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Vibrations of Evaporating Liquid Drops*

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An unusual type of vibration of flattened spheroidal liquid drops, occurring under certain conditions, is described and illustrated. Appropriate liquids placed on a sufficiently hot horizontal surface float on a film of their own vapor, which vapor insulates the drop so that it evaporates quite slowly, this phenomenon being known as the Leidenfrost effect. It is under these conditions that drops have been found capable of sustaining large amplitude regular radial vibrations whose driving force is probably related to the radial flow of vapor below the drops. The vibrating drops are approximately polygons which may have any of 2, 3, 4, . . . n sides, seen by the eye as drops with 4, 6, 8 . . . $(2n+2)$ regular lobes around the circumference. Methods of exciting and recording the effect are described.

DURING a routine operation on another project, one of the authors (NJH) noticed that the behavior of water drops on a hot plate appeared to be more complicated than that generally described as the Leidenfrost effect. Subsequent investigation has lead to the observation that evaporating liquid drops floating on a film of their vapor undergo a series of regular, predominantly radial, large amplitude vibrations in the horizontal plane under appropriate conditions. This effect occurs to an extent which has been either unknown or is so little known as to merit reporting at this time to bring it to the attention of theoretical investigators in the field of waves and vibrations.

A few words on the Leidenfrost effect might be appropriate since many modern physicists do not recognize it under this name. It has been known since before the time of Faraday and is usually mentioned only in older, more obscure physics texts. The effect is essentially that of liquid drops which float on a film of their own vapor when placed on a sufficiently hot horizontal surface. The liquid does not make contact with the hot surface, and if the hot-plate be allowed to cool somewhat, the vapor film is not maintained, whereupon the drop contacts the plate and almost immediately evaporates. The usual descriptions of the effect include mention of the low thermal conductivity of the vapor film which keeps the liquid well below boiling even when the hot-plate is considerably above the boiling temperature of the liquid, the frictionless appearance of the random horizontal motion of the suspended drop, and other related phenomena. Such drops have been spoken of as being in the spheroidal state, which is of course simply the liquid state under the influence of the vapor film, gravity, surface tension, heat, and other factors. The drops are described as round in the horizontal plane and flattened spheroids in the vertical plane. (The effect has probably been noticed in greater or lesser degree by anyone working around an old-style kitchen stove.)

Observations of the vibrational phenomena which can occur with such suspended drops have been apparently obscured in the past by translational and rotational motion of the drop and possibly by the lack of modern stroboscopic and photographic tools. The effect can be seen by the unaided eye when such extraneous motion is minimized and evaporation proceeds. These vibrations give rise to beautiful symmetrical patterns which, because of the persistence of vision, appear (in the higher modes) to the eye as solid geometric figures with an even number of smooth lobes around the circumference of the drop. (The figure is in the horizontal plane; the drops depart greatly from round in the vertical plane under the influence of gravity.) The vibrating drops as seen stroboscopically or with high speed photography appear as either sausage shapes (the basic mode), triangles, squares, pentagons, etc., up to figures with a number of sides too great to be resolved. The corners are rounded and the sides are either approximately straight (see illustrative sketches—Figs. 1(a), 1(b), 1(c)) or indented (Figs. 1(d), 1(e), 1(f)) depending upon the part of the cycle at the instant of observation. Thus the drops have either 2, 3, 4, 5... $(n+1)$ sides, which become corners one-half cycle later with the result that they are seen by the eye or by low speed photography as figures with either 4, 6, 8, 10... $(2n+2)$ lobes, respectively (Figs. 1(g), 1(h), 1(i)). One does not ordinarily conceive of water drops as being square, but under the conditions of this experiment they can be polygons (vibrating) of any type.

