Video measurement of the muzzle velocity of a potato gun

Article *in* Physics Education · August 2011

DOI: 10.1088/0031-9120/46/5/017

Anthony Pollman
Naval Postgraduate School

Video Measurement of the Muzzle Velocity of a Potato Gun

Christopher Jasperson and Anthony Pollman
Physics Department
United States Naval Academy
Annapolis, MD 21402-1363

Using first principles, a theoretical equation for the maximum and actual muzzle velocities for a pneumatic cannon were recently derived. For a fixed barrel length, these equations suggest that muzzle velocity can be enhanced by maximizing the product of the initial pressure and volume of the propellant gas and decreasing the projectile mass. The present paper describes the results of experiments conducted to verify the validity of these theoretical equations. A high-speed video camera was used to quantify muzzle velocity for potatoes of varying mass exiting a pneumatic cannon for gauge pressures ranging from 310 to 830 kPa. The experiments verified that a friction modified version of the theoretical equation is qualitatively and quantitatively accurate for potato masses above 100 g.

INTRODUCTION

Potato cannons are popular for physics demonstrations, science fairs, and even father-son projects. They typically operate by one of three mechanisms: compressed air [1], explosive propellant [2], or vacuum breaking [3, 4]. Choice of operating mechanism is dictated by purpose or personal preference.

Construction of a compressed air or pneumatic potato cannon is fairly straight-forward. Gurstelle [5] outlines detailed specifications and construction in a book popular with hobbyists and professionals alike. The instructions are clear and complete, but due to the intended audience the physical principles involved are not explicitly addressed. Without that physical background, quantifying the performance expectations (such as range) before construction and optimizing the design after construction are nearly impossible.

Maximum range is a fundamental, useful performance criterion. Thus, maximum ranges for common weapons systems are listed in many military manuals [6] and are required to be memorized by members of the armed services [7]. For a smooth bore cannon in the absence of air resistance, range is dictated by firing angle and muzzle speed. The present work was motivated by the desire to optimize performance for a smooth bore, pneumatic potato cannon used in a college physics course to reinforce projectile motion theory and build student intuition.

THEORY

Modeling the potato as a frictionless piston without air leakage, Mungan [8] used Newton's second law, the ideal gas law, and the first law of thermodynamics to derive an analytic solution for the limiting (maximum) and actual muzzle speed for a potato exiting the barrel of a pneumatic cannon. For completeness, a summary of Mungan's derivation follows.

Begin by modeling the potato and barrel as a simple piston-cylinder system as shown in Figure (1). The left end of the cylinder is closed and right end of the cylinder is open to the atmosphere allowing the projectile to exit. In Figure (1), A is the cross-sectional area of the piston, m is the mass of the piston, x is the distance from the closed end to the piston face, n is the number of moles of gas in volume V (which is equal to Ax), p is the pressure of the gas, T is the temperature of the gas, and F is the force that the gas exerts on the piston face.

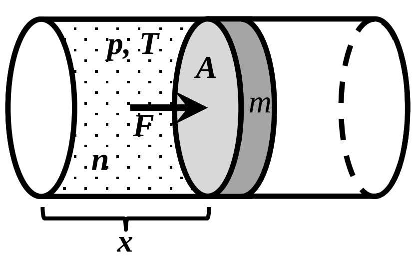

Figure 1. Sketch modeling a pneumatic potato gun as a piston-cylinder system in order to derive a governing equation for muzzle velocity. Piston (potato) is moving from left to right in the cylinder.

Mungan's derivation begins with the ideal gas law,

$$PV = nRT \tag{1}$$

where *R* is the universal gas constant. The force on a piston of cross-sectional area of *A* is *F* = *PA*. Solving for *p* and noting *V* = *Ax* allows us to re-write equation (1) for our piston-cylinder system,

$$Fx = nRT \tag{2}$$

Invoking Newton's second law, we can replace *F* in equation (2) with *ma* where m is the mass and *a* the acceleration of the piston. Prior to firing, the piston (projectile) is at position x_0, has velocity v_0, and the gas has temperature T_0. Thus initial acceleration of the piston can be written as,

$$a_0 = \frac{nRT_0}{mx_0} \tag{3}$$

In the absence of heat exchange with the surroundings, the first law of thermodynamics dictates that the internal energy, *U*, of the gas decreases as it does work on the piston. For an ideal gas, *U* = *ncT* where *c* is the molar specific heat at constant volume (3*R*/2 for a monoatomic and 5*R*/2 for a diatomic gas near room temperature). The work done on the piston by the gas can be written as *F dx* = *p dV* = (*nRT* / *x*) *dx*. This type of work is commonly known as "pressure-volume" or "PV" work. A balance relating the rate at which internal energy is transferred to work can be written as,

