What do you measure when you measure resistivity?

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(Received 13 September 1991; accepted for publication 9 October 1991)

Resistivity measurement is a weighted averaging of local resistivities. We develop a formalism to calculate the weighting function, applying it to square van der Pauw samples and to linear and square four-point probe arrays. In each case, some regions of the sample are negatively weighted, but these regions can be reduced or eliminated by van der Pauw averaging. We discuss negative weighting, which we feel is responsible for spurious reports of superconductivity above room temperature. We show how a square four-point array can be more effective at measuring local resistivity than a linear one. Finally, we show how to apply our formalism to anisotropic materials.

I. INTRODUCTION

Resistivity is one of the most useful measurements for characterizing materials. To measure the resistivity ρ one makes one or more four-point resistance measurements on a sample of the material, and then converts resistance(s) into resistivity by means of some geometrical factor. This factor may depend on the shape and size of the particular sample and the position of contact leads. Ideally it should be both easy to calculate and infinitely accurate. Geometrical factors are well known for the most popular measurement techniques, including moveable four-point probe arrays,[1–5] bridge, and van der Pauw techniques[6] for isotropic materials, and Montgomery's method[7] for anisotropic materials.

For materials with nonuniform resistivity, use of the geometrical factor will result in the calculation of an average resistivity. We shall consider this act of averaging and how it depends on the local values of resistivity. We will start with materials of isotropic, although slightly nonuniform resistivity, considering one van der Pauw geometry and two fixed-probe geometries. We will finally show how one can generalize the results to anisotropic materials.

II. METHOD

Calculating the geometrical factor generally assumes the uniformity of the local resistivity in the sample. If the material's composition varies in the volume of the sample, then any measurement of resistivity gives only an average value, with some regions being weighted more strongly than others. This average can be written as

$$\rho_m = \int\int \rho(x,y) f(x,y)\, dx\, dy, \tag{1}$$

where $\rho(x,y)$ is the local resistivity and $f(x,y)$ is a weighting function which measures the sensitivity of the measurement to the local resistivity. This weighting function depends on the sample shape and the arrangement of current and voltage leads. Knowledge of the function $f(x,y)$ for various geometries would allow one to choose the geometry that would minimize errors due to inhomogeneities.

To calculate the weighting function at a point (x_0,y_0), we consider how the voltage across two voltage probes changes when we perturb the resistivity at that point while keeping the total current fixed. If

$$\rho(x,y) = \rho_0[1 + \epsilon\delta(x - x_0)\delta(y - y_0)], \tag{2}$$

then

$$f(x_0,y_0) = \frac{1}{\epsilon}\frac{\Delta\rho}{\rho} = \frac{1}{\epsilon}\frac{\Delta V}{V},$$

where V is the potential difference between the voltage probes and ΔV is the change in this quantity due to the perturbation.

To solve for the potential in an inhomogeneous material, we start with the steady-state continuity equation

$$\nabla\cdot\mathbf{J} = 0,$$

where $\mathbf{J}$ is the current density. Equating $\mathbf{J}$ to $\mathbf{E}/\rho = -\nabla\Phi/\rho$ leads to

$$\nabla^2\Phi = \frac{\nabla\rho\cdot\nabla\Phi}{\rho}. \tag{3}$$

The potential Φ due to the perturbation of Eq. (2) is thus equivalent to the potential due to a point dipole located at point (x_0,y_0). We will consider the limit of small perturbation, such that the denominator in Eq. (3) can be replaced with the unperturbed resistivity. For an ideal point perturbation, as in Eq. (2), this means choosing $\epsilon \to 0$ in such a way that $\epsilon\delta(x - x_0)\delta(y - y_0) \to 0$ as well. For an actual physical sample, this means that the relative variation of the local resistivity from its average value $\Delta\rho/\rho$ is much less than one.

We have solved Eq. (3) for a square van der Pauw sample both numerically—using the discretizing technique mentioned in Ref. 8—and exactly—using a Green's function technique. We chose to have current flow between electrodes on two adjacent corners and to measure voltage

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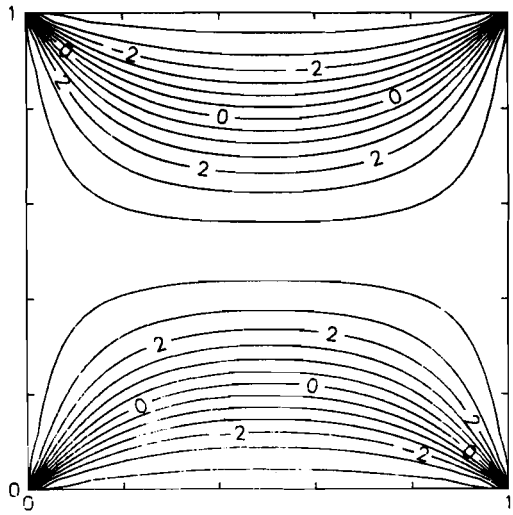

FIG. 1. Contour map of the weighting function for resistivity measurement. Current enters and leaves the sample through the left-hand corners. The voltage probes are on the right-hand corners.

between electrodes on the other two. We calculated the unperturbed electric field at an interior point and then calculated the effect of the dipole of Eq. (3) on the electrical potential for either approach. The calculations were performed on a 386/33 with a 30387 coprocessor using PASCAL.

III. RESULTS

The weighting function $f(x,y)$ is shown as a contour graph in Fig. 1. The current flows between the left-hand corners and the voltage drop is across the right-hand corners. The function has a maximum in the center region, where the weight is over three times greater than it would be if all regions were equally weighted. There are two regions of the sample in which the function $f(x,y)$ is negative. We have confirmed this experimentally with a 0.001-in. thick square of brass shim stock, using solder to alter $\rho(x,y)$. The existence of regions of negative weighting means that the measured resistivity ρ_m may lie outside the range of local values occurring in the sample. Furthermore, gross inhomogeneities can cause unphysical results. In the same way that a student can get a final grade of 100% for less than perfect work in a class in which some grades are negatively weighted, one can measure a zero resistance state in a normal conductor if negative weighting exists. This may explain several reports of zero resistance at high temperature made since 1987.

A simple model confirms how negative weights can occur while measuring resistivity. Imagine four resistors forming the network in Fig. 2. We can use this to model the square van der Pauw sample with the voltage V measured across R_b when a current (I) flows through the network. The measured four-point resistance of this network equals

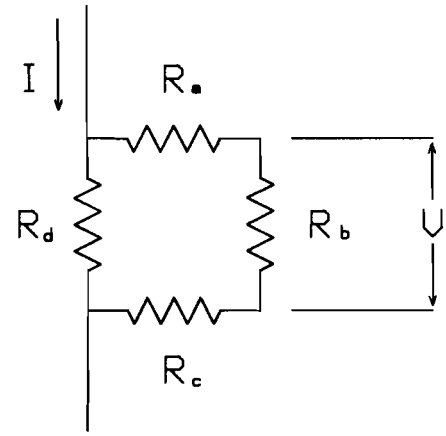

FIG. 2. Four-resistor model of a four-point resistance measurement. The measured resistance R_m is the ratio of the voltage drop across R_b to the total current flowing through the network.

$$R_m=\frac{V}{I}=\frac{R_bR_d}{R_a+R_b+R_c+R_d}.$$

The derivative of this quantity with respect to either R_a or R_c is negative, so that increasing either one reduces R_m. This corresponds to the negative-weight regions of Fig. 1.

It is possible to eliminate the effects of negative weighting. Since the van der Pauw technique requires swapping one current lead with the voltage lead opposite to it and averaging the two resistances, we have averaged the results from Fig. 1 with the results obtained by rotating the plot by 90°. This new $f(x,y)$ function is plotted as a contour plot in Fig. 3. The area of the material that is most sensitive to inhomogeneities [$f(x,y)>3$] has diminished from about 30% of the total area to about 4%. The area of the material where $f(x,y)<0$ has vanished.

