Modeling the exit velocity of a compressed air cannon

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The use of compressed air cannons in an undergraduate laboratory provides a way to illustrate the connection between diverse physics concepts, such as conservation of momentum, the work-kinetic energy theorem, gas expansion, air drag, and elementary Newtonian mechanics. However, it is not clear whether the expansion of the gas in the cannon is an adiabatic or an isothermal process. We built an air cannon that utilizes a diaphragm valve to release the pressurized gas and found that neither process accurately predicts the exit velocity of our projectile. We discuss a model based on the flow of air through the valve, which is in much better agreement with our data.

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Although the description of the internal dynamics of a cannon is complicated, recent proposals have focused on modeling the dynamics of a simplified cannon that uses the expansion of compressed gas to accelerate a projectile. These proposals disagree on whether the gas expansion should be described as an adiabatic[1] or an isothermal[2] process. These models give different predictions for the exit velocity of a projectile as a function of the initial gas pressure. Because we wish to develop an undergraduate physics experiment that uses a compressed gas cannon to illustrate conservation of momentum[3] and the work-kinetic energy theorem,[4] we wanted to develop an accurate model which predicts the internal dynamics of the cannon. Also this work can be extended to investigate projectile motion and air drag using elementary Newtonian kinematics.[5]

We constructed a compressed air cannon to measure the exit velocity of a projectile as a function of the initial reservoir pressure. The models in Refs. 1 and 2 fail to address how the gas becomes pressurized prior to firing the cannon. Our implementation of a compressed air cannon requires the use of a valve to create a reservoir of high-pressure gas, which is then released to accelerate the projectile. We propose a model that takes into account the air flow through the valve. Our data are in better agreement with our model than with prior proposals.

We begin by reviewing the adiabatic and isothermal gas expansion models. We want to know the exit speed v of a projectile of mass m, launched from a cannon with initial gas pressure P_0. We model the cannon as a reservoir of volume V_0 connected to a long barrel of cross-sectional area A loaded with the projectile (see Fig. 1). As the pressurized gas in the reservoir expands, the gas provides a force to propel the projectile along the length L of the barrel before it exits the barrel. The total force on the projectile in the barrel is assumed to be the sum of the force from the gas in the reservoir $AP(x)$, the force from the air in the barrel at atmospheric pressure AP_{atm}, and a small linear frictional force f between the projectile and the wall of the barrel [see Fig. 1(a)]. The equation of motion is

$$F = m\frac{d^2x}{dt^2} = mv\frac{dv}{dx} = AP(x) - AP_{\text{atm}} - f. \tag{1}$$

The cannon reservoir volume increases as the projectile moves down the barrel: $V(x) = V_0 + Ax$. If we assume adiabatic expansion as illustrated in Fig. 1(b), we know that

$$P(x)(V_0 + Ax)^{\gamma} = P_0V_0^{\gamma}, \tag{2}$$

where $\gamma = 7/5$ for diatomic gases such as air and $V_0 = Ax_0$. Thus, in the adiabatic case, we have

$$mv_{\text{ad}}\frac{dv_{\text{ad}}}{dx} = A\left(\frac{P_0V_0^{\gamma}}{(V_0+Ax)^{\gamma}} - P_{\text{atm}}\right) - f, \tag{3}$$

which yields an exit velocity at $x = L$ of

$$v_{\text{ad}} = \sqrt{\frac{2}{m}\left(\frac{P_0V_0}{\gamma-1}\left(1-\left(\frac{V_0}{AL+V_0}\right)^{\gamma-1}\right) - ALP_{\text{atm}} - Lf\right)}. \tag{4}$$

The second model, illustrated in Fig. 1(c), models the expansion of the gas to be quasistatic and isothermal. Because $P(x)(V_0 + Ax) = P_0V_0$, the exit velocity is

$$v_{\text{is}} = \sqrt{\frac{2}{m}\left(P_0V_0\ln\left(1+\frac{AL}{V_0}\right) - ALP_{\text{atm}} - Lf\right)}. \tag{5}$$