In describing this phenomena, it is convenient to make a comparison with a steady state, high amplitude, standing wave system on a string under tension between two fixed points. Figure 2 compares the standing wave system for a string of eight loops with the equivalent wave system around the circumference of a vibrating drop. (The horizontal line in Fig. 2(a) and the circle in Fig. 2(b) are drawn in to represent the nonvibration reference level.) For convenience, the analogous vibration peaks of the string and liquid edge are numbered alike. Thus positive peak amplitudes at a certain time in each case are numbered 1, 3, 5, 7 and negative peak amplitudes 2, 4, 6, 8 at the same time. One half-cycle later the positive peaks are 10, 12, 14, 16 and the negative peaks 9, 11, 13, 15. For the string, a high speed photograph taken at the instant of minimum amplitude

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would show a straight horizontal line; presumably one would see a circle at the same instant for the drop. Since random high speed photographs are not likely to be taken at exactly the time of minimum (or maximum) amplitude, the drops take on the sausage-triangle-square-etc. appearance described previously.

The drop of Fig. 2, seen at high speed as a square, corresponds to the string with a total of 8 loops (4 positive and 4 negative). Although the string can vibrate in modes with any number of total loops from 1 to n, (high speed view) the drop cannot vibrate in modes which have an odd number of loops at any instant, e.g., no drop vibration equivalent to 7 string loops (4 positive and 3 negative or vice versa) exists; the next drop mode lower than the square of 8 loops is the triangular drop analogous to 6 total instantaneous loops for the string. The drop must have an equal number of positive and negative peaks at any instant since otherwise two positive (or negative) peaks would have to exist next to each other. Moreover, there is no mode for an unrestrained drop equivalent to the string with a total of 2 loops at a given instant; the "sausage" mode is the lowest for the drop and corresponds to the string with a total of 4 loops (2 positive and 2 negative).

Commenting further on Fig. 1 as an aid to interpretation of the photographs which follow, we find that g, h, and i also indicate how the first three modes can appear to the eye using edgewise illumination while j, k, and l show how the same vibrating forms can appear when lightly colored and illuminated from above. In these latter cases the contrast between the center and the lobes is the result of the center portion, common to all phases of the drop, being viewed continuously whereas the two sets of lobes are seen alternately and hence for a smaller total interval.

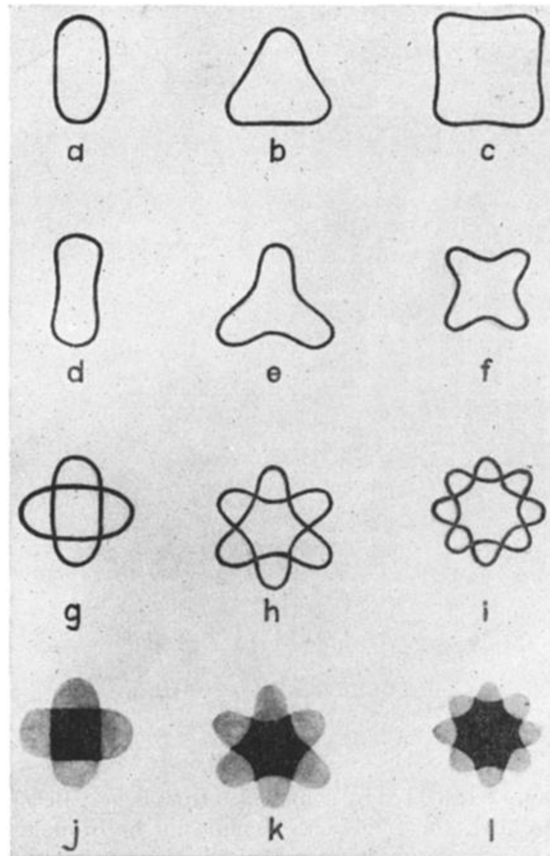

FIG. 1.

A systematic quantitative examination of the effect has not been completed, but a general idea of the frequencies involved can be seen from the following typical individual results. A large drop has been seen to vibrate as a "sausage" and to stay in this mode until quite small, while the frequency changes continuously from about 4 to 10 cycles/sec; the vibrating triangle has had a frequency range from 13 to 16 cycles/sec while decreasing in size; similarly the square from 15 to 30 cycles/sec, the hexagon from 36 to 54 cycles/sec, etc.

