A Measurement of Surface Tension by Means of Stationary Waves on a Vertical Jet

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INTRODUCTION

The various problems connected with liquid jets have been a prominent subject of investigation for many years. Outstanding names among the many who have conducted research along these lines are those of Rayleigh(1), Pedersen(2), Bohr(3), and Tyler(4). The present investigation, however, a continuation of the work of Satterly and McPherson(5), has as its basis a phenomenon which was first noted by Maass(6).

It is a well-known fact that when a liquid jet, issuing vertically from a circular orifice, is allowed to strike a solid barrier, or falls into a vessel of the same liquid, stationary waves are set up on the surface of the jet. These waves may be used to find a value for the surface tension of the liquid. Lamb's(7) hydrodynamical theory may be applied to such a cylinder of liquid, and the value of the surface tension T is given by the relation:

$$T = \frac{\rho v^2 . 2\pi r^2}{\lambda} \; \frac{I_0\left(\frac{2\pi r}{\lambda}\right)}{I_1\left(\frac{2\pi r}{\lambda}\right)} \; \frac{1}{\left(\frac{2\pi r}{\lambda}\right)^2 - 1}$$

where r and λ are the radius and wave-length respectively of the jet at any point, and v is its velocity. I_0 and I_1 are Bessel's functions whose values are given by

$$I_0(z) = \left(1 + \frac{z^2}{2.2} + \frac{z^4}{2.4.2.4} + \ldots\ldots\right)$$

$$I_1(z) = \frac{z}{2}\left(1 + \frac{z^2}{2.4} + \frac{z^4}{2.4\,.\,2.2\,.\,2.3} + \ldots\ldots\right)$$

The mean velocity of the jet is given by $v = \frac{w}{\rho\pi r^2}$ where w is the mass of liquid of density ρ delivered per second. Also in the experiments r and λ are found by measurements on a photographic plate, so that we must include a magnification factor M. Making these substitutions, we find

$$T = \frac{2w^2M^3}{\pi\rho} \frac{\lambda}{(4\pi^2r^2-\lambda^2)r^2} \frac{I_0\left(\frac{2\pi r}{\lambda}\right)}{I_1\left(\frac{2\pi r}{\lambda}\right)}$$

where r and λ are now the photographic magnitudes of the radius and wave length.

Maass investigated the surface tension of water in this way, allowing the jet to fall into a vessel of the same liquid, and found quite good values for T. In the present work, the water jet was allowed to strike the spherical surface of an upturned distillation flask, which produces more strongly marked waves. Also a different method of lighting the jet is used, which may be applied equally well to transparent or opaque liquids.

Experiments with Water

The apparatus was arranged as shown in Fig. 1. Water from a large overflow tank A, flows through the siphon U, and issues from

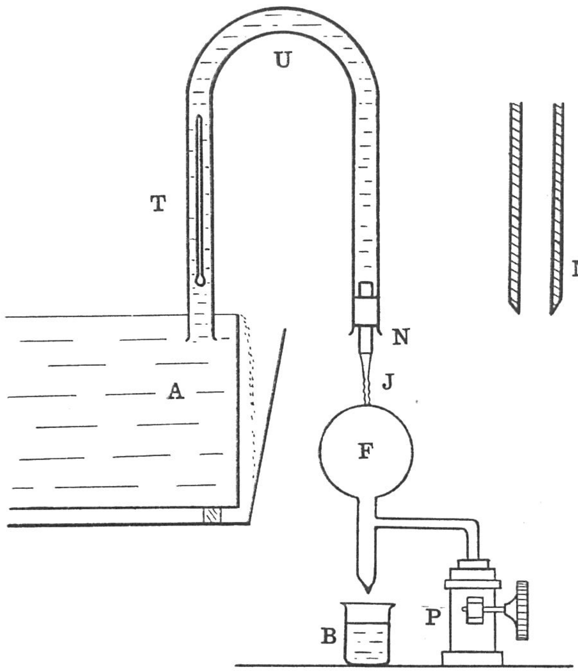

Fig. 1

the nozzle N, forming the jet at J. The liquid falls upon the flask F, which is supported by a rack and pinion arrangement P, by means of which the jet length may be conveniently altered. Since the water wets the end of a glass nozzle, producing a certain amount of dead water and tiny eddy currents, a brass nozzle with a bevelled end, as shown at M, was used. The temperature was taken by the thermometer T. By raising or lowering the whole U tube, the rate of flow could be easily altered. This latter was found by weighing the water collected in the beaker B for a given length of time as measured by a stop-watch reading to 1/10 of a second.

