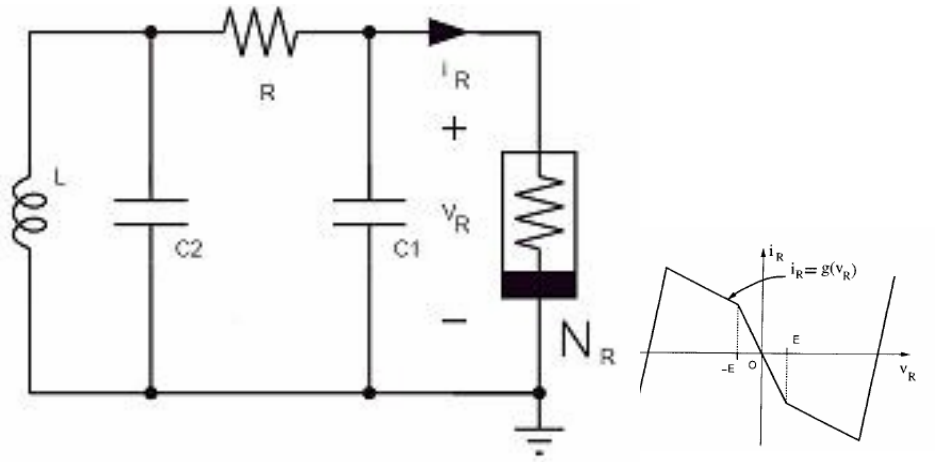
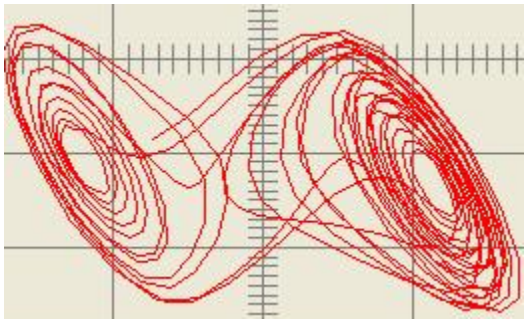
Chua's Circuit: The Paradigm for Generating Chaotic Attractors



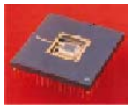


EE129 Fall 2007

Bharathwaj Muthuswamy
NOEL: Nonlinear Electronics Lab
151M Cory Hall
Department of EECS
University of California, Berkeley
mbharat@eecs.berkeley.edu

Outline

- Introduction to Chaos
  - What is Chaos?
  - Chaos in Nature
  - Hallmarks of Chaos
    - Non-periodic behavior in time domain
    - Sensitive dependence on initial conditions
- Proving the existence of Chaos
- Easy Chaos: Chua's circuit
- Building and working with Chua's circuit
  - References
  - Simulating Chua's circuit: MultiSim
  - Turning your PC into an oscilloscope: Osqoop
  - Chua's circuit for high school students
  - Interesting MATLAB experiments
- Questions

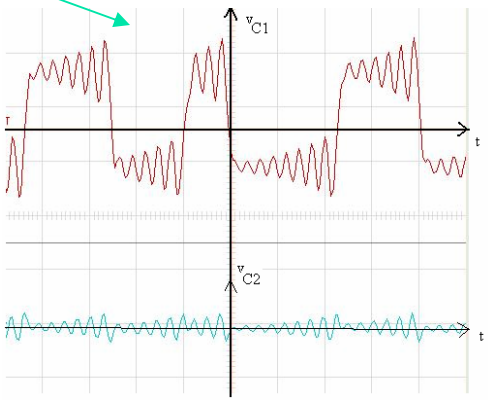


Introduction to Chaos: What is Chaos?

- There is **NO universal agreed-upon definition of Chaos**
- Loosely speaking, a Chaotic system is a deterministic system that exhibits random behavior. Example - Chua's circuit:

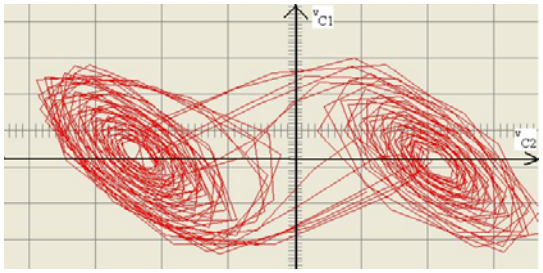
Set of ordinary **differential equations** with a **simple nonlinearity** BUT the **system behavior is complex**:

$$C_1 \frac{dv_{C_1}}{dt} = G(v_{C_2} - v_{C_1}) - g(v_{C_1})$$
$$C_2 \frac{dv_{C_2}}{dt} = G(v_{C_1} - v_{C_2}) + i_L$$
$$L \frac{di_L}{dt} = -v_{C_2}$$
$$g(v_R) = m_0 v_R + \frac{1}{2}(m_1 - m_0)\left[|v_R + B_p| - |v_R - B_p|\right]$$