Both of these models, however, are over-simplified descriptions of real cannons. A real pneumatic air cannon has a valve between the reservoir and the barrel to allow pressurization of the reservoir before firing the projectile. Although we can imagine a perfect valve that does not have any appreciable effect on the air which flows past it, such a valve is difficult to realize in practice. Because the air flow through a real valve is a function of the pressure drop across the valve, it is unreasonable to ignore the effect of the valve, and thus the pressure in the barrel is not necessarily the same as the pressure in the reservoir.[6]

We propose a model, shown in Fig. 1(d), which takes into account the flow rate of air through the valve. According to Ref. 7, the molecular flow rate Q through the valve is a function of the ratio

$$r \equiv \frac{P(t) - P_b(t)}{P(t)}, \tag{6}$$

where $P(t)$ is the pressure in the reservoir and $P_b(t)$ is the pressure in the barrel of the cannon at time t. When the pressure difference is large enough, this ratio saturates to

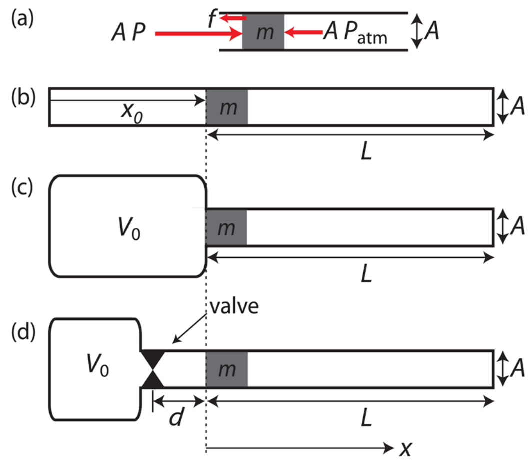

Fig. 1. (a) The forces on the projectile in the barrel of a compressed air cannon. The pressure P is due to the compressed gas. (b) The explosive gas expands adiabatically,[1] which is described by Eq. (4). (c) The gas from a pressurized reservoir expands isothermally[2] [see Eq. (5)]. (d) The expansion of the gas is limited by a valve with a finite flow factor.

$r \rightarrow r_{\max}$, which is limited by the geometry of the valve and is typically between 0.2 and 0.9. There are thus two flow regimes. In the nonchoked regime where $r < r_{\max}$, or equivalently $P(t) < P_b(t)/(1 - r_{\max})$, the flow is modeled empirically as a function of the pressure differential between the tank pressure and the barrel pressure:[7]

$$Q = BP(t)C_v\left(1 - \frac{r}{3r_{\max}}\right)\sqrt{\frac{r}{G_gTZ}}. \tag{7}$$

In the choked regime, $P(t) \geq P_b(t)/(1 - r_{\max})$, the flow is limited by the geometry of the valve:

$$Q = \frac{2}{3}BP(t)C_v\sqrt{\frac{r_{\max}}{G_gTZ}}. \tag{8}$$

In Eqs. (7) and (8) $G_g = 1$ is the specific gravity of air, $T \approx 293\,\mathrm{K}$ is the temperature in the reservoir which is assumed to be constant, and $Z \approx 1$ is the compressibility factor. The flow coefficient C_v of the valve is a dimensionless parameter which describes the flow capacity of the valve. The final parameter $B = 3.11 \times 10^{19}\sqrt{\mathrm{K}}/(\mathrm{Pa \cdot s})$ is an engineering constant which converts the quantity into units of molecules per time.[7]

We model the gas expansion in the barrel and the tank using the ideal gas law:

$$P(t)V_0 = N(t)k_BT \tag{9}$$

$$P_b(t)A(d + x(t)) = N_b(t)k_BT, \tag{10}$$

where N and N_b are the number of molecules in the tank and barrel, respectively. The number of molecules in the tank and barrel are governed by the flow of molecules between them through the valve:

$$\frac{dN}{dt} = -Q \tag{11}$$

$$\frac{dN_b}{dt} = Q. \tag{12}$$

Equation (1) governs the position of the projectile. This equation, when combined with Eqs. (7)–(12) and the initial conditions, is numerically solved to give

$$v_{\text{valve}} = \left.\frac{dx}{dt}\right|_{x=L} \tag{13}$$

as a function of initial pressure P_0.