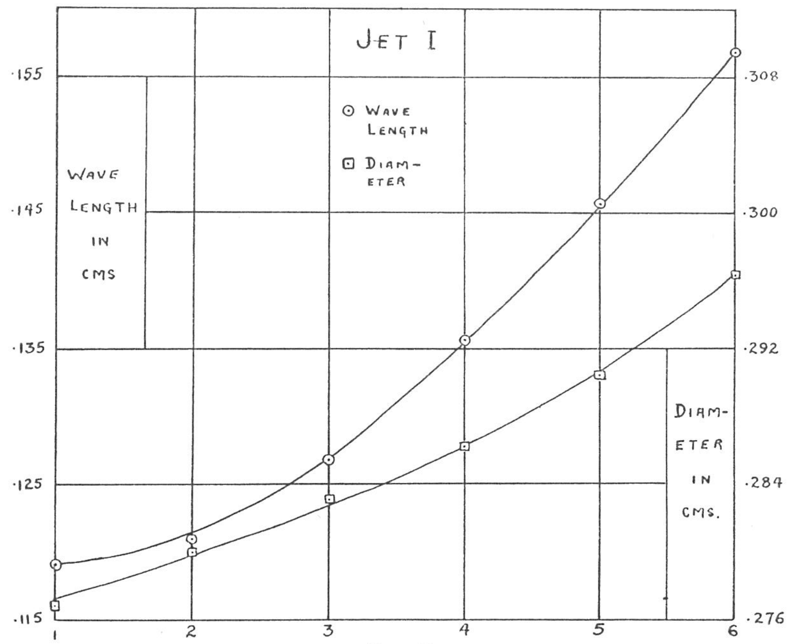

FIG. 2

In order to find r and λ, photographs of the jet were taken and measured. The arrangements for lighting the jet were as follows: A parallel beam of light, furnished by a Pointolite lamp and a condensor lens, was allowed to fall on the jet in such a way that light was reflected back to the camera lens from the surface of the ripples. If the source of light, the jet, and the camera lens are all at the same horizontal level, light is reflected into the camera only from those portions of the jet where the tangent to the surface is vertical. The result is that if we reduce the camera aperture sufficiently, a sharp spot is formed on the image at each crest and at each trough. The distance between alternate spots yields the wave-length. At the same time, a ground glass screen illuminated from behind, is placed at the back of the jet, producing an image of the jet in sharp relief, so that a good measurement of the diameter may be obtained. The mean diameter and wave-length are found for each wave, and substituted separately in the formula.

The mean diameter could in all cases be found at once by a careful setting of the microscope cross-hair, since the amplitude of the waves is small. The magnification factor was found by taking the ratio of the photographic to the actual magnitude of the diameter of the nozzle.

Since the wave-lengths and diameters are small quantities, the values of each were plotted against the natural numbers, a smooth curve was drawn between the points, and corrected values read from the curve.

Table I shows the results obtained for two typical jets of water. In Fig. 2 is shown the curve by which the measurements for jet *I* were corrected. The jet is shown in Fig. 5 (Plate I). The vertical order is from the bottom to the top of the jet.

TABLE I

Jet	Rate of Flow gms/sec.	Magnification	$t°$ C.	Corrected λ cms.	Corrected d cms.	T dynes per cm.	Mean T for stream corrected to 15° C.
	2.89	1.119	12.3	.1190	.2773	72.8	73.4
				.1214	.2797	71.9	
I				.1269	.2826	72.5	
				.1354	.2862	74.0	
				.1456	.2906	75.5	
				.1571	.2963	76.0	
II	1.85	1.126	17.7	.1200	.2216	71.2	72.4
				.1249	.2241	71.2	
				.1307	.2270	71.3	
				.1383	.2303	71.7	
				.1473	.2342	72.0	
				.1583	.2387	72.6	
				.1735	.2440	74.2	

Other jets gave mean values for T ranging from 70.5 dynes/cm. to 76.0 dynes/cm. at 15° C. Although the probable error is large, this is in fair agreement with the latest accepted statical value, which is about 73.5 dynes/cm. at 20° C.

Experiments with Mercury

It was found impracticable to allow the mercury jet to fall on a spherical glass surface, since the mercury ran off in disconnected streams causing unsteadiness in the jet, which made photography impossible. The jet was therefore allowed to fall into a dish of mercury, which was kept at a constant level by means of a siphon. The apparatus was arranged as shown in Fig. 3. D and D_1 are two large

