian physical phenomena. What about the time dimension?

- As a novel type of non-reciprocal material that satisfies the above two requirements, what are the non-Bloch properties of spacetime-modulated materials (STM)?

### Non-reciprocal hopping brings NHSE

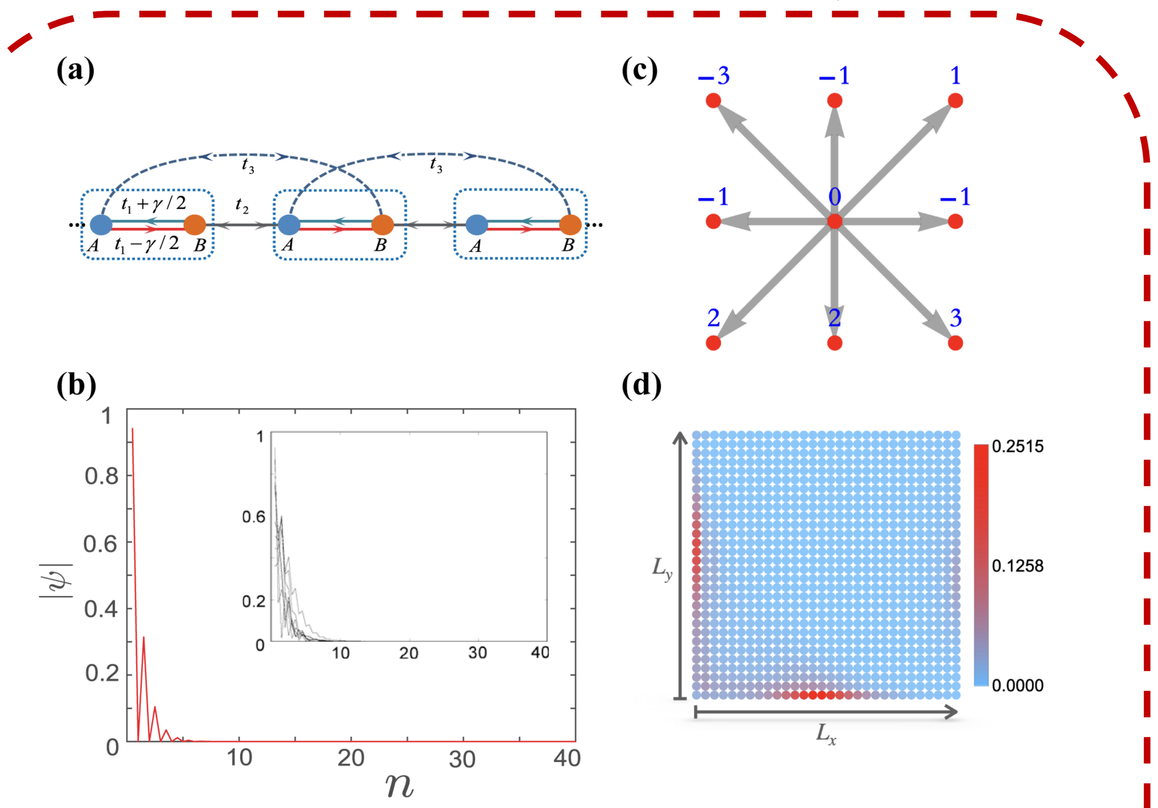



**Figure 1. The lattice model with non-reciprocal hopping and its energy spectrum under open boundary conditions (OBC). (a) and (b) are the one-dimensional cases, while (c) and (d) are the two-dimensional cases.**

