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Self-Induced Vibration of a Water Drop Placed on an Oscillating Plate

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Vibration of a water drop placed on a vertically oscillating plate was observed, and the amplitudes and frequencies of the plate were measured with which the drop showed large amplitude vibrations. A special cloth was used so that the drop did not wet the plate. The results are compared with a simple theoretical prediction based on the Mathieu equation. The experimental data are shown to lie within the unstable growing region derived from the theory.

KEYWORDS: liquid drop, self-induced vibration, parametric excitation, forced vibration, effect of variation of gravity, mathieu equation

§1. Introduction

Some leaves do not become wet and repell the water, so that a water drop put on them has a shape of circular disk. The contact angle between the drop surface and the part of the leaf below the drop is often more than 120°. Now, when the leaf holding a drop oscillates vertically, the drop begins to vibrate in the horizontal direction. Some experimental studies of this phenomenon have been made, and appearances of polygonal plane shapes have been reported.[1-3] Matsuda *et al.* observed a water and a mercury drops on a Taro leaf,[1] Smithwick & Boulet observed small mercury droplets and Sugino & Tosaka measured vibration frequency of a mercury drop.[2,3] It is an interesting problem to inquire the mechanism of the drop vibration. This phenomenon is considered to be a kind of parametric excitation, because the horizontal motion of the drop is excited through vertical oscillation and because the frequency of the drop vibration is about the half of that of the plate oscillation.[1-3] In order to examine this conjecture we undertook an experiment to excite large amplitude vibration by the use of a plate coverd with a water-repellent cloth and an audio speaker to drive the plate. Studies of self-induced vibration of liquid drop have been made also in other situations, such as thermal and chemical inequilibria. Adachi & Takaki,[4] Takaki & Adachi,[5] and Yoshiyasu *et al.*[6] investigated self-induced vibrations of rapidly evaporating drops of liquid nitrogen and oxygen, which were put on a horizontal plate with the room temperature. On the other hand, a mercury drop in an electrolyte solution shows a self-induced vibration, as was observed by Keizer *et al.*[7] The effect of evaporation on the vibration mechanism is studied by Tokugawa *et al.*[8] On the other hand, excitation mechanism by an oscillating plate is not understood well. In order to study the mechanism, it would be nessesary to make an experiment to investigate how the drop vibration depends on the frequency and the amplitude of the plate. It is the purpose of this paper to report results of this experiment and to compare with a simple analysis of parametric excitation. In the following sections the experimental method and the results are presented in §2 and 3. A simple analysis based on the Mathieu equation is given in §4. Conclusions and discussion are given in the last section.

§2. Apparatus and Experimental Methods

As shown in Fig. 1, a plastic plate, covered with a repellent texture (RECTAS, TEIJIN Co.), was attached horizontally to a loud speaker cone. Frequency f_e of signal generator to drive the speaker was varied within 20–67 Hz. A water drop of diameter 0.66–1.7 cm was placed on the plate, and its plane and side views were recorded at the same time by the use of a CCD-video camera (MACLORD MOVIE M21, NATIONAL PANASONIC) and a mirror. The amplitude A of plate oscillation (peak to peak) was measured by means of a high speed video camera (HVC, PHOTRON Co.), where an

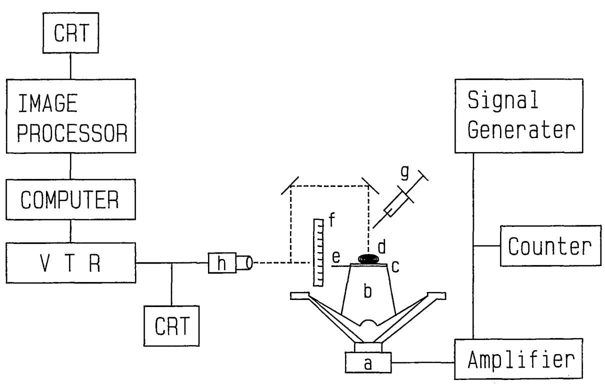

Fig. 1. Experimental apparatus and block diagram of controlling system. a: speaker, b: table, c: water-repellant cloth, d: drop of water, e: needle, f: scale, g: syringe, h: high speed video camera or ccd-video camera