Water was used for the present experiments although there are other suitable liquids. (Liquid air exhibits the Leidenfrost effect but none is available here to investigate whether it vibrates as do more ordinary liquids.) It was used with and without various detergents and coloring materials such as dyes, inks, and solid suspensions. Hot surfaces used included iron and aluminum plates and porcelain and fused silica dishes which were carefully leveled and heated with a gas burner. In order to minimize horizontal motion of the drops, slight concavities of various depths and diameters in the heating surfaces were tried with varying degrees of success. Rotational motion was less when translation was minimized; the low speed photos were usually taken by waiting for intervals when both interfering motions were absent.

Illumination was either from overhead spotlights or a concentric fluorescent tube with the drop in the center and in the same plane. The latter provided edgewise illumination which was excellent for observation but not quite bright enough for the best photography. When using blackened drops in a fused silica dish illumination was from below, which provided good edge contrast. With a concentric lamp the drop was placed in a slight concavity at the top of a vertical steel 1-inch

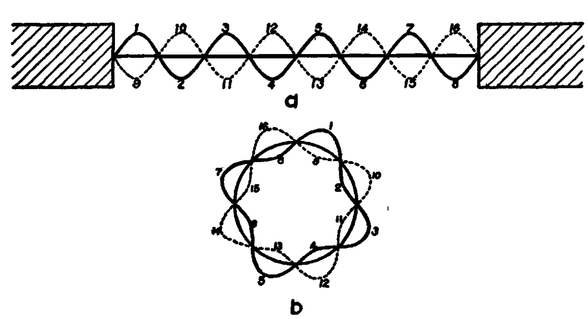

FIG. 2.

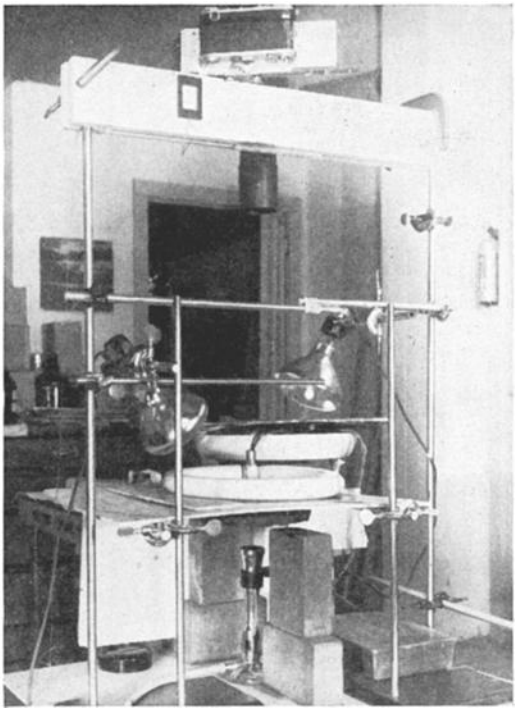

FIG. 3.

diameter bar heated by conduction from below. Because of the heat, the camera lens could not be brought as close as was desirable for adequate image size without cumbersome protective measures, but by using the objective and prism of a war surplus elbow telescope together with the shutter of a Contax camera a convenient image was obtained with the assembly well away from the heat region. Figure 3 shows the over-all view of this arrangement which was used for most of the accompanying photographs; various lamps are in position and the back of the Contax can be seen at the top.

When the surface is hot enough, water is added with a dropper until the drop is not quite big enough to slip to one side of the concavity. As it slowly evaporates, it will usually (but not always) approach a condition where the smooth round drop becomes wrinkled on the surface and appears to go into a strong resonance (although it might be a forced vibration), sometimes with sufficient amplitude to destroy the drop by loss of a lobe or by the ejection of many small drops (water-drop model of nuclear spallation?). Usually the vibrating drop appears as a strikingly beautiful stationary or slowly rotating solid figure maintaining its form either fleetingly or for an interval of the order of minutes, after which it suddenly returns to round or to a figure with a different number of lobes. Much greater sided figures sometimes appear fleetingly between such transitions, accompanied by complicated surface patterns which have proven very difficult to record with clarity in a photograph. When viewed with a stroboscope adjusted slightly off the frequency of drop vibration, the drop appears to be alive, "breathing" gently in and out as the form slowly exchanges sides and corners.

