Co-ordination of Randomly Packed Spheres

In following up earlier work on a geometrical model of liquid structure[1], we have been examining more closely the precise mutual co-ordination of spheres when arranged at random and more-or-less closely packed. As can be seen from Dr. Scott's communication, which came to hand during this investigation, there are at least two types of random packing: random close packing with an occupied volume of 0·64, and random loose packing with one of 0·60. The first of these packings we had already studied with wax balls contained inside a rubber balloon. Realizing, however, that the arrangement studied was liable to be distorted from that of random close packing owing to the compression, it seemed desirable to repeat the experiment with a larger number of approximately rigid spheres. Accordingly, these were now replaced with ¼-in. ball bearings, about 1,000–5,000 in number in different experiments, well shaken down and compressed by winding round with thick rubber bands. This assembly appeared to be rigid, its occupied volume of 0·62 indicated a reasonable approximation to Scott's close-packed density, considering its limited size.

Table 1. Analysis of Close and Near Contacts
(*a*) From assembly in random close packing

Total number of contacts	Number of close contacts: 0	1	2	3	4	5	Total contacts	Close contacts
3	—	—	—	—	—	—	—	5
4	—	—	—	—	—	—	0	13
5	5	0	2	—	—	—	7	72
6	12	20	5	1	—	—	38	**132**
7	31	**47**	25	3	1	—	107	**133**
8	33	**61**	**46**	13	3	1	**157**	95
9	19	**52** *	**37**	21	6	2	**137**	22
10	3	3	10	4	6	3	29	4
11	0	1	—	—	—	—	1	—
Number of balls							476	

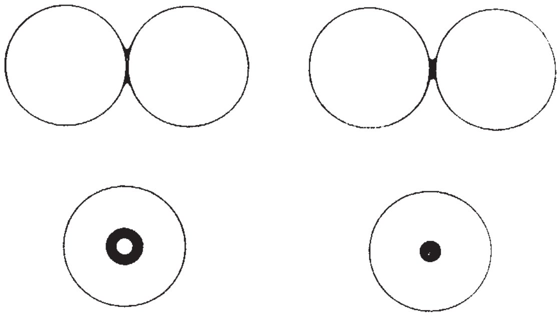
* Centre of distribution. The cases marked in bold type contain between them more than half the balls.

(*b*) From assembly in random loose packing

Total number of contacts	Number of close contacts: 0	1	2	3	4	Total contacts	Close contacts
3							21
4	6	1	—	—	—	7	78
5	10	10	10	—	—	30	**128**
6	**17**	**32**	26	8	—	83	**108**
7	17	**47** *	**45**	24	2	**135**	57
8	15	**28**	**28**	**39**	12	**122**	24
9	4	5	11	14	2	36	4
10	—	—	3	1	2	6	0
11	—	—	—	1	—	1	0
Number of balls						420	

* Centre of distribution. The cases marked in bold type contain between them more than half the balls.

The mass of balls prepared in this way while still compressed was soaked in black japan paint. After standing for some time the paint was drained off. The object here was to mark the nature of the contacts and distinguish between them. Surface tension drew the paint to the places where the balls touched (*a*), or nearly touched (*b*), the first giving rise to a black ring with a clear centre and the latter to a black spot (Fig. 4). Careful experiment showed that the near contacts were registered by this method up to a radius of 5 per cent greater than the diameter of the ball. After thoroughly drying the paint, the mass of balls was unwrapped, with an appearance resembling caviare (Fig. 5). When broken apart they showed many of the small clusters referred to as pseudo-nuclei[2], and a certain number of holes, none of which was observed large enough



(*a*) (*b*)

Fig. 4. Diagram of method of marking (*a*) close and (*b*) near contacts between spheres. The areas of adherent black paint are marked

Fig. 5. Portion of random close packed ball assembly showing marks of further contacts

to contain another ball. About 400–500 of these balls, taken from the centre of the sphere to avoid the surface effects noted by Dr. Scott, were separately examined and a number of close and near contacts counted. They were sorted according to the numbers of both kinds of contacts, the results being shown in Table 1 (*a*). These give effectively the distribution of co-ordination types characteristic of random packing. The distribution, as can be seen, is extensive and skew; but the modes are clearly marked and do not differ much from the mean. A modal co-ordination lies between 8 and 9, the largest number having between 6 and 7 close contacts and between 1 and 2 near contacts. The means are 8·5 total, and 6·4 close contacts.

A later experiment using the slow rolling method of Dr. Scott on a mass of ball bearings supported on a highly irregular base gave the conditions for random loose packing with an occupied volume of 0·6. The results of the sorting here give lower co-ordinations all round. The modes were between 7 and 8 for the total co-ordination and between 5 and 6 for the close contacts; the means are 7·1 total and 5·5 close contacts. In the looser random packing it was evident that there were a number of large holes, and that the co-ordinations of particular balls were highly unsymmetrical; for many of them, all the contacts being on one side, the other side facing the hole.