Time domain

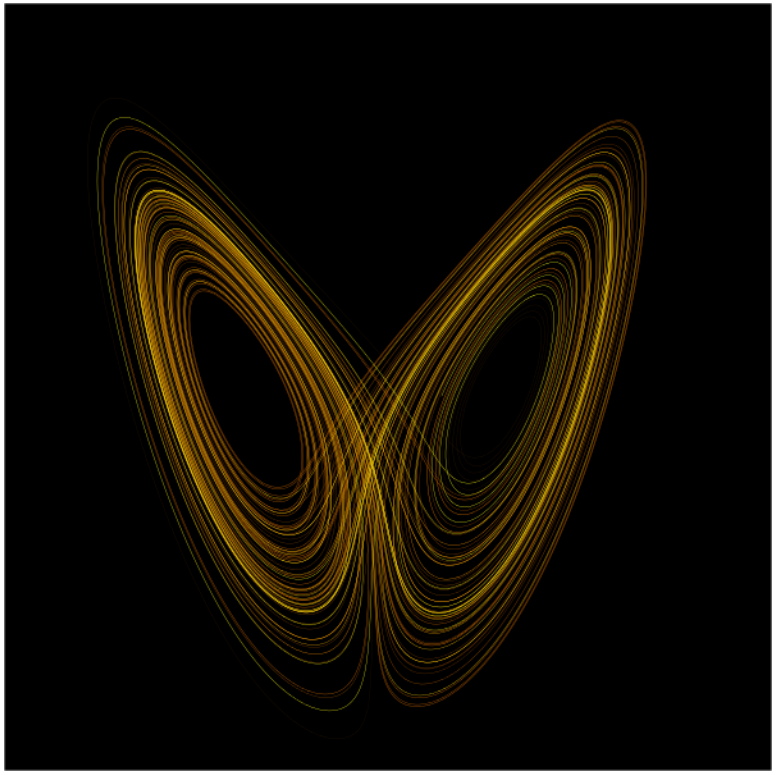
Phase space



Chaos in Nature

- Weather

(Reference: "Deterministic Nonperiodic Flow". Lorenz, Edward N. *Journal of Atmospheric Sciences*. pp. 130 – 141, 1963)



A plot of the Lorenz attractor for r = 28, σ = 10, b = 8/3
(Reference: Chaos Theory, Wikipedia Entry. Online at: http://en.wikipedia.org/wiki/Chaos_theory)

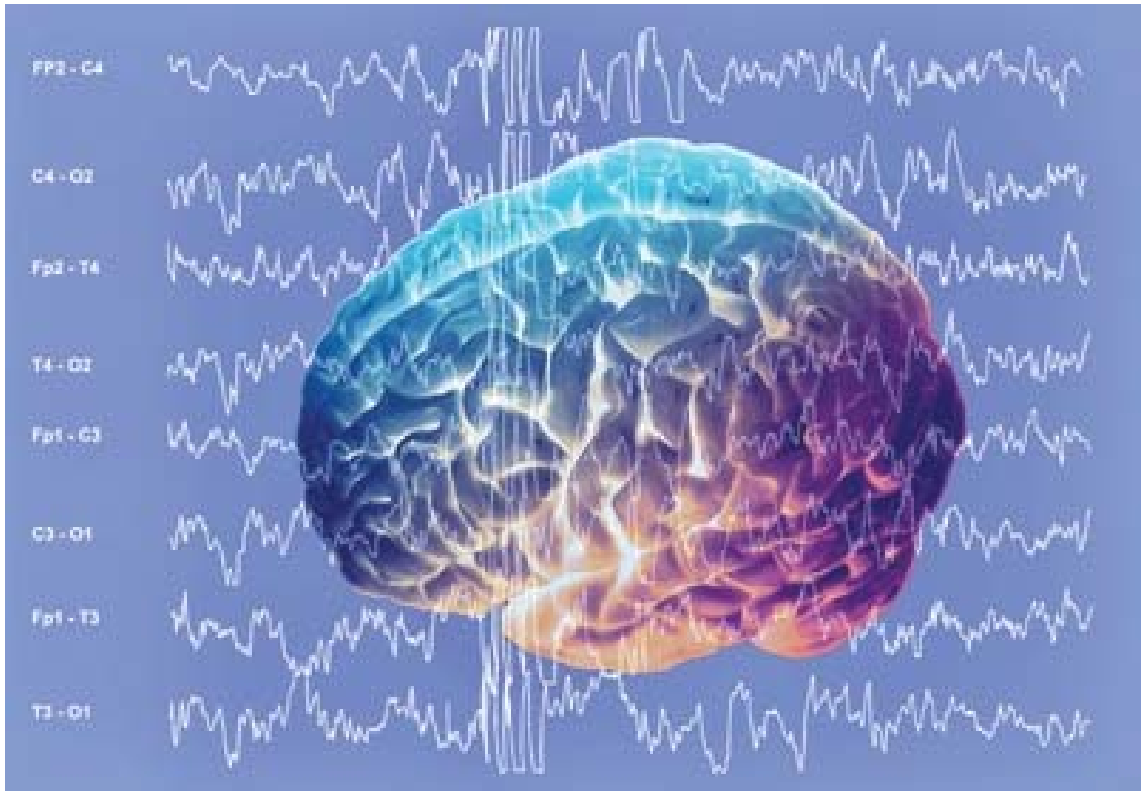
Chaos in Nature

- Trajectory of planetary orbits:
  (Reference: "The role of chaotic resonances in the Solar System". Murray, N. and Holman M. *Nature.* pp. 773 – 780, vol. 410, 12 April 2001)
  - Irregulary shaped satellites like Hyperion (moon of Saturn) tumble chaotically.
  - Chaos in the orbits of giant planets (Jupiter, Saturn and Uranus) – the location of these planets cannot be predicted on a time scale longer than a few tens of millions of years.
  - Quote from the paper above:

"The worried reader
may find some comfort in that the
accompanying analytic theory
predicts that no planet will be ejected
before the Sun dies."