Figures 4 through 13 are photographs of drops vibrating in various modes under a variety of coloring and lighting conditions. For convenience these conditions and drop types are summarized in Table I.

From what was said of the sketches of Fig. 1, together with the coloring and lighting summary above, these figures need little further comment. All numbers of lobes from 4 up, in steps of 2, have been observed to the limit of resolution by the eye, although not all have been photographed. Figures 9–12 viewed in sequence bring to mind the usual example in elementary calculus of a many-sided figure having an area which is a first approximation to that of a circle and which becomes equal as the number of sides approaches infinity. (Drops which look at first glance to be round can often be seen to have the tiniest of serrations around the edge suggesting that the drop is on its way to "vibrating into roundness.")

A number of hypotheses suggest themselves when considering the mechanism by which these vibrations are supported but further experimental work is in order to elucidate them. It was thought that the driving force might originate as the drop touched the plate and continued to do so at each vibration, but it was found by connecting an electric circuit from the drop to the plate that no such contact was being made through the vapor film while vibrations were occurring, except for quite occasional and sporadic ones which could not support continuous steady state vibrations. It was found, by the use of a fine light powder, that the vapor has a significantly high radial velocity outward from between the drop and hot plate. This might give rise to an edge flutter like that produced as a result of the Bernoulli effect when blowing under a piece of paper on a flat surface, which vertical vibrations then excite the radial vibrations when the proper drop mass is approached during evaporation. But if they exist, such vertical vibrations at the edge have a very much smaller amplitude than do the radial vibrations. Eddies might be formed and cast off as the vapor flows upwards around the drop, thus providing a driving force. Bubbles of vapor can be seen rising through the center of the drop when the drops are large but, to the eye, this does not occur in a manner which gives rise to a periodic driving force, nor is it visible after the drops have vibrated awhile. Experiments have been designed to test these and other conjectures.

Still other unexplained phenomena accompany the Leidenfrost effect and the radial vibration effect, e.g., a round nonvibrating drop (or possibly its surface) is often seen to rotate very rapidly in any plane, occasionally stopping and reversing the direction of rotation; side forces are present which cause drops on a flat plate continuously to move back and forth horizontally; complicated but regular patterns appear and disappear


Fig. 4(a).

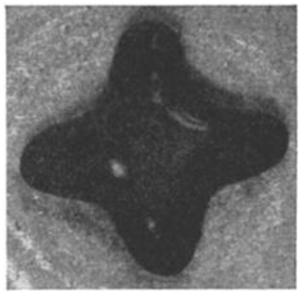
Fig. 6(b).

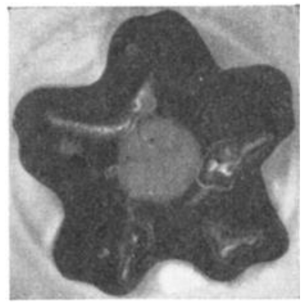
Fig. 7.

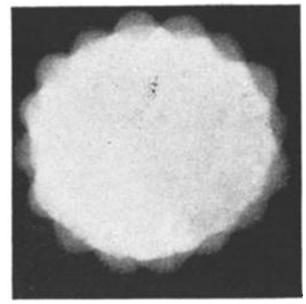
Fig. 9.

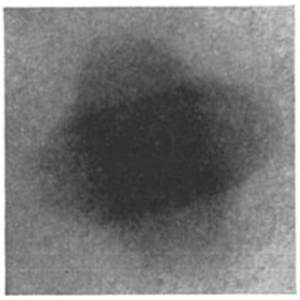
Fig. 4(b).

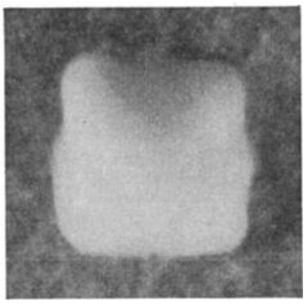
Fig. 6(c).