The meaning of these preliminary observations seems fairly clear. In any random packing we would expect each sphere to be in cose contact with at least four others, a necessary condition of stability, and at most twelve. This absolute upper limit is extremely improbable. The most probable average would seem to be six, as each sphere may be

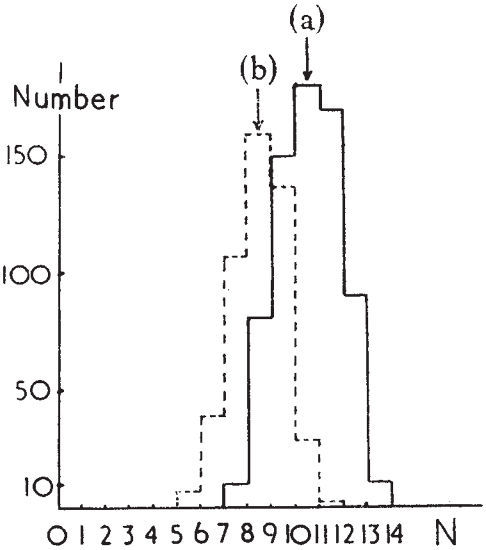


Fig. 6. Histogram of contact numbers derived (*a*) from calculated random model and (*b*) from assembly of spheres

considered in general to rest on three others and in turn to support another three. The number of near contacts, however, is not determinate, except that the sum of the close and point contacts may not exceed twelve, and is very unlikely to approach that number.

By the process of shaking and pressing, the arrangements of the spheres are progressively altered so as to give the minimum volume, and hence the largest number of near contacts. The figure for the occupied volume of random close packing—0·64—must be mathematically determinable, although so far as we know undetermined. If it is one of minimum volume, then it is also one of minimum energy, corresponding to any monotonously diminishing form of centrally attractive forces. For, by definition, any variation of this structure, while not altering the number of point contacts, must necessarily increase the distances between near contacts and thus diminish the energy.

The method used for marking near contact in the model would necessarily fail to note those between 1·05 and say, 1·25 of the minimum value, and these would to a measurable extent contribute to the total energy and should be included as physical neighbours in the sense of the first communication[1]. We may consequently take the mean physical co-ordination of random close packing to be a little more than the value of 8·5 indicated. It probably lies between 10 and 11 (Fig. 6).

The difference between random close packing and regular crystalline close packing fits very well with the observations of rare gas liquids. The increase in volume when passing from regular to irregular is almost exactly the same—16 per cent observed in argon—while the static contribution to the internal energy should correspond to a latent heat of melting between one-quarter and one-sixth of the energy of evaporation as against the observed value of 1/5·5 observed in argon.

The mathematical status of physical random loose packing is not so evident, though it may represent the approximation of the liquid at higher temperatures. The change-over would correspond roughly, on the basis of volume, to a rise in temperature of 1/5 of the interval between the melting and critical points. This will also give a configurational change of internal energy of about 0·18 c./mol. as against 0·16 observed.

Further investigations are in hand to study the statistical contribution of the angles between the contacts between the spheres and also to examine even looser aggregates which can only be obtained in the model by the use of rough or sticky spheres.

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FAILURE TO DEMONSTRATE INFECTIVITY OF NEWCASTLE DISEASE VIRUS NUCLEIC ACID PREPARATIONS

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FOLLOWING the isolation of the infectious ribonucleic acid (RNA) moiety from the tobacco mosaic virus (TMV)[1,2], infectious RNA fractions have been extracted from numerous animal viruses[3-8]. In contrast to the intact viruses, the infectivity of the extracts is ribonuclease (RNAase) sensitive, thermolabile, and is sedimented very slowly in the ultracentrifuge.

At the time that the present work was initiated no information on the infectivity of RNA preparations from the myxoviruses had been reported ; however, recently Maassab[9] and Portocala *et al.*[10] described such fractions derived from influenza viruses. After repeated failures to detect infectious RNA from chick embryo-adapted influenza viruses, we made additional attempts to obtain infectious RNA fractions from Newcastle disease virus (NDV). Employing the cold phenol technique[1], the NDV extracts so prepared lacked infectivity in chick embryos. The existence of numerous variables in this type of study has become increasingly obvious from the reported results of others. Thus, exhaustive attempts were made to extract an active NDV RNA fraction by varying the conditions which are summarized as follows :

Egg-adapted strains (Iowa 23 and Calif. 11914) were used in the form of crude infected allantoic fluid, partially purified concentrated allantoic fluid,