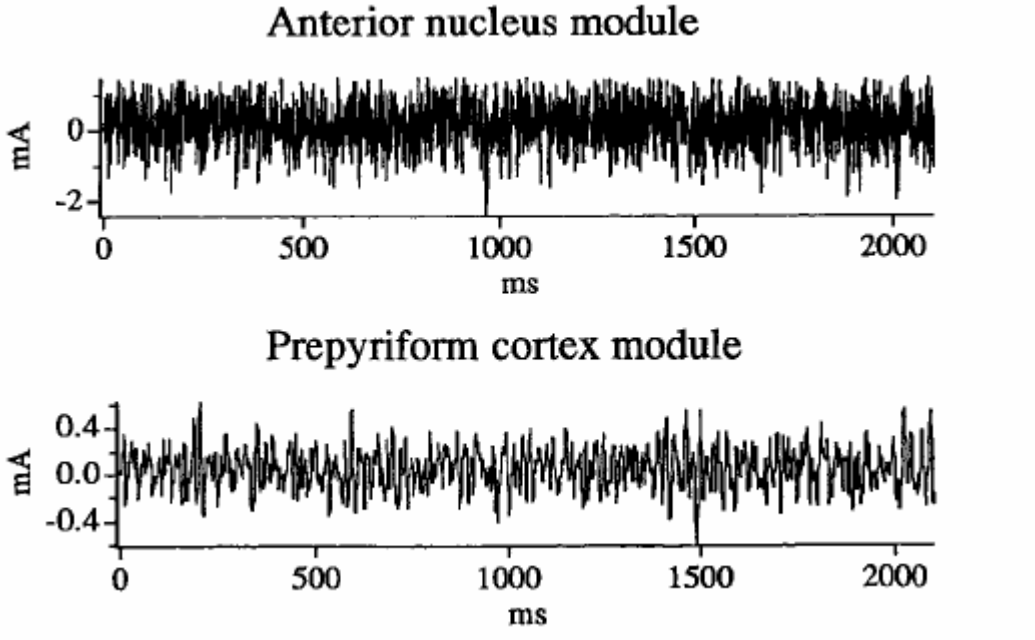
Chaos in Nature

- Brain waves
  (Reference: Rhythms of the Brain. Buzsaki, Gyorgy. *Oxford University Press*. 2006)



Hallmarks of Chaos

- **Non-periodic behavior in the time domain**

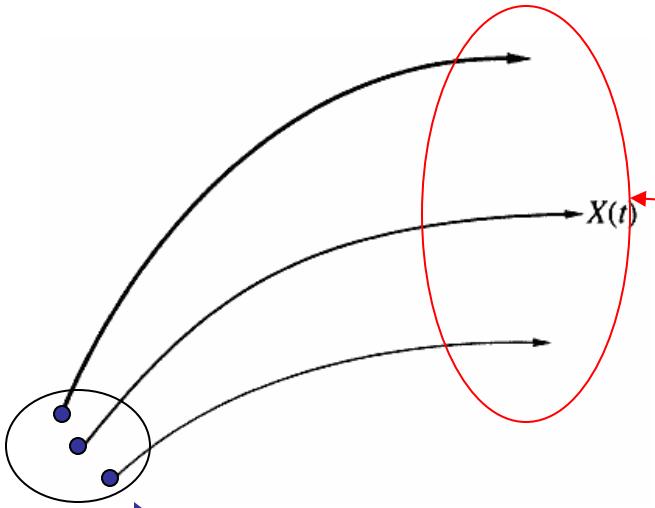


Time series data from neural nodes implanted in rat cortex

Reference: "Taming Chaos: Stabilization of Aperiodic Attractors by Noise". Freeman, W. et. al. *IEEE Trans. On Circuits and Systems – I: Fundamental Theory and Applications.* Vol. 44, No. 10, Oct. 1997

Hallmarks of Chaos

- **Sensitive dependence on initial conditions**



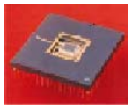
Eventually, the paths diverge (mathematically quantified by the Lyapunov exponent)

Three initial conditions x_{10}, x_{20} and x_{30} (they are arbitrarily close to each other)