### Geometry dependent skin effect occurring in higher dimensions

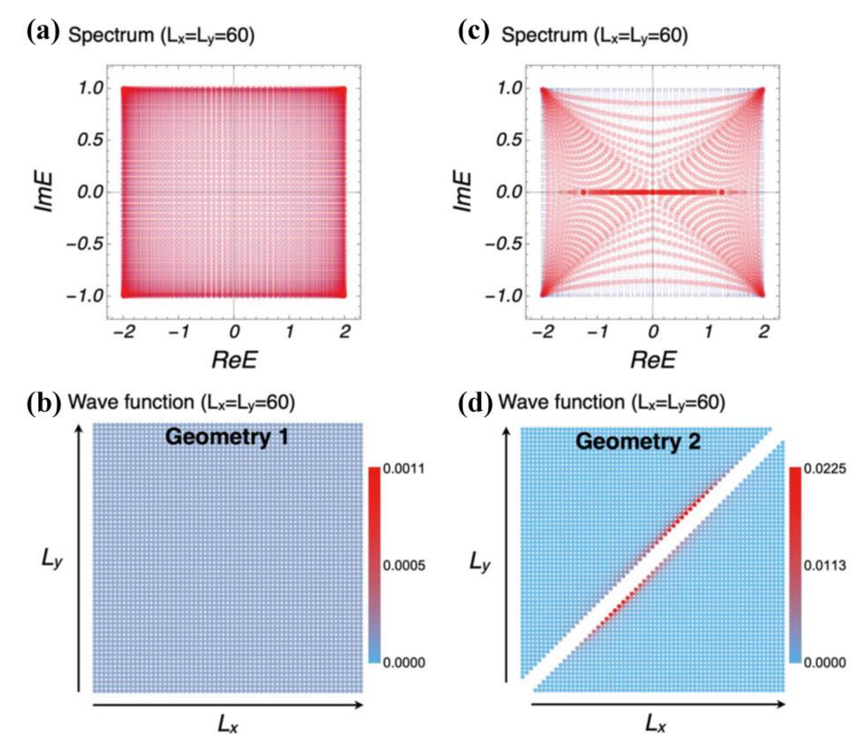
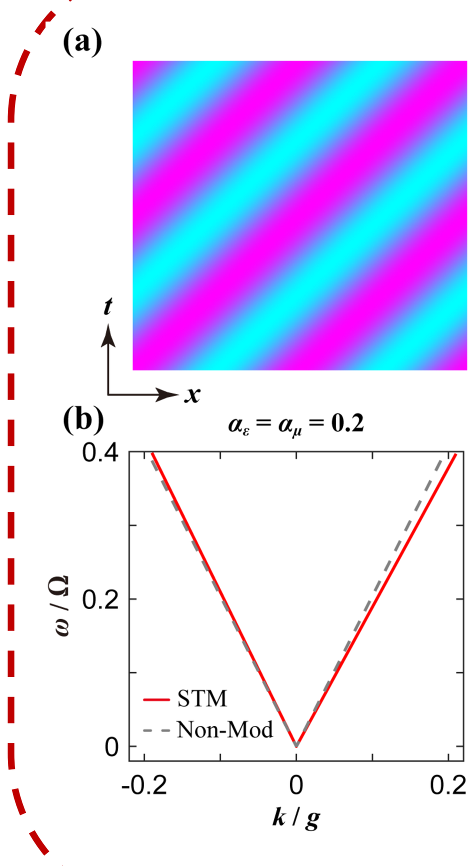



**Figure 2. The OBC energy spectrum and eigenstates under different boundary shapes. (a) and (b) are for a square shape, while (c) and (d) are for a triangular shape.**

### A novel non-reciprocal Medium: spacetime-modulated materials




- Constitutive relation

$$\varepsilon = \varepsilon_r\left[1 + 2\alpha_\varepsilon \cos(gx - \Omega t + \phi_\varepsilon)\right]$$

$$\mu = \mu_r\left[1 + 2\alpha_\mu \cos(gx - \Omega t + \phi_\mu)\right]$$

**Figure 3. (a) Space-time distribution of material parameters. (b) Low-frequency dispersion of the STM, with the gray curve representing the material dispersion without modulation.**

## Results and methods

### The effective medium of spacetime-modulated materials

- Constitutive relation of effective medium

$$\begin{pmatrix} \boldsymbol{D} \\ \boldsymbol{B} \end{pmatrix} = \begin{pmatrix} \boldsymbol{\varepsilon}_{\text{eff}} & \boldsymbol{\eta}_{\text{eff}} \\ \boldsymbol{\zeta}_{\text{eff}} & \boldsymbol{\mu}_{\text{eff}} \end{pmatrix}\begin{pmatrix} \boldsymbol{E} \\ \boldsymbol{H} \end{pmatrix}$$

$$\boldsymbol{\varepsilon}_{\text{eff}} = \text{Diag}(\varepsilon_{\text{eff},x}, \varepsilon_{\text{eff},x}, \varepsilon_{\text{eff},x})$$

$$\boldsymbol{\mu}_{\text{eff}} = \text{Diag}(\mu_{\text{eff},x}, \mu_{\text{eff},x}, \mu_{\text{eff},x})$$

$$\boldsymbol{\eta}_{\text{eff}} = \boldsymbol{\zeta}_{\text{eff}}^{\text{T}} = \begin{pmatrix} & & \\ & & \xi_{\text{eff}} \\ & -\xi_{\text{eff}} & \end{pmatrix}$$