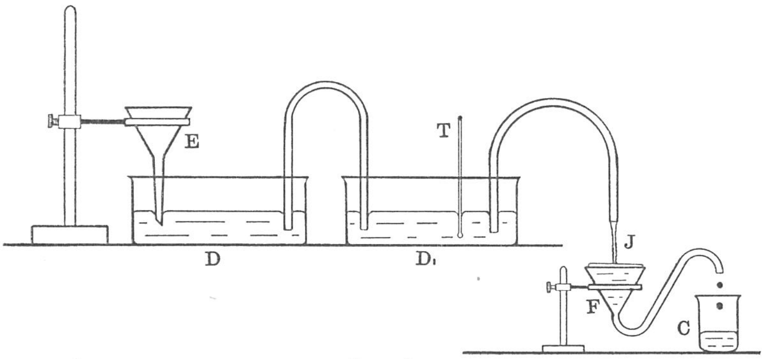

Fig. 3

dishes about 25 cms. in diameter, which are filled to a depth of about 1.5 cms. with mercury. A wide siphon connecting the two serves the purpose of keeping the levels the same in the two dishes without transmitting disturbances from one to the other. The large sizes of D and D_1 serve to keep the head steady over a moderate period of time. The jet is formed at J and falls into a funnel F. From F the mercury siphons into C and is transferred back to D by way of the funnel E. A thermometer T immersed in D_1 measures the temperature. Glass nozzles were used and were carefully selected for circularity of bore. The apparatus was carefully cleaned and dried, and the mercury itself was purified by passing it in finely divided drops first through dilute acid, and then through distilled water, then carefully drying it in an oven.

Table II shows the results obtained for a typical mercury jet. The jet is shown in Fig. 6 while the graph by which its measurements were corrected is shown in Fig. 4.

The mean value of T corrected to 20° C. is 562 dynes/cm. Other jets gave values ranging from 548 to 577 dynes/cm. with a mean for six jets of 563 dynes/cm. at 20° C. Each of these jets was photographed within half an hour or so of purification of the mercury. On allowing the mercury to stand in the dishes exposed to the laboratory

TABLE II

Rate of flow = 9.18 gms/sec.
Magnification = 1.142
Temperature = 25.4° C.

Corrected λ cms.	Corrected d cms.	T dynes per cm.
.0767	.1388	560
.0795	.1403	557
.0830	.1418	563
.0870	.1436	568
.0915	.1459	561
.0970	.1485	559
.1040	.1516	560
.1126	.1553	557
.1237	.1596	561

air, it was found that T feil to 518 after one week, to 505 after two weeks and to 499 after three weeks. Although these values are not to be trusted too far since the variation in the individual values is considerable the value for T seems to tend asymptotically to a value of about 495 dynes/cm.

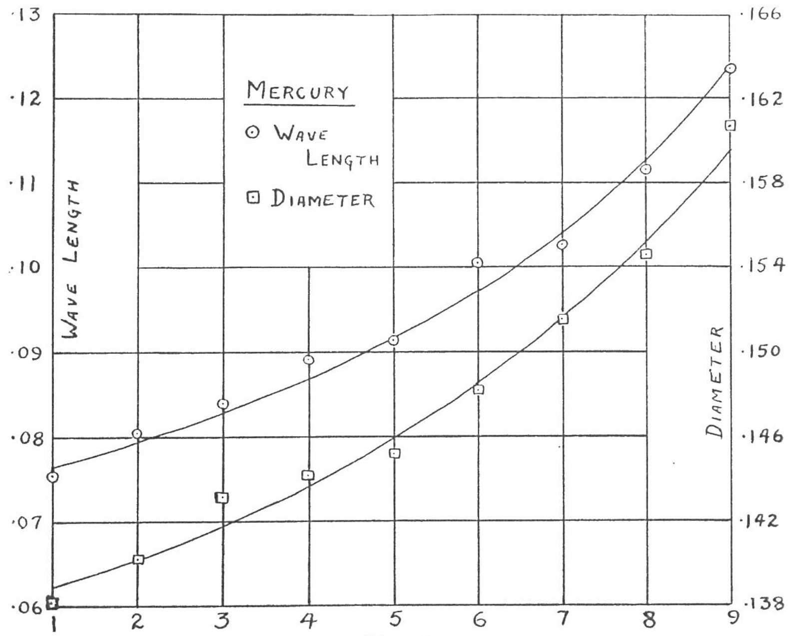

FIG. 4

The values recently obtained using static methods vary considerably, but all are much lower than the value found by the present method. Popesco (8) and others (9) using the sessile drop method found the surface tension of a drop freshly formed in a gas to be as much as 100 dynes higher than that of a drop freshly formed in a vacuum. Popesco's explanation is essentially that the atoms are prevented from orienting themselves to a position of minimum energy by the presence of the gas, while in a vacuum this orientation takes place immediately producing a lower value for the surface tension. This is a possible explanation of the high values found using the jet method.

Another possible explanation is that the surface layer of the jet forms a "skin" through which the liquid inside flows. This skin would have a smaller velocity than the main column of liquid due to friction with the surrounding air. Our measurements of the jet velocity, since they are based on the rate of flow, would give an average value of the velocity over the whole jet, whereas the value used should be the velocity of the surface layer. If this explanation be the true one, the ratio of the two values of the surface tension would give us the relation between the surface velocity and the mean velocity. The great difference in the intrinsic properties of mercury and water might make it quite possible for either of these effects to be present in the former but not in the latter. Probably also the formula of Lamb is not strictly applicable to the jets employed.