Reference: http://www.keldysh.ru/departments/dpt_17/eng/ndeng.html

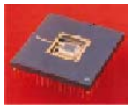
Proving the existence of Chaos

- Mathematically very challenging:
  - Lorenz's system was proved to be chaotic nearly **40 YEARS** after Lorenz's observations:
    - The Lorenz Attractor Exists. Tucker, Warwick. **Ph.D. Thesis**, 1998. University of Uppsala.
- One way to prove chaotic behavior: define a **homeomorphism** to the **Cantor set** using a **Smale horseshoe**



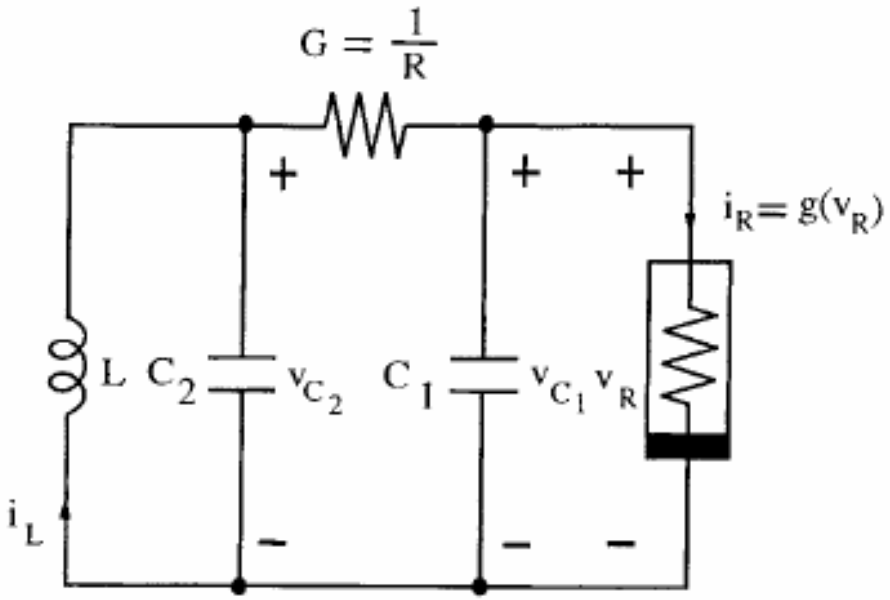
Proving the existence of Chaos

- But for the purposes of EE129:
  - Central concept: **Poincare-Bendixson Theorem**
    - In a nutshell, the consequence of the theorem is that a continuous time autonomous dynamical system CANNOT be chaotic in the plane (2-dimensions).



Easy Chaos: Chua's circuit

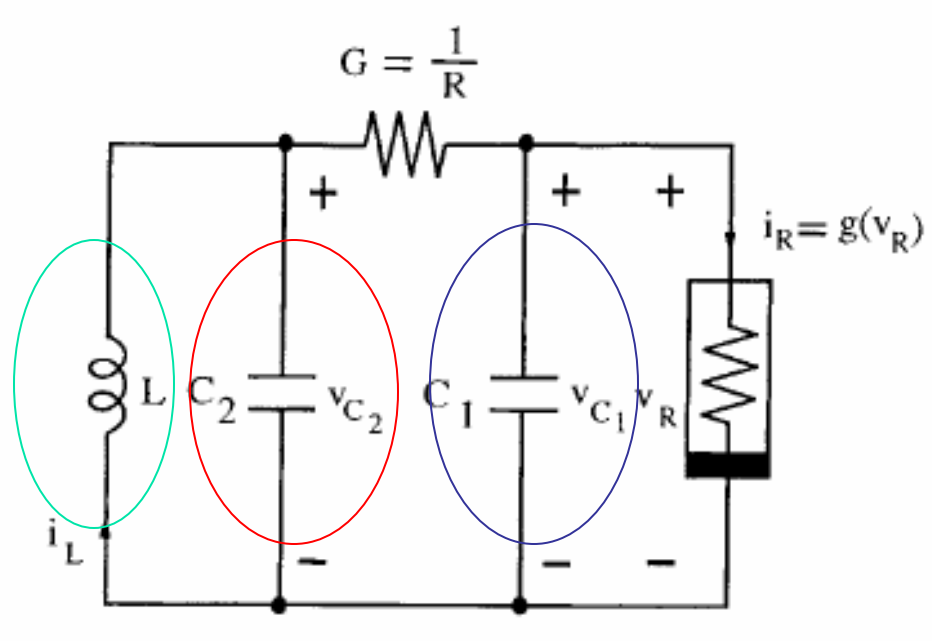
- Designed using **systematic nonlinear circuit techniques** by Leon O. Chua in 1983



Excellent Reference:
"The Genesis of Chua's circuit". Chua, Leon O. *Archiv fur Elektronik und Uebertragungstechnik,* July 1992, vol. 46, (no. 4): 250-257.

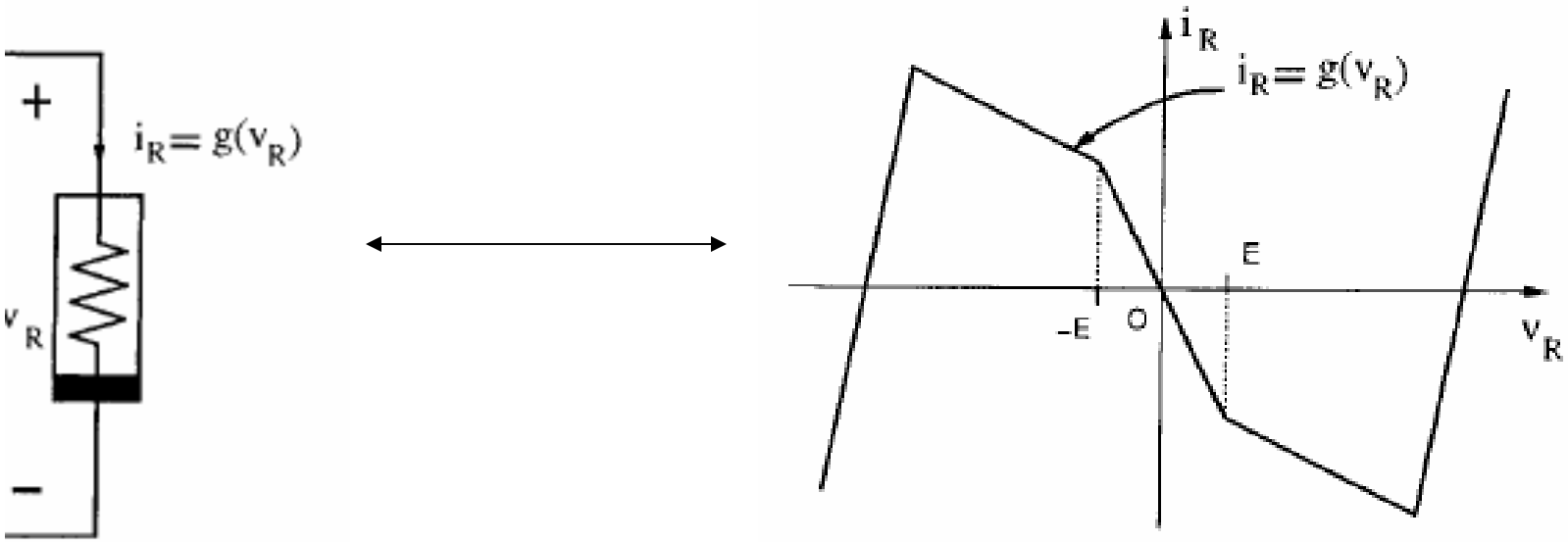
Easy Chaos: Chua's circuit

- **A consequence** of the **Poincare-Bendixson theorem** - we need **three independent energy storage elements**. Hence:



Easy Chaos: Chua's circuit

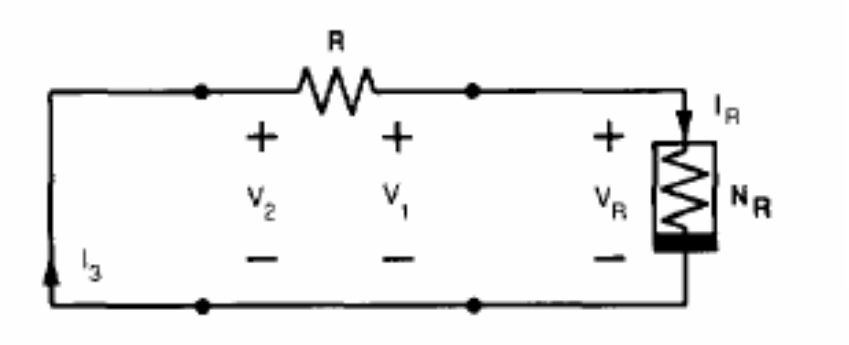
- Nonlinearity "designed" by Leon so that a proof of Chaos is "easy". For details, please refer to "The Genesis of Chua's Circuit" paper.



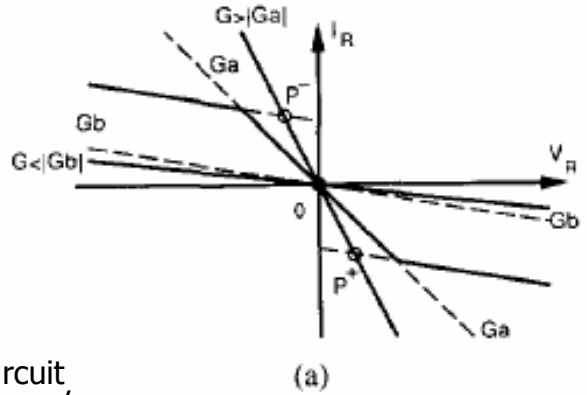
- **KEY: We need at least two unstable equilibrium points – one to provide stretching dynamics and the other to provide folding.**

Easy Chaos: Chua's circuit

- Existence of these equilibrium points can be seen from **DC load line**:



DC Load line



Reference: "Three Steps to Chaos – Part II: A Chua's Circuit Primer". Kennedy, Michael P. *IEEE Trans. On Circuits and Systems – I: Fundamental Theory and Applications.* Vol. 40, No. 10, Oct. 1993