To test this model, we constructed an air cannon using a steel air tank with a volume of $V_0 = 4.196 \pm 0.010$ L (all measurements given to the 95% confidence interval) as the pressure reservoir. We attached a silicon cell pressure transducer (Omegadyne Model PX309-100GV), a thermocouple (Omega Model TC-K-NPT-E-72), a solenoid-actuated diaphragm valve (Granzow Model 21HN5KY160-14W), and an air intake hand valve to the tank as shown in Fig. 2. We used a seamless stainless steel (304/304 L) threaded pipe for the barrel with a diameter of 1.913 ± 0.013 cm and a total length of 91.6 ± 0.2 cm. We measured the exit velocity using two optical photogates: one positioned the end of the barrel, the other $\ell = 24.6 \pm 0.9$ cm away from the first. The diaphragm valve opens when a current of 440 mA activates a solenoid in the valve. Data acquisition was triggered when an ammeter connected to the solenoid actuator circuit read an increasing current across the 5 mA level.

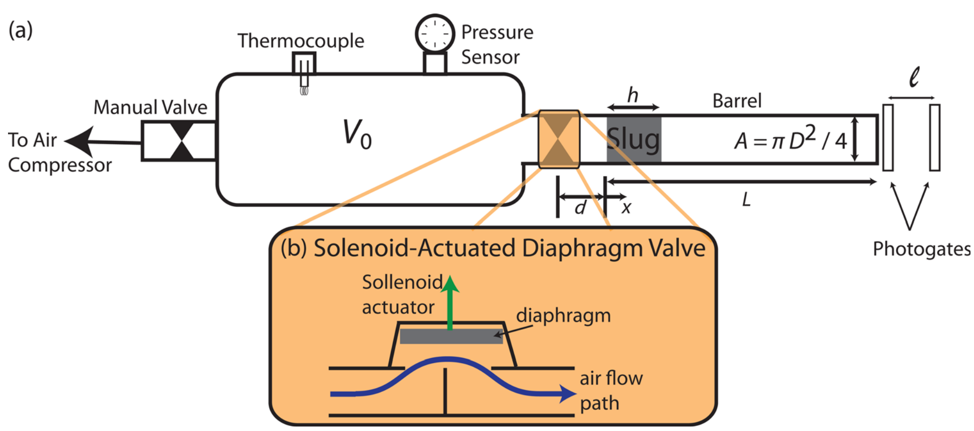

Fig. 2. (a) A schematic of the air cannon. The tank is a reservoir with a volume $V_0 = 4.196 \pm 0.010\,\mathrm{L}$ at initial pressure P_0. (b) The tank is discharged using a diaphragm valve with the associated flow factor C_v. The pressurized air propels the projectile, whose height and mass are $h = 4.8019 \pm 0.0006\,\mathrm{cm}$ and $m = 19.40 \pm 0.05\,\mathrm{g}$, a distance $L = 88.25 \pm 0.11\,\mathrm{cm}$ out of the barrel of cross-sectional area $A = 2.87233 \pm 0.00003\,\mathrm{cm}^2$, according to Eq. (1). The exit velocity of the projectile is determined using the two photogates at the end of the barrel spaced $\ell = 24.6 \pm 0.9\,\mathrm{cm}$ apart.

