A theoretical study on the conversion efficiencies of gradient meta-surfaces

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Motivations:
In a recent work (Sun S. et al., Nat. Mater., 11 (2012) 426), nearly 100\% efficiency of the conversion of an incident propagating wave (PW) to an obliquely outgoing PW or even a surface wave (SW) is demonstrated in an ideal gradient meta-surface (GM).

However, practical systems might have non-equal and \( \varepsilon_r(x) \) and \( \mu_r(x) \) profiles and sometimes use supercells to truncate the profiles to avoid using too large values of \( \varepsilon \) and \( \mu \).

Here, based on non-ideal GM systems, we systematically studied the factors that influence the efficiencies of such conversion processes (both PW-PW and PW-SW).

I. Efficiency issues of model GMs

(A) Methods

\[ R_{\text{PW-PW}} = \sin^2 \left( \frac{k_d}{\varepsilon_r/k_\theta} \right) \]

- To study the conversion efficiency for the PW-PW process, we calculate the reflectance:

\[ R_{\text{PW-PW}} = \rho_k \sqrt{1 - \left( \frac{k_d}{k_\theta} \right)^2} \]

- For the PW-SW process, \( k_{SW} = \xi > k_\theta \).

Use an eigen-SPP guide material with \( k_{SW} = k_{GM} \) and calculate the power flow ratio as the PW-SW conversion efficiency.

(B) PW-PW conversion efficiency

- How to generate the systems we want to study:

We assume \( \varepsilon_r(x) = 1 + \alpha \cdot \varepsilon_\theta x / 2k_\theta d \) and then retrieve \( \mu_r(x) \) by letting the calculated reflection phase

\[ \Phi(x) = \cos^{-1} \left( \frac{\varepsilon_r(x) + \mu_r(x)}{\varepsilon_r(x) \mu_r(x)} \right) \]

satisfy the given

\[ \Phi(x) = \Phi_0 + \xi x \]

\( \alpha \in [0, 1] \), a parameter to measure the degree of impedance mismatch: from completely impedance-mismatched to impedance-matched

(C) PW-SW conversion efficiency

\[ R_{\text{PW-SW}} = \frac{1}{2} \cdot \frac{1}{1 + \beta} \cdot \frac{1}{1 - \beta} \]

Where \( \beta = \frac{\mu_r(x)}{\varepsilon_r(x)} \)

II. An improved model GM with enhanced efficiency

- The GM system working as a PW-SW converter can work even with a very small total length.

III. Conversion efficiencies of realistic GM systems

The PW-PW conversion efficiency for a realistic GM

\[ R_{\text{PW-PW}} = \frac{\int_{\Omega} P_i(\Omega) d\Omega}{\int_{\Omega} P_0(\Omega) d\Omega} \]

Conclusions:

- We know the key factor affecting the conversion efficiency is the super periodicity scattering through the study of model GM.
- Our improved model GM can describe the realistic GM better.
- Probable application in miniaturized situations where a grating coupler is not suitable.

We found that while intra-supercell impedance-mismatch can hardly affect the conversion efficiencies, the scatterings caused by inter-supercell discontinuities can have non-negligible effects on the PW-SW conversion efficiency.

References: