A directionally radiating antenna is highly desirable, in order to save unnecessary energy loss along unwanted radiation directions\[^1\]. A simple method is to put a ground plane on the back of an antenna to shield the backward radiations of the antenna, for example, using photonic crystal(PhC) can significantly modulate the radiation efficiency and directivity of the antenna\[^2\]. However, due to the Bragg mechanism\[^3\] such systems are typically bulky for microwave applications. Recently, there has been much interest to employ metamaterials to control the radiation behaviors of antennas\[^4, 5\]. For instance, directional emission can be achieved by simply putting a point source inside a metamaterial with zero refractive index\[^4\], or inside a subwavelength cavity formed by specifically designed metamaterials\[^5\]. However, similar to PhCs, such systems are still bulky and complex, which limit their applications in microwave technologies. Based on the concept of transformation optics, an antenna substrate has been designed to support high radiation directivity and efficiency\[^6\], but the complicated distributions of permittivity and permeability make it difficult to realize in practice.

In this paper, taking a single sheet of planar metamaterial as an antenna ground plane, we applied a dyadic Greens function(GF) approach to analytically study the radiation properties of small antennas put vertically or horizontally on the ground plane, and analyzed the conditions under which the antenna would radiate directionally and efficiently. We found that the metamaterial ground plane should exhibit certain incidence angle-dependent reflection phases in order to support directional emissions. Our theory can be employed to explain a previous experiment for the horizontal antenna geometry\[^7\], and guides us to design a realistic structure to support directional emissions in the vertical antenna geometry, which is subsequently demonstrated by our experiments as shown in the following figure\[^8\].

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