This process for calculating $f(x,y)$ can be applied to four-point probe array measurements. Probe arrays are electrodes that are attached to a holder that keeps them a

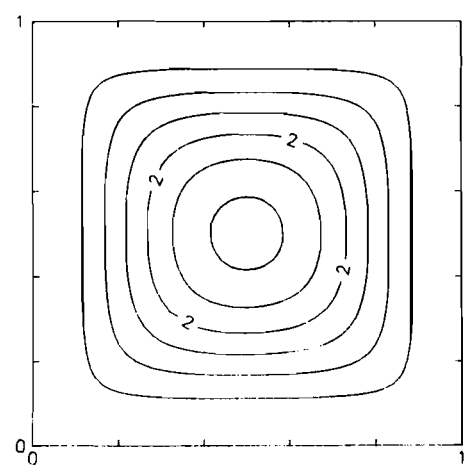

FIG. 3. The weighting function $f(x,y)$ for an averaged van der Pauw measurement on a square sample. Contours are spaced 0.5 apart.

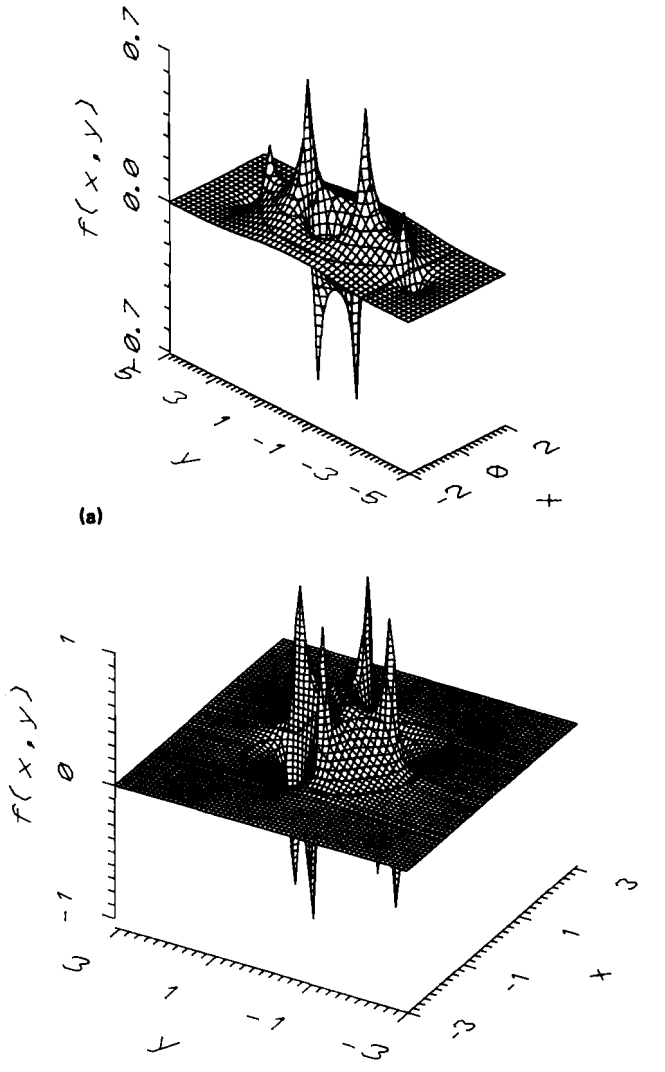

FIG. 4. The weighting function $f(x,y)$ for a single resistance measurement on (a) a linear four-point probe array and (b) a square four-point probe array.