$$nc\,\frac{dT}{dt} = -\frac{nRT}{x}v \tag{4}$$

Replacing *nRT* with equation (2), and recalling that *F* = *ma*, equation (4) can be re-written as,

$$\frac{c}{R}\,\frac{d(ax)}{dt} = -a\,\frac{dx}{dt} \tag{5}$$

Mungan went on to show how equation (5) can be manipulated and integrated in several fairly sophisticated ways in order to obtain an expression for the limiting and actual muzzle speed for a projectile exiting the barrel of a smooth bore pneumatic cannon, namely,

$$v = v_\infty\sqrt{1 - \tilde{x}^{-R/c}} \tag{6}$$

where $$v_\infty = \sqrt{\frac{2ncT_0}{m}} \tag{7}$$

is the limiting or maximum theoretical velocity, $\tilde{x}$ is 5.2 for a diatomic gas (dry air), *R* is the gas constant, *c* is the molar specific heat at constant volume (5*R*/2 for a diatomic gas near room temperature), *n* is the number of moles of gas, T_0 is temperature of the gas, and *m* is the mass of the projectile (potato).

Mungan's analysis found that the speed of the potato projectile levels off after the volume of the gas has expanded to a few times its initial, compressed value. With this in mind, an optimum barrel length can be inferred for a known tank volume and pressure or Denny's equation can be used [9]. Once the barrel length is fixed, the velocity and hence range can be increased by maximizing the product of the initial pressure and volume of the gas and/or decreasing the projectile mass.

EXPERIMENTAL APPARATUS

A pneumatic potato cannon with a tank volume of 9040 mL and barrel volume of 3640 mL (5.08 cm ID, 1.80 m long) was used to fire potatoes of varying mass at gauge pressures of 310, 550, 690, 760, and 830 kPa. At least 15 potatoes were fired at each pressure. The cannon was constructed from commercially available, schedule 40 PVC. Maximum pressure was fixed at 830 kPa (120 psig) in order to ensure a factor of safety of 2.

Each potato was weighed before being loaded into the muzzle. The beveled end of the muzzle cut the potatoes to size and ensured an airtight fit. The "scrap" from each potato was subsequently weighed, and the mass of the projectile was obtained by taking the difference between the pre-cut and scrap measurements.

Barrel inner diameter was dictated by the size of an *average* potato. Once the diameter was fixed, the barrel length was chosen based on space restrictions and by performing trade-off estimates of the ideal tank-to-barrel volume ratio needed to obtain maximum velocity [8] in the presence of friction [9]. The overall goal was to maximize muzzle speed and thus range. Friction estimates were obtained by pulling a potato through the barrel at constant velocity with a force plate and data acquisition system attached. Taking friction to be independent of the speed, Mungan's equation was modified to account for reduced speed due to friction.

Speed was determined by recording a potato exiting the muzzle with a high-speed video camera operating at 1000 frames per second. An array of three fluorescent lights provided the illumination. Length scale (pixel count) was calibrated before every shot. Each data point was generated using a series of about 8 frames (more for slower, less for faster moving potatoes). Knowing the distance travelled between frames and the exposure rate, speed was calculated. Figure 1 is a schematic representation of the experimental setup. Figure 2 is an example of a single frame from the raw video data.

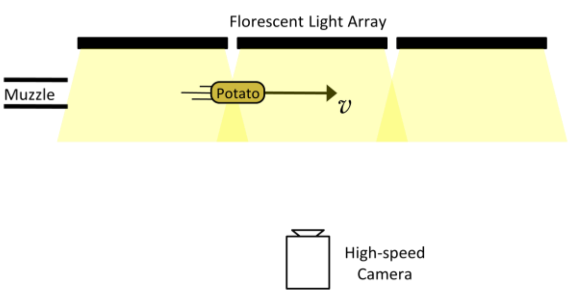

Figure 1. Experimental apparatus used to determine muzzle speed for a potato leaving a pneumatic cannon.

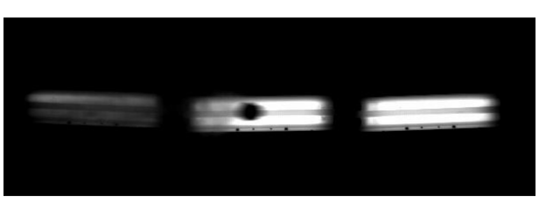

Figure 2. Single frame from high-speed video camera showing potato moving from left to right while being illuminated from behind.

EXPERIMENTAL RESULTS

Figure 3 is a plot summarizing all experimental results. The plot and best fit curves were generated using Microsoft Excel. The curves are "power" fits (of the form $y = Ax^b$) and intended only to give a general idea about the nature of the data.

