

## Electrohydrodynamic Instabilities in Dielectric Liquids Induced by Corona Discharge

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### Abstract

Corona discharge from a needle is used to induce electrohydrodynamic (EHD) instabilities in a flat thin layer of dielectric liquid. We encounter two types of instabilities. The first one appears only for very insulating liquids and it is linked to the space charge induced in the liquid. This instability happens to be the classical EHD instability induced by unipolar injection.

At higher voltages we observe a second instability in highly insulating liquids that is also observable for more conducting liquids. This second instability is due to the existence of a charged free surface.

### Introduction

Surface instabilities in liquids due to electric forces have been widely observed [1,2,3]. There is a large variety of convection cells, patterns of deformation and conditions for the occurrence of instability. We consider in this paper instabilities induced by a corona current which is forced to pass through a liquid layer.

### Experiments

Fig. 1 shows the experimental set up. A point electrode, whose tip has a radius of about  $10 \mu\text{m}$ , is placed above a flat circular electrode of 8 cm diameter. A ring in Plexiglas of 5 cm diameter, thickness and width of 1 mm, is glued to the electrode. The tip is connected to a high voltage source and the plate to an electrometer, so as to measure the current through the point-plate gap for each voltage. We dispose a fixed amount of liquid onto the flat electrode with the help of a pipette. The volume of disposed liquid serves to control the thickness of the layer, denoted by  $d$  in what follows. This thickness is

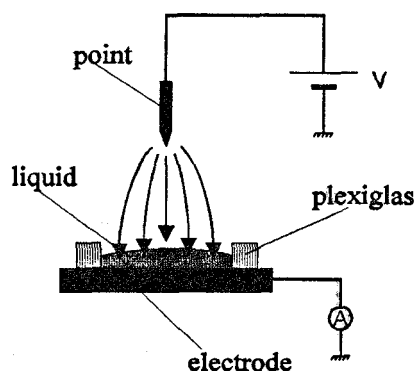


Figure 1: Experimental set up.

typically between 0.2 and 1 mm. The distance from the point to the plate is another variable in the set up. We will denote it by  $L$  and ranges from 2 to 50 cm. Two liquids have been extensively used: silicone oil of viscosity 50 centistokes and castor oil. In this study conductivity is the most important property. For silicone oil, conductivity is less than  $10^{-13} \text{ S/m}$  and for Castor oil we have measured  $7 \times 10^{-11} \text{ S/m}$ .

When high voltage is applied to the tip there is corona current, provided the voltage is above a certain value: the corona threshold. In our set up this is about 2 kV. An almost flat surface of liquid, except for a meniscus near the Plexiglas ring, appears after having drop some liquid onto the electrode. If the liquid is silicone oil we observe a first instability just above the corona inception. The surface starts to deform until a hexagonal pattern is clearly visible for voltages above a certain value. In this pattern the cells are, in order of magnitude, of the same size as the liquid layer thickness.

Increasing the voltage we may observe a second in-

stability, provided the distance  $L$  between the tip and the plate is not too large, i. e. , the electric field is high enough. The pattern that appears in this case has a typical length scale five to six times larger than the first instability. And while a circular region, centered right below the tip, is destabilized, the outer part of the liquid remains undeformed. We will refer to this instability as *rose-window* instability, because its similarity to a rose-window in a Gothic cathedral.

These instabilities are clearly linked to the conductivity of the liquid used. In castor oil (more conducting than silicone oil) we do not observe the first instability, but we do observe the second. For even more conducting liquids, as pure water or ethanol, we do not observe instabilities in the range of voltage available in our set up.

### Space charge instability

Electrohydrodynamic instability in non conducting liquid is a well known phenomenon (see [3,4]). Corona discharge is suitable to induce EHD motion in a liquid layer. Referring to Fig. 1, the ions injected at the point, due to corona discharge, are carried by the electric field onto the liquid surface. They accumulate near the surface, because their mobility in liquid are several orders of magnitude smaller than in air. This is equivalent to the injection of ions achieved by other processes (membranes, electron guns, etcetera).