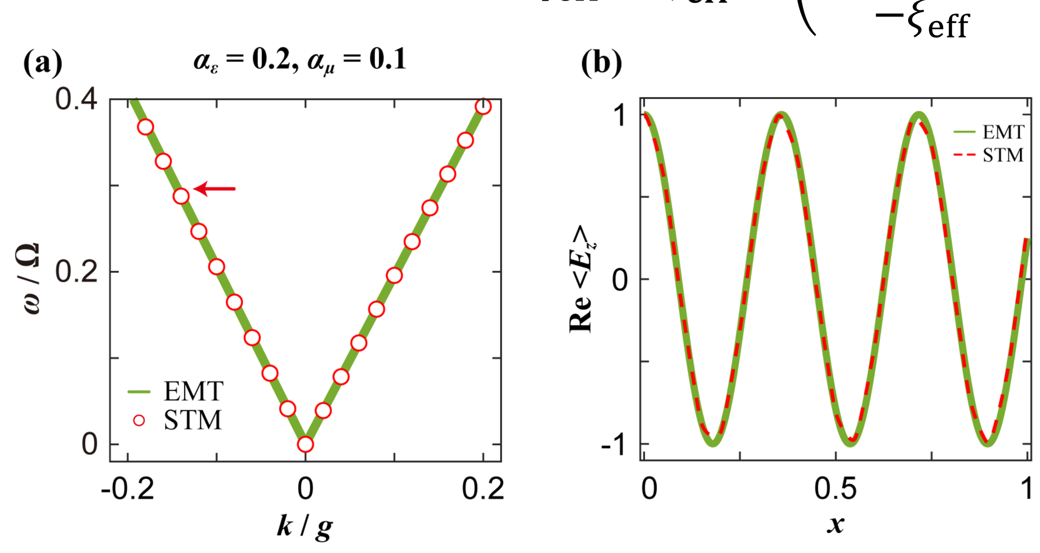



**Figure 4. Comparison of numerical results between the effective medium and the STM. (a) Dispersion relation, (b) Eigenfield.**