- We will see how we can obtain this nonlinearity later using op-amps

Easy Chaos: Chua's circuit

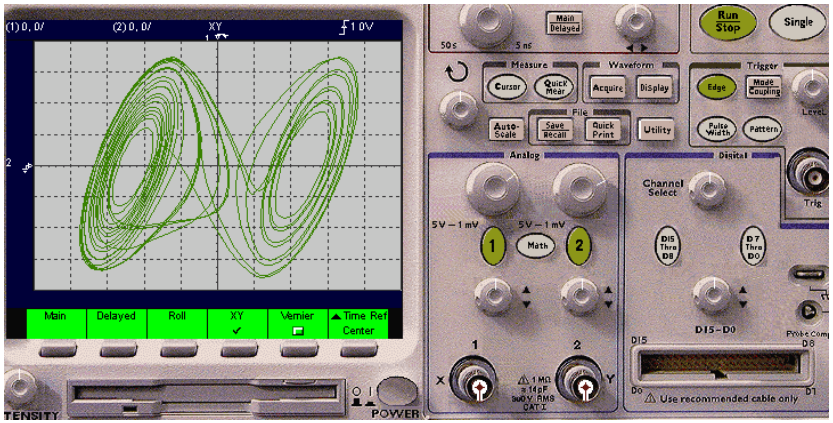
- Some properties of Chua's circuit:
  - Chua's circuit is the **simplest** possible electronic circuit that can exhibit chaotic behavior. Reference: "The double scroll family, Parts I and II". Chua et. al. *IEEE Trans. On Circuits and Systems*. Vol. CAS-33, no. 11, pp. 1073-1118, 1986.
- Applications of Chua's circuit:
  - Music:
    - "Reading Complexity in Chua's Oscillator through Music. Part I: A New Way of Understanding Chaos". Bilotta, Eleonara et. al. *International Journal of Bifurcation and Chaos*. Vol. 15, No. 2, pp. 253 – 282. 2005.
  - Communications:
    - "Chaotic Digital Encoding: An Approach to Secure Communication". Frey, D.R. *IEEE Trans. On Circuits and Systems II: Analog and Digital Signal Processing*. Vol. 40, #10, pp. 660 – 666. Oct. 1993.

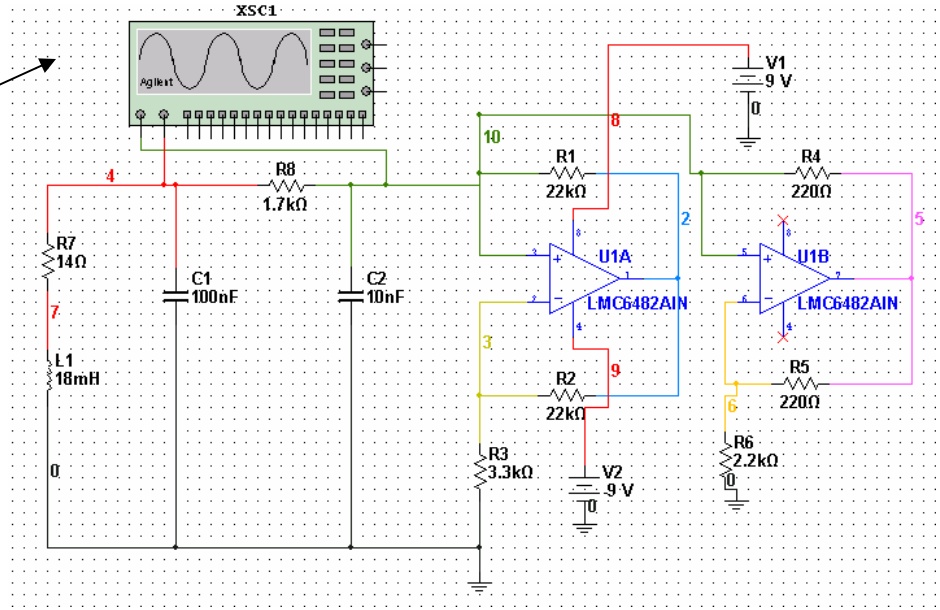
Building and working with Chua's circuit

- Now, we will see how **easy it is** to build Chua's circuit with **readily available components**!
- References
  - NOEL – Chaos in Chua's Circuit homepage http://nonlinear.eecs.berkeley.edu/chaos/chaos.html
  - Chaos Wiki: http://robotlab.itk.ppke.hu/~wiki/mediawiki-1.9.3/index.php/Main_Page
  - "Chua's Circuit for High School Students". Gandhi, Gaurav., Muthuswamy, Bharathwaj and Roska, Tamas. *To appear in the International Journal of Bifurcation and Chaos in Dec.* 2007 – preprint copy online
  - Also read (available online):
    - "The Genesis of Chua's Circuit"
    - "Three Steps to Chaos. Part 2 – Chua's Circuit Primer"