There are two parameters that control the stability of a liquid layer subjected to unipolar injection [5,6]. The first one results from the balance between the electric energy pumped to the system and the energy dissipated by the viscous forces:  $T = \epsilon V / (K \eta)$  where  $\epsilon$  is the dielectric constant of the liquid,  $K$  is ion mobility,  $\eta$  is liquid viscosity and  $V$  is the voltage difference across the liquid layer. The second parameter measures the relative strength of the injection, and is the ratio between the injected charge ( $q_0$ ) and the electrode surface charge,  $C = q_0 d^2 / (\epsilon V)$ , where  $d$  is the liquid layer thickness. If  $C \ll 1$ , weak injection, the electric field is scarcely modified by the injected charge. If  $C \gg 1$ , strong injection, the electric field is close to zero near the injecting surface. This situation is the so called Space Charge Limited Current (SCLC).

For strong injection  $C \gg 1$ , the parameter  $T$  determines whether there is instability or not. Below a certain value  $T_c$ , which depends on the nature of the surfaces, there is no motion and the current density

is given by:

$$j = \frac{9}{8} K \epsilon \frac{V^2}{d^3} \quad (1)$$

When  $T > T_c$  the liquid is put into motion in the form of hexagonal convection cells. The most unstable mode corresponds to a cell of diameter [3]:  $\lambda = 0.63d$ . The value of  $T_c$  is, for a free injecting surface [6],  $T_c = 95$ .

On the other hand there is an experimental requirement on the conductivity of the liquid used to observe this EHD instability. This is due to the fact that a space charge can only exist in a distance where the time of flight of the injected ions is less than the relaxation time of the charges in the volume of the liquid, that is:

$$\frac{d^2}{KV} < \frac{\epsilon}{\sigma}. \quad (2)$$

If this condition is not fulfilled the behavior would be ohmic, and there would be neither space charge nor instability.

The relaxation time for silicone oil is greater than 20 seconds, and  $K \sim 5 \times 10^{-10} \text{ m}^2/\text{Vs}$ , so  $d^2/KV \sim 10\text{s}$  for  $V_l = 200 \text{ V}$  and  $d = 1\text{mm}$ . Therefore, experiments with silicone oil are usually in conditions to have space charge effect. On the contrary, for Castor oil the relaxation time is about 0.4 seconds, so charge is relaxed before the ions could have time to fly a significant distance. This is the reason why we do not observe the first instability in Castor oil.

Our experimental conditions are those of the SCLC. This is a consequence of the fact that the mobility of ions in the air  $K_a$  is much higher than in the liquid  $K_l$ , along with the constancy of the electrical current through the surface of the liquid:  $K_a q_a E_a = K_l q_l E_l$  implies  $q_l \gg q_a$ .

The voltage drop across the liquid layer is related to the current flowing through it and its thickness by (1). As the instability appears for a critical voltage drop across the layer, it should be found experimentally that it appears for a given value of  $j d^3$ , or  $I \Omega^3$ , where  $I$  is the total measured current and  $\Omega$  the volume of liquid used. We have measured  $I \Omega^3 = 1.9 \pm 0.5 \times 10^{-26} \text{ Am}^3$ , which gives  $V_c = 290 \text{ V}$  and  $T_c = 270 \pm 70$ .

This value is well above the theoretical one  $T_c = 95$ , but there are a few reasons for this discrepancy. First, the theory [6] is valid when the injecting surface is an equipotential. This is not the case in our experimental arrangement. Also, we have assume that the current is uniformly distributed over the liq-

uid surface and, again, this not the case because the injected current has a distribution imposed by the tip-plane geometry. Another factor to be claimed for the discrepancies is the layer thickness, that appears to be non uniform, due to the meniscus near the Plexiglas ring. And last, since the instability is detected by visualization, the measured values represent an upper bound for the instability parameter.