### The non-Bloch properties of spacetime-modulated materials

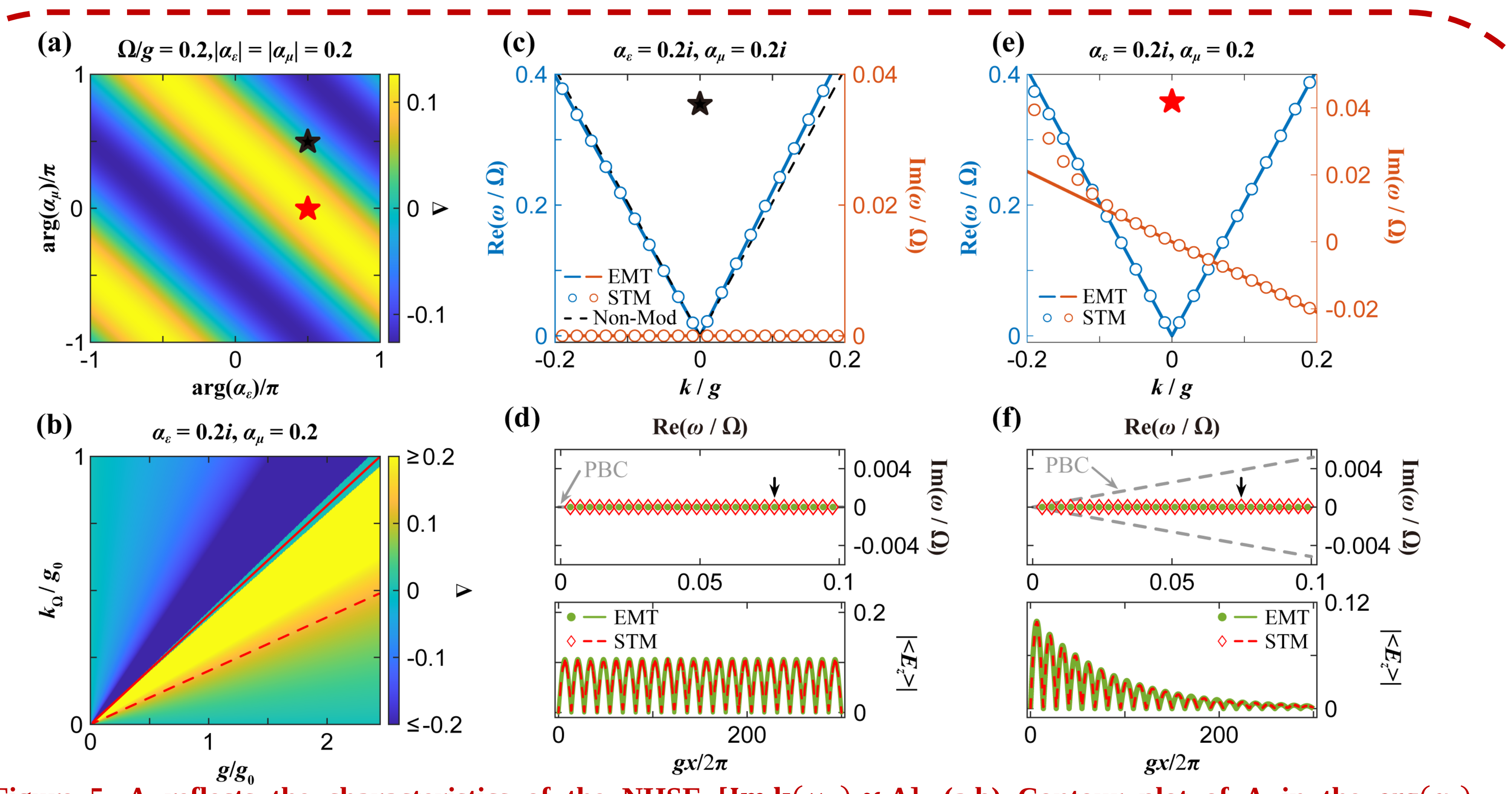
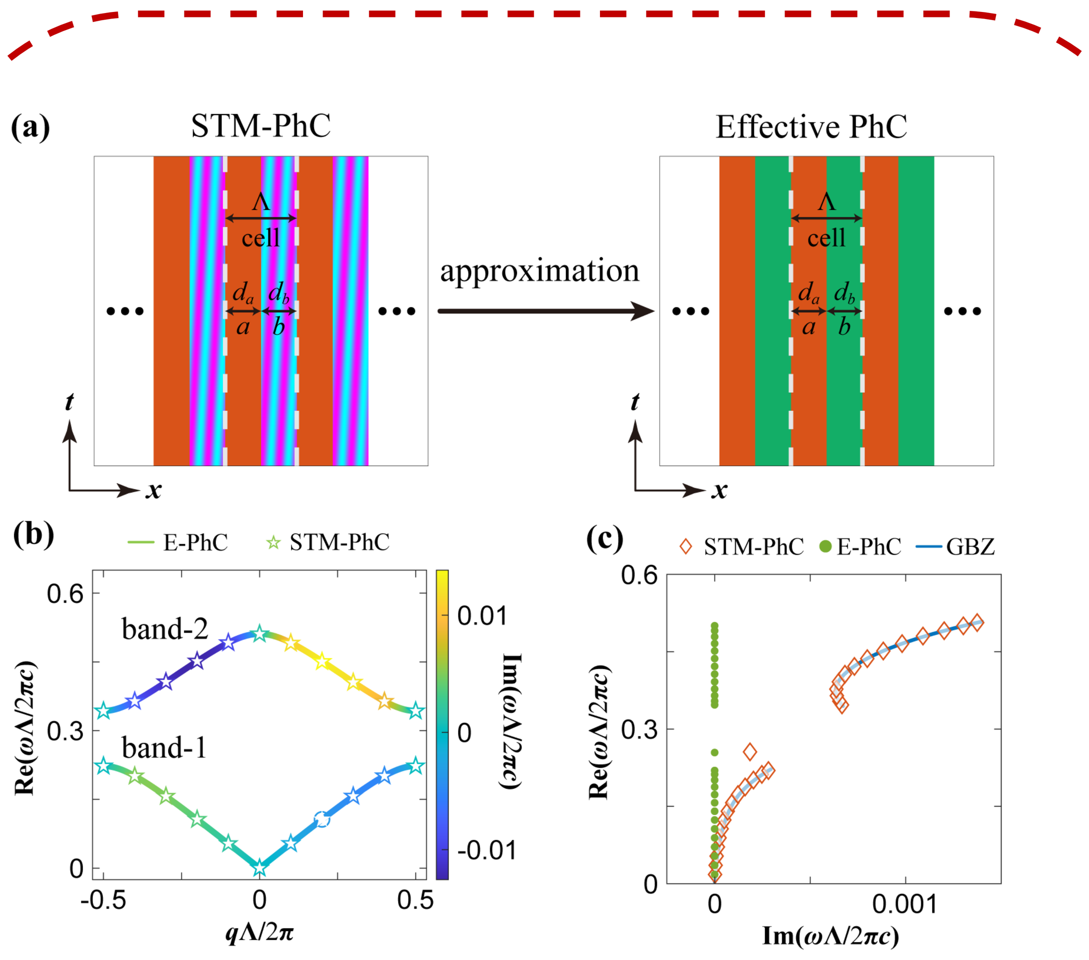
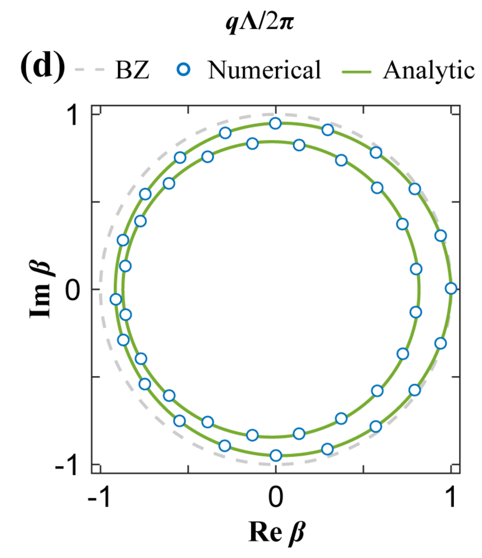