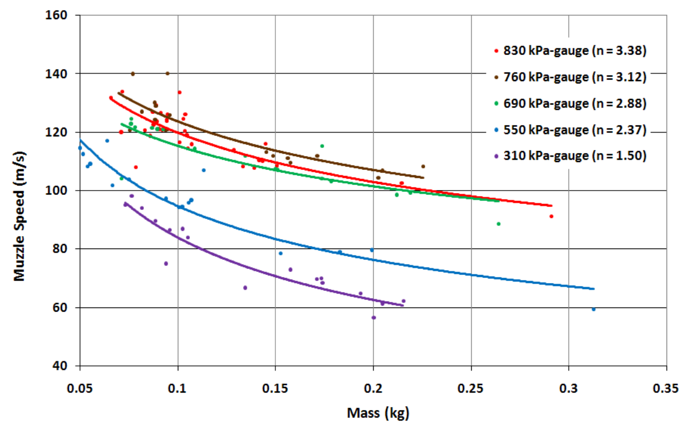

Figure 3. Muzzle speed as a function of potato mass. Colored lines show experimental results for constant pressure (particular valued of *n*).

Inspection of Figure 3 verifies Mungan's hypothesis that velocity can be increased by maximizing the product of the initial pressure and volume of the working fluid and/or by decreasing the projectile mass.

Figure 4 is a plot of the data for masses ranging from 100 to 250 g. Due to a fixed barrel diameter, potatoes larger than 250 *g* become undesirably long and tend to tumble after exiting the muzzle. Potatoes smaller than 100 *g* suffer from malfunctions because they often fail to create an airtight seal. Thus, muzzle speeds observed for 100 to 250 *g* potatoes can be considered suitable for analysis and direct comparison to the theoretical muzzle speed equation. An error estimate of +/- 5% was obtained by performing uncertainty analysis and error propagation calculations as outlined in Taylor [10].

As mentioned, friction estimates were obtained by pulling a potato through the barrel at constant velocity with a force plate and data acquisition system attached. Assuming kinetic friction to be independent of the speed, we can write,

$$P_e = \frac{F_e}{A} = \frac{(F - f)}{A} = P - \frac{f}{A} \quad (8)$$

where P_e is the effective pressure, F_e is the effective force, F is the ideal force expected from a given gauge pressure, A is the cross-sectional area of the barrel, P is the gauge pressure, and f is the kinetic friction force. The kinetic friction force was estimated to be between 1.00 and 2.00 *N*. This seems like a small force, but potatoes are carbohydrates and the barrel can be slightly moist from the loading process. It is well known that carbohydrates can be used as lubricants. Perform the following thought experiment: jam a potato thru the beveled end of a PVC tube with open ends – then raise the pipe to a vertical position such that the potato is prepared to slide the length of the pipe to the ground. Then ask: how much mass should I place on top of the potato in order to have it slide the length of this pipe? When framed in this way and taking into account the slippery nature of the potato and the smoothness of the PVC, 1.00 or 2.00 *N* or approximately 100-200 *g* seems reasonable. Keep in mind that static friction would have to be initially overcome. Plugging 1.00 *N* into equation (8), we can solve for $P - P_e$. Plugging this into equation (1) allows us to solve for the Δn associated with this ΔP, which in turn can be plugged into equation (6) to find the velocity change due to the friction force. For P = 830 *kPa* and f = 1.00 *N*, the velocity is reduced by approximately 15.0 *m/s* due to friction in the barrel. This result is shown as the lower dotted lines in Figure 4. For P = 830 *kPa* and f = 2.00 *N*, the velocity is reduced by approximately 21.0 *m/s* due to friction in the barrel. This effect is not shown in Figure 4.

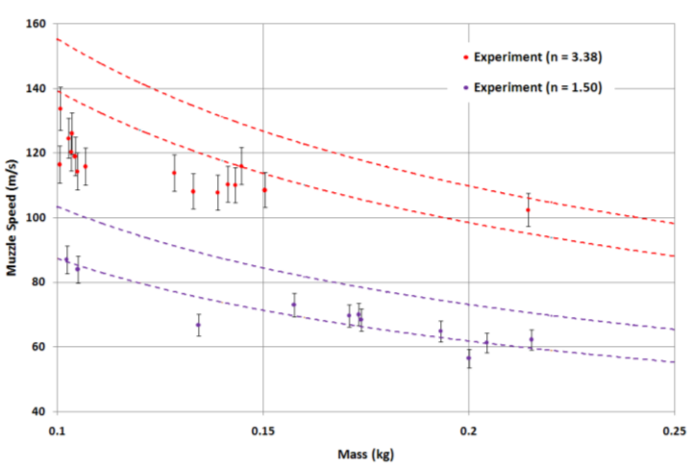

Figure 4. Muzzle speed as a function of potato mass. Sets of dashed lines show the theoretical estimate (upper) and the friction modified theoretical estimate (lower) for n = 3.38 and n = 1.50, respectively.