Building and working with Chua's circuit

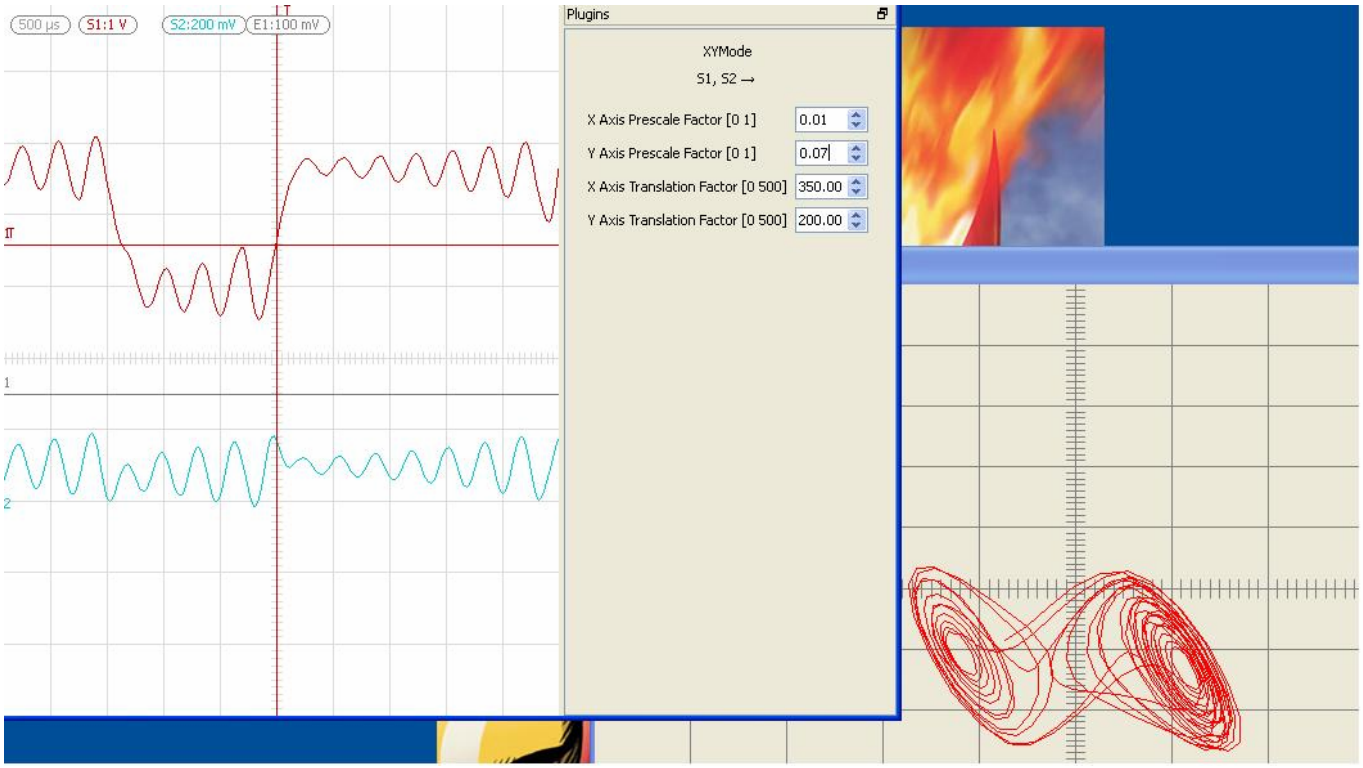
- Lecture demo - **Simulating Chua's circuit: MultiSim**