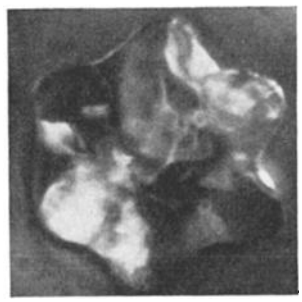
Fig. 8(a).

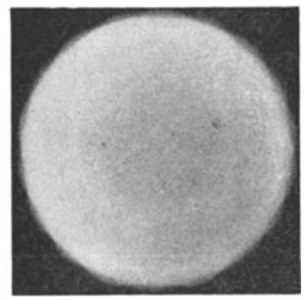
Fig. 10.


Fig. 5(a).

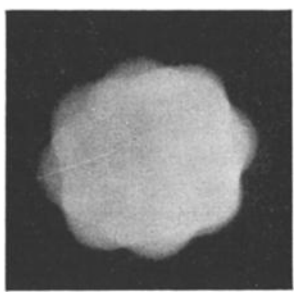
Fig. 6(d).


Fig. 8(b).

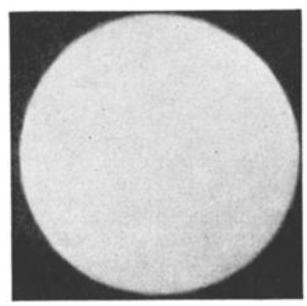
Fig. 11.

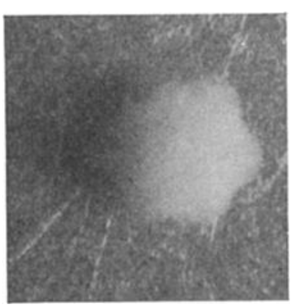
Fig. 5(b)

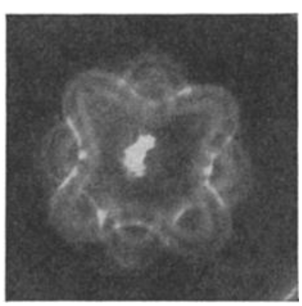
Fig. 6(e).

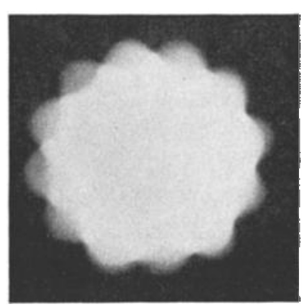
Fig. 8(c).

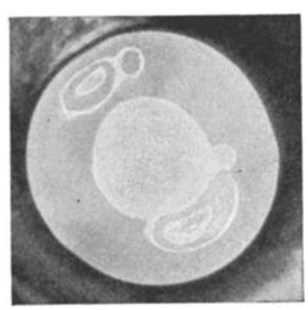
Fig. 12.

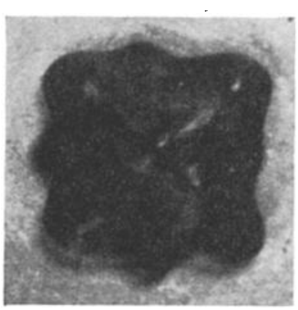
Fig. (6a).

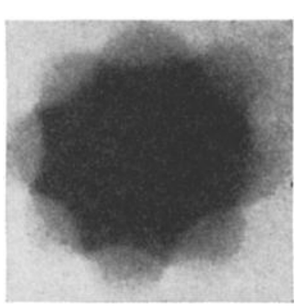
Fig. 6(f).

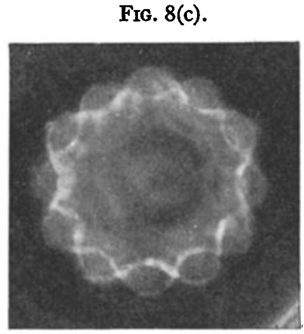
Fig. 8(d).