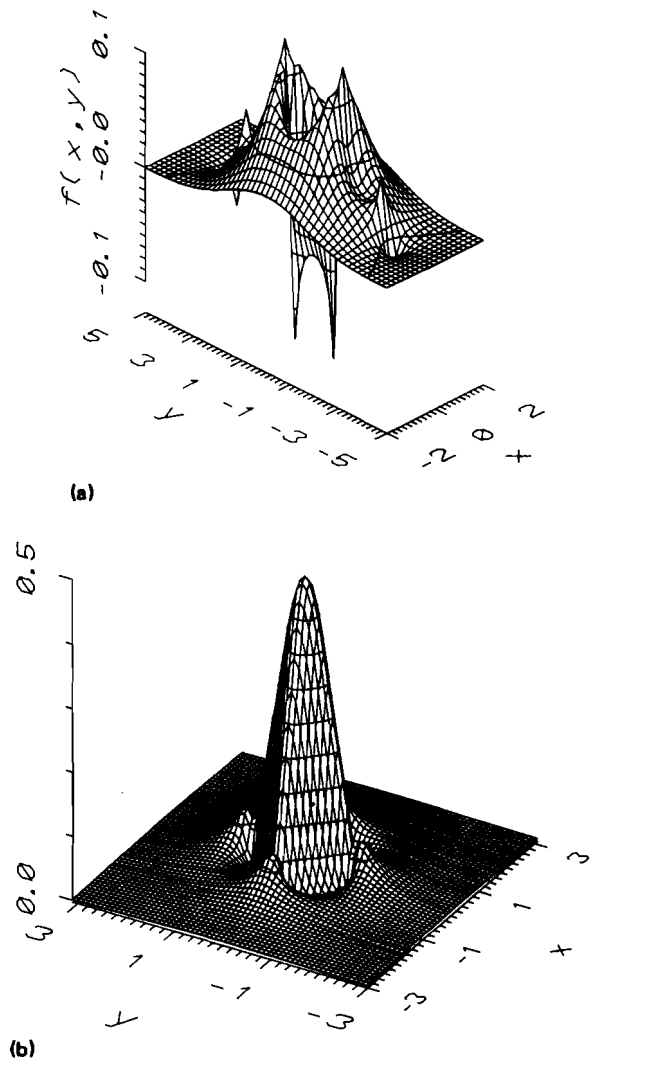

FIG. 5. The weighting function $f(x,y)$ for resistance measurements averaged over two configurations of current and voltage probes for (a) a linear four-point probe array and (b) a square four-point probe array.

known, fixed distance from each other, but which can be positioned anywhere on the surface of the sample to be measured. The most studied of these are the linear and square arrays. We calculated $f(x,y)$ for both of these geometries on an infinitely large sample by analyzing the effect of an electric dipole placed at the point of interest. The function $f(x,y)$ is displayed in Fig. 4 for both geometries. In both cases, there are singularities at the current and voltage probes, and regions of negative weighting. The finite size of actual physical probes will eliminate the singularities near the probes, but the measurement of resistivity is still very sensitive to variations in these four regions.

Making the second resistance measurement as prescribed by van der Pauw and averaging these two resistances will reduce the singularities for both geometries, as shown in Fig. 5, eliminating them for the square array. It will also reduce the negative weighting regions, eliminating them for the square array. Comparing these two figures, one sees that the square array measures the resistivity in a more local region. However, one needs to average over two independent measurements to eliminate singularities and negative weighting.

Van der Pauw observed that, for a given configuration of voltage and current leads on a sample of thickness t and resistivity ρ, for which the measured resistance was R_m, that the quantity $R_m t/\rho$ is invariant under conformal transformations.[6] Likewise the local quantity

$$f(x,y)\,dx\,dy$$

is invariant under mapping from one geometry to a conformally equivalent one. Since the Montgomery method relies on conformal mapping to model an anistropic material as an isotropic one, our formalism can be readily adopted to calculating the weighting function for Montgomery measurements on anisotropic materials.

We are presently studying the weighting function for various measurement geometries. This work, combined with our present knowledge of the relative sensitivity of different geometries to the effects of finite contact size[6,9,10]

and misplacement,[6,8,11,12] will aid researchers in choosing the sample shape best-suited to reducing measurement error.

It is a pleasure to thank O. G. Brandt and B. P. Watson for helpful discussions, particularly those pointing us to our four-resistor model and to the Green's function method.

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