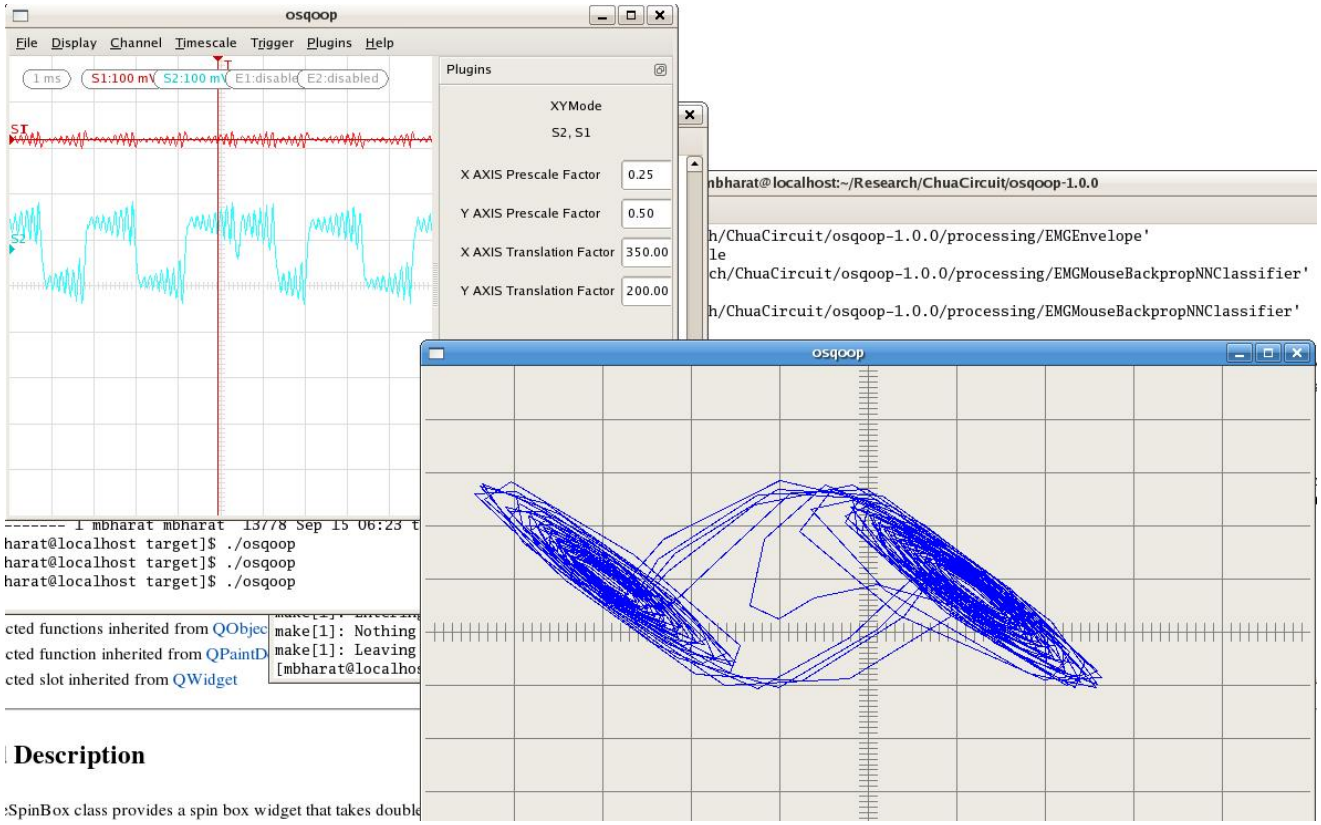
Building and working with Chua's circuit

- Lecture Demo - **Turning your PC into an oscilloscope: osqoop**



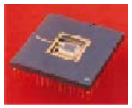
Building and working with Chua's circuit

- Lecture Demo - **Chua's circuit for high school students**



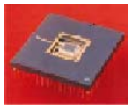
Building and working with Chua's circuit

- **Chua's circuit for high school students. Debugging Tips:**
  - **Checking the functionality of the nonlinear resistor**
  - **Tuning the circuit**

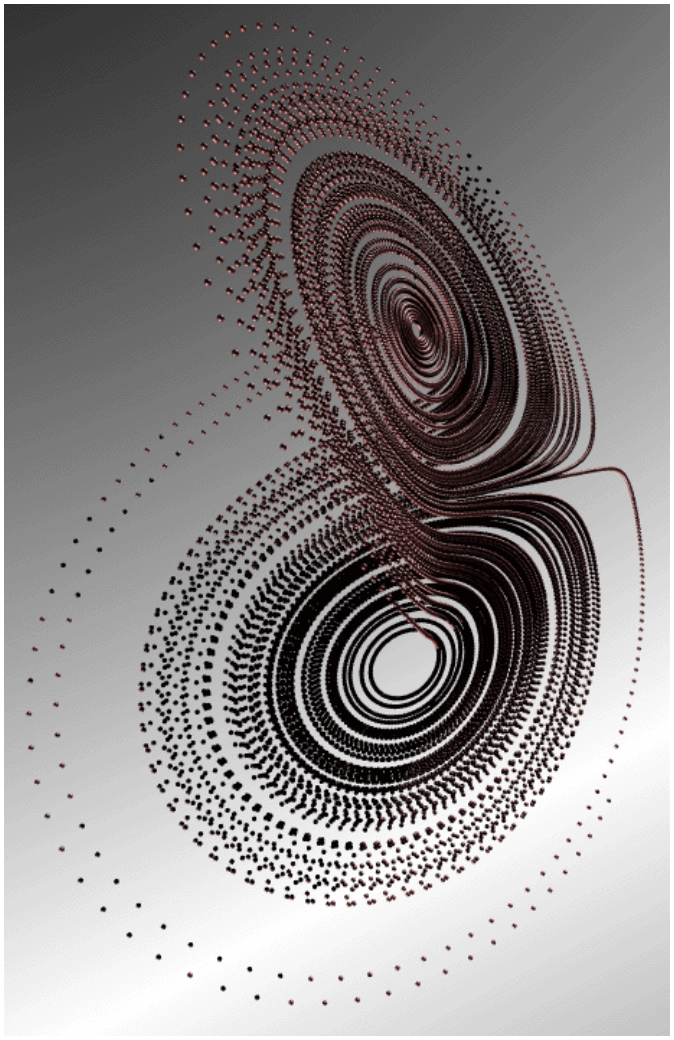


Building and working with Chua's circuit

- Lecture Demo - **Interesting MATLAB experiments** using Chua's circuit:
  - Sample data from sound card
    - Compute Fourier Transform
    - Compute Autocorrelation coefficient. Compare this to autocorrelation coefficient of white noise.



Questions…



Reference:
http://mathstat.helsinki.fi/mathphys/paolo_files/lorenz11.gif