Examination of Figure 4 reveals that the experimental data is qualitatively similar to the theoretical estimate for the operating range. However, the theoretical equation overestimates muzzle speed. This is not surprising because the theory assumed a frictionless barrel. The friction modified theoretical estimate shows acceptable agreement for lower operating pressures. However, for smaller potatoes at higher operating pressures, the friction modified estimate still generally overestimates the experimental results. This slight discrepancy is related to the divergence predicted by the theoretical equation. Inspection of the equation reveals that velocity will diverge as mass approaches zero. Considering the combined effects of divergence and seal malfunctions, it is no surprise that the theoretical equation and data do not match well for masses less than 100 g.

SUMMARY

An experiment to validate a theoretical muzzle velocity equation was undertaken. Using a high-speed video camera, muzzle speed was determined for potatoes with masses ranging from 50 to 310 g and tank gauge pressures from 310 to 830 kPa. The theoretical equation shows good agreement with the data for masses above 100 g.

CAUTIONARY NOTE

A 200 g potato with a velocity of 100 m/s has 1000 J of kinetic energy. This is nearly twice the average energy of a .45 caliber handgun [11]. A student studying Systems Engineering at the United States Naval Academy accidently shot herself in the face with an explosive propellant potato gun in the spring of 2010. The shot fractured portions of her nasal, maxillary, and zygomatic bones (the "eye socket"). She was subsequently found unfit for service due to the disabilities sufferer and was discharged from the service. (Optional) Please see attached high-speed video of a potato fired into a concrete block and watermelon as a warning of the potential danger involved (End Optional).

Due to the potential for injury, thoughtful experimental procedures with built in safety checks and proper supervision are essential. Safety equipment such as ear plugs and safety glasses should be used at all times. Maintenance and regular "pressure" checks are required. Never operate at pressures outside the factor of safety or beyond design limits. Calculations or engineering estimates should be performed anytime procedures or operating conditions are changed. And finally, potato guns should be handled like any other weapon; they should always be treated as if they are loaded and never pointed at anything that you do not intend to shoot.

ACKNOWLEDGEMENTS

The authors would like to thank Gary Bishop, Jeff Walbert, and Bob Catalani for their technical support, expert advice, and patience.

REFERENCES

[1] B. Taylor, "Recoil Experiments Using a Compressed Air Cannon," *Phys. Teach.* **44**, 582-584 (2006).

[2] M. Courtney & A. Courtney, "Acoustic Measurement of Potato Cannon Velocity," *Phys. Teach.* **45**, 496-497 (2007).

[3] E. Ayars & L. Buchholtz, "Analysis of the Vacuum Cannon," *Am. J. Phys.* **72**, 961-963 (2004).

[4] R. Peterson, B. Pulford, & K. Stein, "The Ping-Pong Cannon: A Closer Look," *Phys. Teach.* **43**, 22-25 (2005).

[5] W. Gurstelle, *Backyard Ballistics: Build Potato Cannons, Paper Match Rockets, Cincinnati Fire Kites, Tennis Ball Mortars, and More Dynamite Devices*, Chicago Press Review: Chicago (2001).

[6] *Commander's Tactical Handbook*, U.S. Marine Corps Reference Publication 3-11.1A, Washington D.C. (1998).

[7] Marine Corps Common Skills, Training and Readiness Manual (NAVMC 3500), Vol. 1 & 2, Washington, D.C. (2007).

[8] C. Mungan, "Internal ballistics of a pneumatic potato cannon," *Eur. J. Phys.* **30**, 453-457 (2009).

[9] M. Denny, "The Internal Ballistics of an Air Gun," *Phys. Teach.* **49**, 81-83 (2011).

[10] J. Taylor, *An Introduction to Error Analysis: The Study of Uncertainties in Physical Measurements*, University Science Books: Mill Valley, CA (1982).

[11] *Muzzle Energy*, Available: http://en.wikipedia.org/wiki/Muzzle_energy

Christopher Jasperson is an undergraduate student in applied physics at the United States Naval Academy. He was recently selected for service in the Marine Corps and is interested in becoming an artillery officer or pilot. This paper was published with data he gathered during a portion of his Capstone Project.

Anthony Pollman is a Marine Captain assigned to the Naval Academy as a physics instructor. He obtained an M.S. in nuclear engineering from Purdue University in 2004. He is currently pursuing a Ph.D. in mechanical engineering from the University of Maryland, College Park. His research interests include thermalhydraulics, computational fluid dynamics, nuclear reactor safety, and "less than lethal" weapons systems.