In spite of all these reasons, the size of convective cells, the fact that we need a liquid with very low conductivity, and the correlation with the parameter  $I\Omega^3$  allow us to conclude that this first instability is the instability due to unipolar injection in the case of Space Charge Limited Current.

### Surface charge instability

The second instability is observable in both ohmic and non-ohmic liquids. From the point of view of the theoretical analysis, the ohmic case is easier to understand. Let us analyze it first.

Consider the model problem of Fig. 2. Between two parallel electrodes there is an air gap and a liquid layer of thickness  $L$  and  $d$ , respectively, with  $d \ll L$ . A voltage difference  $V$  is applied between both electrodes. While the transport of charge in air is due to the existence of a space charge, that we consider in the limit of SCLC, the liquid is ohmic. The equations are, in the air:

$$j = K_a E q, \quad \frac{dE}{dz} = \frac{q}{\epsilon_0} \quad (3)$$

and in the liquid:

$$j = \sigma E, \quad \frac{dE}{dz} = 0 \quad (4)$$

Here the current density  $j$  is constant and the same in both media. The boundary conditions are that the electric field is zero at the injector (this is equivalent to suppose that we have SCLC), and that there is an imposed voltage,

$$E(z = -L) = 0, \quad V = \int_{-L}^d E(z) dz \quad (5)$$

The charge in the liquid is a surface charge and it is given by  $\sigma_s = \epsilon E(z = 0^+) - \epsilon_0 E(z = 0^-)$ . From (3) the electric field in the air can be obtained, and from (5) the voltage is:

$$V = \frac{2}{3} \sqrt{\frac{2jL^3}{K_a \epsilon_0}} + \frac{j}{\sigma} d \quad (6)$$

The surface charge density is given by:

$$\sigma_s = \epsilon \frac{j}{\sigma} - \frac{3\epsilon_0 V}{2L} \quad (7)$$

where we have used  $d \ll L$ . If the liquid is a good conductor this charge density is of the opposite polarity that the injector and this situation has already been analyzed in the literature [7]. However, if the conductivity is low enough in order to have  $\epsilon j L / (\epsilon_0 \sigma V) \ll 1$ , the charge density becomes of the same polarity that the injector and can be approximate by:

$$\sigma_s = \epsilon \frac{j}{\sigma} \quad (8)$$

Eq. (6) and (8) help to understand the potential instability of the liquid surface. The surface is subjected to a pressure which is proportional to  $\sigma_s$ . Therefore, if  $d$  increases, and the voltage is kept constant,  $j$  decreases (from (6)) and the pressure onto the surface decreases. Alternatively, if  $d$  decreases, the pressure increases. This leads to the amplification of any perturbation of the liquid surface. Two other phenomena oppose to the deformation: the effect of the gravity and the surface tension. The balance between the three effect determines the instability threshold.

Melcher [7] has analyzed the instability of a liquid surface subjected to an electric field. Supposing that the surface is deformed in a sinusoidal wave  $z = \eta_0 \exp i(\omega t - kx)$ , he found the following relation between  $\omega$  and  $k$ :

$$\omega^2 = \left(g + \frac{\gamma k^2}{\rho} - \frac{P_e k}{\rho}\right) k \quad (9)$$

where  $P_e$  is the electrical pressure onto the surface and  $\gamma$  the surface tension.

Instability appears when  $\omega^2 < 0$ , that is,  $P_e < \rho g/k + \gamma k$ . The most unstable mode is  $k_c = \sqrt{\rho g/\gamma}$  and the instability threshold is, then:

$$P_e = 2\sqrt{\rho g \gamma} \quad (10)$$

If we take  $\gamma = 0.01$  N/m, this gives  $P_e = 20$  N/m<sup>2</sup> and  $\lambda_c = 1/k_c = 6$  mm.