**Figure 5. Δ reflects the characteristics of the NHSE [Im k(ω₀) ∝ Δ]. (a,b) Contour plot of Δ in the $\arg(\alpha_\varepsilon) - \arg(\alpha_\mu)$ plane (a) and the $g - k_\Omega$ plane (b). (c,e) The PBC BS in the long-wavelength limit when (c) $\alpha_\varepsilon = \alpha_\mu = 0.2i$ and (e) $\alpha_\varepsilon = 0.2i, \alpha_\mu = 0.2$. (d,f) The OBC spectra (top) and fields (bottom) when (d) $\alpha_\varepsilon = \alpha_\mu = 0.2i$ and (f) $\alpha_\varepsilon = 0.2i, \alpha_\mu = 0.2$.**

### The eigenenergy spectra and GBZ of spacetime-modulated photonic crystals




- Hermitization

Transformation

$$\hat{L}\psi(x) = \frac{\omega}{c}\kappa_{\text{E}}(x)\psi(x) \longrightarrow \hat{L}_s Q(x) = \frac{\omega}{c}Q(x)$$

Analytical GBZ formula:

$$\beta_{n_i}(q) = \exp\left[i\int_{uc}\frac{\omega^s_{n_i}(q)}{c}\xi_{\text{E}}(\nu)d\nu + iq\Lambda\right]$$

▲ $\omega^s_{n_i}(q)$ is the PBC energy of $\hat{L}_s$

**Figure 6. (a) Schematic of the approximation of the effective photonic crystal. (b) its PBC band structure, (c) its OBC energy spectrum. (d) Comparison between numerical results based on STM-PhC and analytical formulas based on E-PhC for the GBZ.**