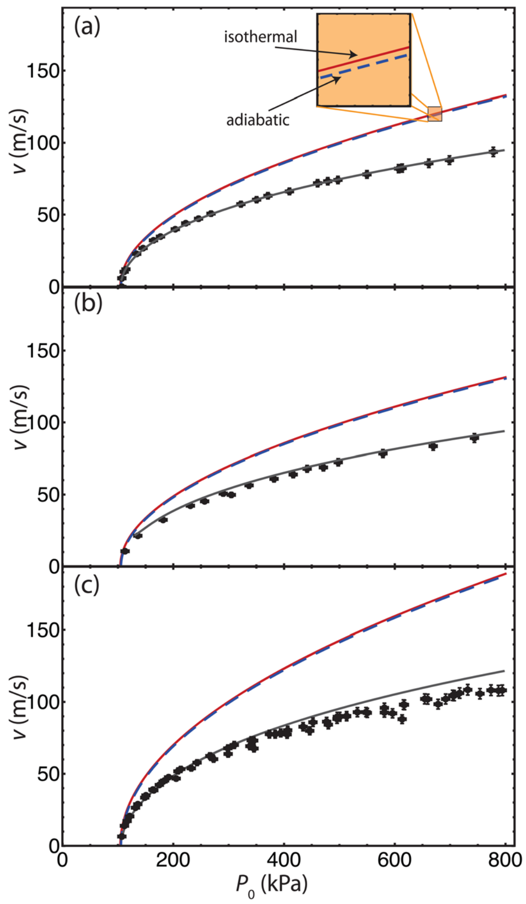

Fig. 3. Exit velocity as a function of initial reservoir pressure P_0 of (a) a plastic projectile with $m = 19.40 \pm 0.05$ g and $h = 4.8019 \pm 0.0006$ cm, (b) an aluminum projectile with $m = 19.90 \pm 0.05$ g and $h = 2.5923 \pm 0.0006$ cm, and (c) a plastic projectile with $m = 9.60 \pm 0.05$ g and $h = 2.3929 \pm 0.0006$ cm. The upper solid curve is the result of the isothermal model and the dashed curve is the result of the adiabatic model. The two models give similar results because the temperature drop associated with the adiabatic expansion is so small. Our data disagree with both. The lower solid curve shown in (a) is a plot of our model fit with the barrel parameters $r_{\max} = 0.80 \pm 0.11$ and $C_v = 1.93 \pm 0.04$. The lower solid curves in (b) and (c) are numerical models using the fit parameters from (a) and the appropriate projectile properties.

We loaded the cannon with a low-friction cylindrical plastic (acetal copolymer) projectile of mass $m = 19.40 \pm 0.05$ g, height $h = 4.8019 \pm 0.0006$ cm, and diameter $D = 1.9124 \pm 0.0006$ cm. The diameter of the projectile was such that it just fit into the barrel of the cannon. We tested whether air could escape from around the edges of the projectile by closing the diaphragm valve and attempting to load the cannon. The projectile was sufficiently airtight that it built back pressure when we tried to insert it.

We loaded the cannon by sliding the projectile into the barrel using a steel rod to push it in to the specific length $L = 88.25 \pm 0.11$ cm. We used an air compressor to pressurize the tank to the desired initial pressure P_0. We typically waited two to three minutes after closing the hand valve to allow the pressure reading in the reservoir to stabilize before opening the diaphragm valve to fire the cannon.

We collected data for the exit velocity v of the projectile as a function of P_0 [see Fig. 3(a)]. Our data disagree with both the adiabatic and isothermal models. When we performed a manual two-parameter fit of our data to the valve flow model, we obtained much better agreement. Our fit yields $r_{\max} = 0.80 \pm 0.11$ and $C_v = 1.93 \pm 0.04$.

In all three models, we ignored the frictional term f because we found that the introduction of a frictional term in the adiabatic and isothermal models corresponds to a horizontal shift of the model. Because our data have a very small horizontal offset, we take $f \approx 0$ for all of our calculations. We also find that a very small initial tank pressure above atmospheric pressure ejects the projectile, suggesting that f is small.

We also performed an experiment with an aluminum projectile of about the same mass and half the length ($m = 19.90 \pm 0.05$ g and $h = 2.5923 \pm 0.0006$ cm) and obtained the data shown in Fig. 3(b). The model, using the same fit parameters, is in good agreement with the data.

Finally, we performed an experiment with a plastic projectile of approximately half the length and mass of the first ($m = 9.60 \pm 0.05$ g and $h = 2.3929 \pm 0.0006$ cm). These data are shown in Fig. 3(c). It is evident that our model overshoots the data for high P_0, using the same fit parameters as before. We believe that this discrepancy is due to quadratic air drag, which has a larger effect at higher velocities (which occur at higher initial pressures). Our model would need to be improved to incorporate this effect. The reason we did not see this problem in Figs. 3(a) and 3(b) is that the larger mass of the projectile means that the acceleration due to drag would be smaller.

In conclusion, our exit velocity data disagree with previous models that ignore the pressure drop across the valve of the air cannon. We have here presented a model that accounts for the flow of gas through the valve and is in better agreement with the data.

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