The rose-window instability appears in castor oil without the previous existence of a space charge instability. Apart from the characteristic pattern, a fingerprint of this instability is that the size of the convection cells is about 5 mm at instability, quite independent of the thickness of the liquid layer. From the theoretical analysis we expect the instability to

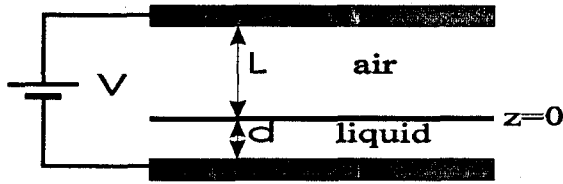


Figure 2: Geometry of the approximate model for the study of the surface charge instability.

appear for  $P_e = \sigma_s E \simeq \sigma_s V/L \simeq 20 \text{ N/m}^2$ . From (8) this quantity is related to  $IV/L$ .

We have measured the instability threshold for different values of  $L$ , ranging between 25 and 50 mm, and  $d$ , ranging from 0.4 to 1.0 mm, and the value obtained is:  $IV/L = 0.04 \pm 0.02 \text{ AV/m}$ . From this the experimental value for  $\sigma_s E$  is  $4 \pm 2 \text{ N/m}^2$ , to be compared with the theoretical prediction 20. Although this difference claims for further improvement of theory and experiments there are two facts that allow us to assert that the essential aspects of the theoretical model are correct: the first one is the size of the cells. The second one is the fact that the value  $IV/L$  has a low dispersion compared to the other parameters that have been changed.

Let us consider the same situation in a non ohmic liquid. The equations in the air are the same as before, while in the liquid we have:

$$j = K_l E q, \quad \frac{dE}{dz} = \frac{q}{\epsilon} \quad (11)$$

where, again,  $j$  is constant.

The boundary conditions are the same that in the ohmic case, but now we have the additional condition of continuity of  $\epsilon E$  through the interface, i. e. , there is not any surface density of charge.

Since the space charge distribution in the liquid corresponds to the case of SCLC, its gradient is very steep. Therefore, most of the charge that exists in the volume is close to the liquid surface. This situation should allow us to approximate the electrical behavior of the liquid layer as if it had an effective surface charge of value:

$$\sigma_s = \int_0^d q(z) dz = \int_0^d \frac{j}{K_l E} dz = \left( \frac{2j\epsilon d}{K_l} \right)^{1/2} \quad (12)$$

where the electric field is obtained solving (11) along with (3).

For silicone oil we have observed the rose-window instability for voltages higher than those needed for the space charge instability. As in castor oil, the size of the cells are about 5 mm. From the theoretical analysis we expect that the instability appears for a given value of the combination of parameter  $\sigma_s E = \sigma_s V/L$ . From 12 is  $\sigma_s E = \sqrt{2\epsilon/K_l S} \sqrt{IdV/L}$ . Experimentally we obtain  $\sqrt{IdV/L} = 4 \pm 1$  in SI units, and  $\sigma_s E = 17 \pm 5 \text{ N/m}^2$ , which compares well with the predicted value  $20 \text{ N/m}^2$ .

## Conclusion

We have encountered two types of instability in a layer of low conducting liquid subjected to corona current. The first instability corresponds to the classic EHD instability of liquids subjected to unipolar injection. Its characteristics are: hexagonal pattern of small cells,  $\lambda \sim d$ ; space charge limited current; the instability appears above of a certain value of  $Id^3$ ; it is not present in ohmic liquids.

The second instability is due to the presence of a surface charge of the same polarity as that of the corona current. It is characterized by: hexagonal pattern of large cells,  $\lambda > d$ ; the instability appears above a certain value of  $\sigma_s E$ ; the value of  $\sigma_s$  is related to the measured values  $I$  and  $V$  in different ways for ohmic or non ohmic liquids.

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