### The topological properties of spacetime-modulated photonic crystals

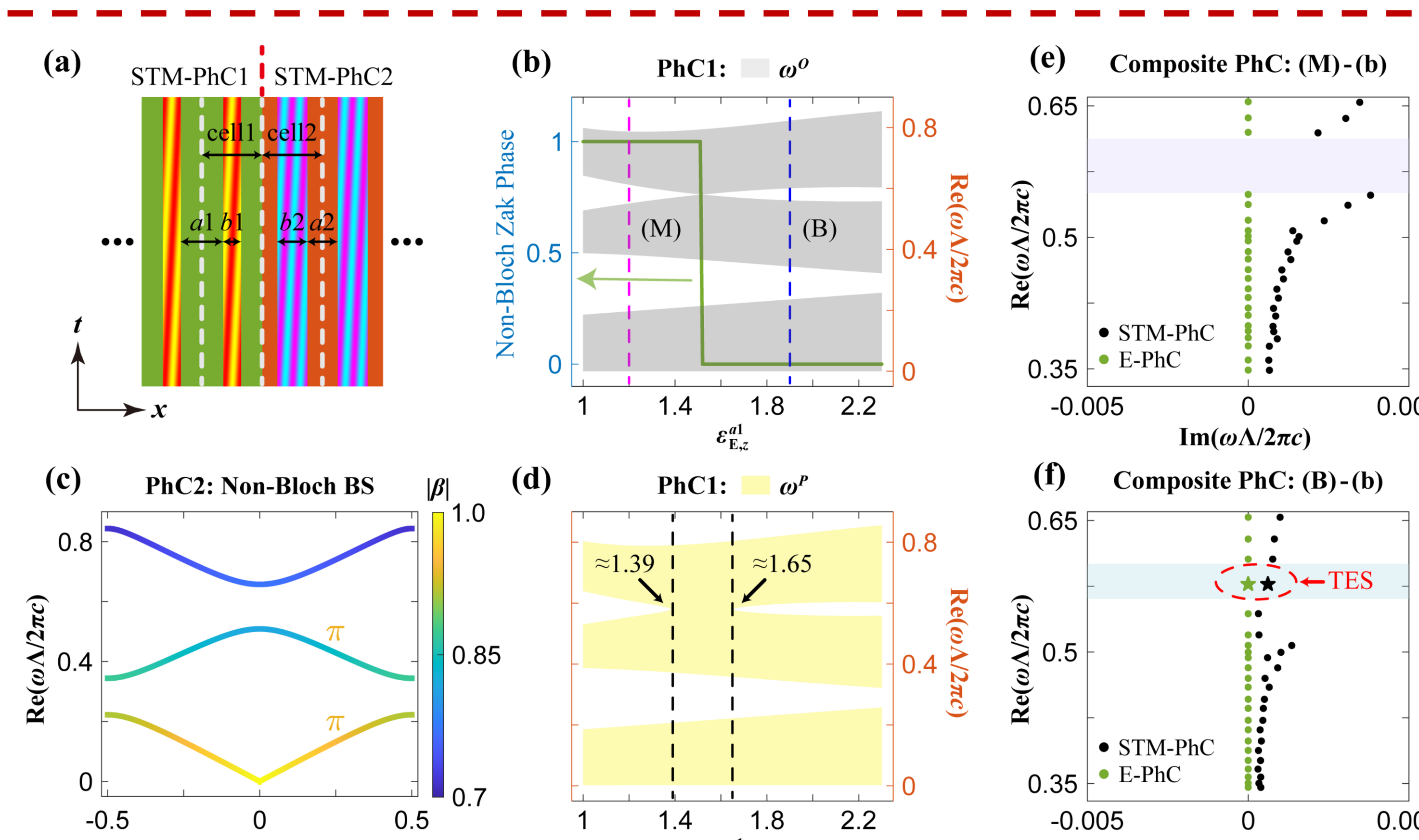



**Figure 7. (a) Schematics of a domain wall formed by two STM-PhCs. (b) The non-Bloch BS of the STM-PhC2, and the non-Bloch Zak phase of each non-Bloch band is indicated nearby. (c) The non-Bloch BS and non-Bloch Zak phase (left y-axis) and the OBC spectra (right y-axis) of the STM-PhC1. (d) The PBC spectra of the STM-PhC1. (e,f) The OBC spectra of the composite PhCs. The PhC in (e) [(f)] is composed of the one defined in (b) and (M) [(B)] marked in (c).**

## Conclusions

1. The EMT extends the non-Bloch band theory, allowing us to directly investigate the non-Bloch properties of spatiotemporal modulation materials.
2. The EMT can help us analytically solve the GBZ of complex system.

## References


1. S. Yao and Z. Wang, Phys. Rev. Lett. 121, 086803 (2018).
2. K. Zhang, Z. Yang, and C. Fang, Nat. Commun. 13, 2496 (2022).
3. H. Ding and K. Ding, arXiv:2401.12536 (2024).