Attempts were made to find the reason for the variation in the values found from different jets. The rate of flow was varied as much as possible, and nozzles of various sizes were tried. Neither of these seemed to have any effect on the value found for T. As a check on the method of lighting in the case of water, a photograph was taken using a low power microscope objective as the camera lens, and with the parallel light passing through the jet, as in Maass' experiments. The result obtained gave a mean of 77 dynes/cm. which is considered to be within the limits of experimental error. Also the water used was tested from time to time with a Du Noüy Cenco tensiometer and the values found for T did not vary by any appreciable amount.

It is concluded therefore that the main cause of the variations is in the determination of the magnification factor and the rate of flow. A small error in either of these will produce a much larger error in the value of T.

Variations in the values found from a single jet are no doubt due to errors in the measurement of d and λ. Although the vertical jet method does not admit of a high degree of accuracy, it nevertheless possesses two distinct advantages. First, the measurement of the surface tension does not depend on the angle of contact; and second, this method affords a means of measuring the surface tension on a surface formed only a small fraction of a second earlier. This latter property is one which none of the static methods possesses, and for that reason the method should prove of considerable interest.

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Plate I

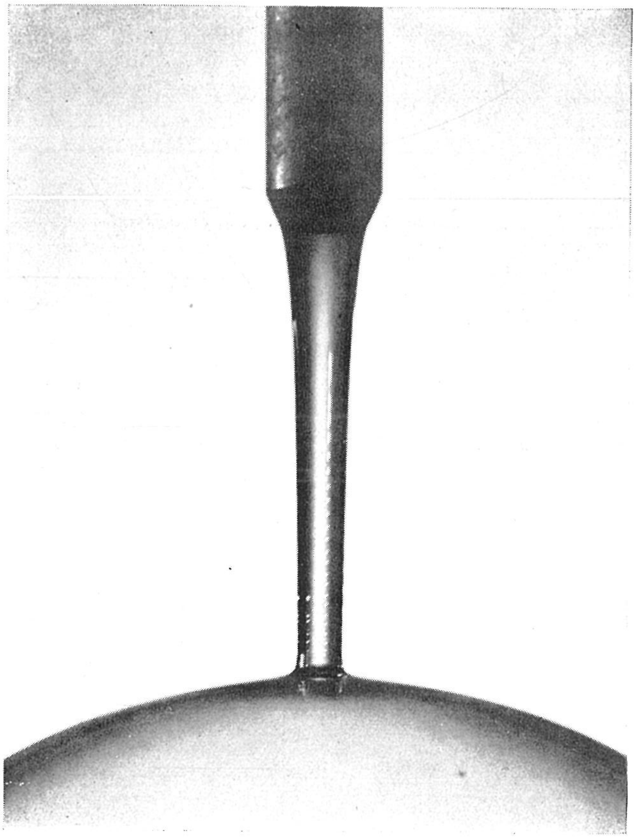

Fig. 5

Plate II

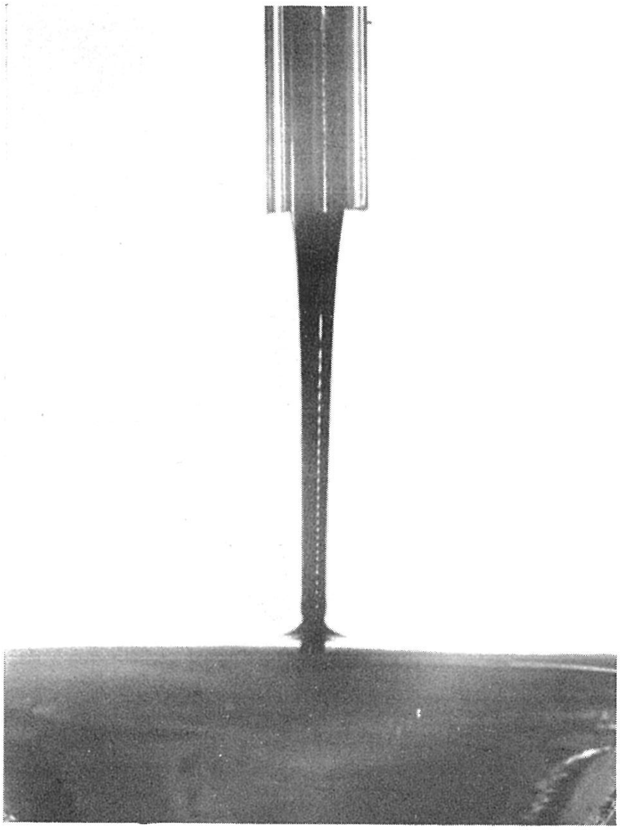

Fig. 6