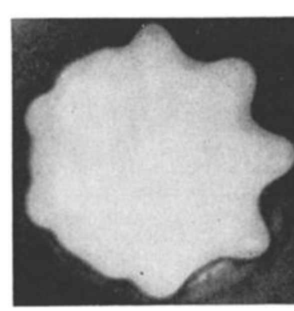
Fig. 13.

TABLE I. Summary of photographs of radial vibrations of evaporating liquid drops.

Fig.	Basic geometry	Shutter speed[a]	Coloring	Lighting	Remarks
4a	"sausage"	high	black India ink	2 spotlights	
4b	"sausage"	low	light blue ink	2 spotlights	
5a	triangle	high	black India ink	2 spotlights	
5b	triangle	low	talcum powder	1 spotlight	
6a	square	high	light blue ink	2 spotlights	
6b	square	high	black India ink	2 spotlights	
6c	square	high	talcum powder	2 spotlights	
6d	square	low	white ink	2 spotlights	
6e	square	low	none	circular fluorescent	
6f	square	low	light blue ink	2 spotlights	
7	pentagon	high	dark blue ink	2 spotlights	rotating
8a	hexagon	high	none	2 spotlights	
8b	hexagon	high	detergent	2 spotlights	
8c	hexagon	low	white ink	2 spotlights	
8d	hexagon	low	detergent	circular fluorescent	
9	octagon	low	white ink	2 spotlights	
10	nonagon	low	white ink	2 spotlights	medium-amplitude
11	duo-decagon	low	white ink	2 spotlights	low-amplitude
12	round	low	white ink	2 spotlights	
13	(see text)	high	white ink	2 spotlights	see text

[a] (High shutter speed means fast enough or more than fast enough to "freeze the motion"—from 1/200th to 1/1250th sec, depending on vibration frequency; low speed is such that one or more complete cycles are recorded—from 1/5th to 1/50th sec.)

on the surface of the drop; vibrating drops often eject small droplets at high speed as if they were a model of a radioactive atomic nucleus; traveling wave groups sometimes seem to move around the edge of the drop without the whole circumference taking on a standing wave appearance (e.g., Fig. 13); Fig. 13 is also unusual in that one-half of the drop is vibrating as if the entire drop were an octagon whereas the other half approaches one-half of a hexagon.

The classical treatment, by Lamb[1] or Rayleigh[2] of the natural vibrations of liquid spheres does not appear to cover the present situation which is complicated by gravity, heat gradients, and vapor flow. Moreover, the classical theory applies only to small vibrations. Ganot,[3] in discussing the Leidenfrost effect, states only that, ". . . taking sometimes the form of a star, . . .," and Tyndall[4] describes an 18-lobed drop without mention of its nature (vibrating nonagon) or of the more general nature of the effect.

[1] Lamb, *Hydrodynamics* (Dover Press, New York, 1945), p. 473.
[2] Lord Rayleigh, *Theory of Sound* (MacMillan and Company, London, 1929), Vol. 2, p. 371.
[3] Ganot, *Physics* (Spottiswoode and Company, London, 1863), p. 341.
[4] Tyndall, *Die Wärme* (Braunschweig, 1894), p. 239.

The basic mode of the drops described herein is of the same general form as that of a liquid sphere and also of a bell. With a bell, the higher modes usually predominate and the same seems to be true of radially vibrating liquid drops; the most frequently seen mode is that with eight lobes (vibrating square). Vortex rings are frequently seen to vibrate radially (in the plane perpendicular to the direction of the motion) with major and minor ellipse axes alternating, a phenomenon which is known to occur with water jets (Savart[5]), which is also the basic mode of a water drop and of a bell. Close observation of such rings blown at random by mouth indicates that they also occasionally appear fleetingly as multi-sided figures which, however, do not vibrate radially with observably appreciable amplitude. (One is therefore tempted to predict the feasibility of the existence of "square" smoke rings in which case the cartoon of such an effect will lose its whimsy.)

This project was financed by The Holter Research Foundation, Inc., a nonprofit, privately supported Montana corporation.

[5] Savart (oft-quoted in older texts but reference not located).