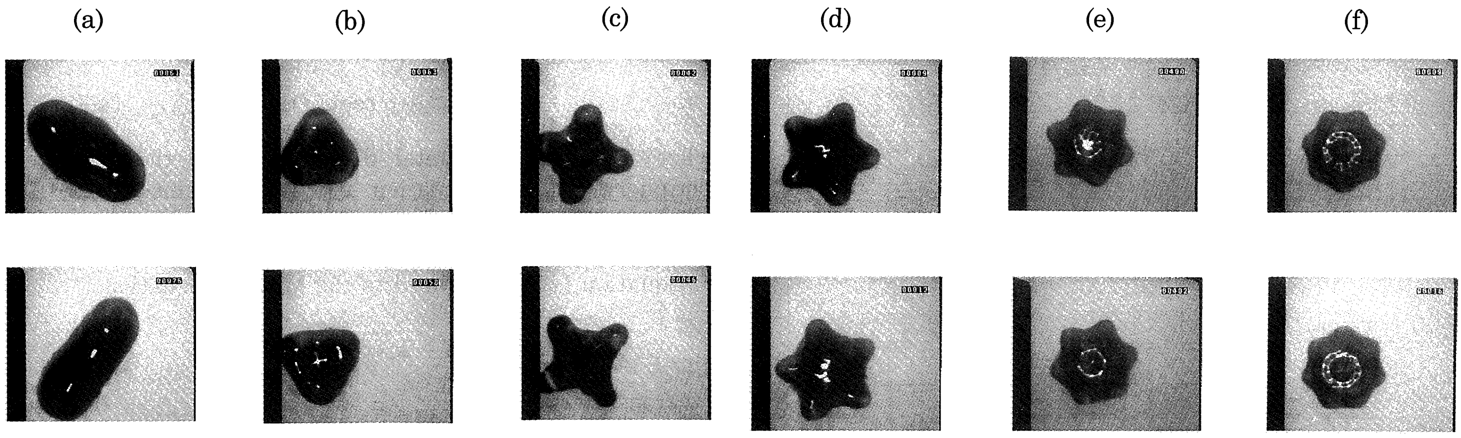

Fig. 2. Examples of instantaneous plane views in self-excited vibration. The lower shapes are those after half period. From (a) to (f) values of n increases from 2 to 7

oscillating needle which protruded from the plate and a scale (CONTACT NON PARALLAX SCALE, GRATICULES LTD.) with meshes of 1/10 mm fixed close to the plate were recorded in the same frame. When the drop showed a large amplitude vibration, the mode number n (number of vertices in the plane-view), the frequency f_n and the mean diameter $2r$ of the drop were measured from video images, which were digitized and processed by the use of an image processing system (DIG 98, DITECT Co.).

A note is given here on water repellent materials. If the contact angle was less than 90°, the drop vibration did not appear. We had used some materials, which allowed large contact angles, such as the fireproof texture (AKUTO-SOKEN Co.). Among them RECTAS cloth was the best choice and the contact angle was more than 120°.

§3. Experimental Results

Water drops at rest had a shape of flattened circular disk. The radius r in the plane view, the thickness h and the contact angle θ were measured for the rest states, as are given in Table I. For any drop the contact angle was more than 120° and increased with the radius. The thickness h was within the region of 0.42–0.45 cm and did not depend much on r. Some examples of large amplitude stationary vibration are shown in Fig. 2.

Relation between the measured frequency f_n of stationary vibration and the frequency f_e of the oscillating plate is shown in Fig. 3. This figure gives a simple relation

$$f_n = \frac{f_e}{2}. \tag{3.1}$$

Appearance of a large amplitude vibration depended also on the value of A which is the amplitude of plate oscillation. As it was increased from zero, the vibration appeared within a certain range of A, as is shown in Fig. 4 for several values of f_e. Below this range the drop remained to be a circular disk, and above it the drop showed a disordered motion or broke into small droplets.

Table I. Sizes of water drop used in the experiment.

volume (cm^3)	radius r_0(cm)	thickness h(cm)	contact angle θ(°)	h/r_0
0.2	0.44	0.42	130	0.95
0.4	0.58	0.45	134	0.77
0.6	0.67	0.45	147	0.67

§4. A Simple Model Analysis

A qualitative understanding of the above data is obtained by the following simple model. A drop, which is flattened by the gravity, can vibrate freely in the horizontal direction with the actions of inertia and surface tension. The angular frequency ω_n of the vibration is derived by Takaki & Adachi[5] in the following form:

$$\omega_n{}^2 = \frac{n(n^2-1)}{\rho R_0{}^3/\sigma} \cdot \frac{1}{1+2\beta H + (n-3)/2GH}, \tag{4.1}$$

where n is a mode number (the number of vertices in the plane view), σ is the surface tention coefficient, ρ is the liquid density and R_0 is an average radius in the plain view. Nondimentional parameters β, G and H are defined as follows:

$$\beta = 0.107, \quad G = \frac{\rho g R_0{}^2}{\sigma}, \quad H = \frac{h_0}{R_0}, \tag{4.2}$$

where h_0 is an average thickness of the drop and g is the gravity acceleration. From the balance of gravity and surface tention at the periphery of the drop, h_0 is given by

$$h_0 = \sqrt{\frac{4\sigma}{\rho g}}, \tag{4.3}$$

hence $GH^2 = 4$. Note that this formula is valid for $R_0 \gg h_0$, hence $H \ll 1, G \gg 1, GH \gg 1$.

Now, effect of a vertical acceleration by the oscillating plate, is equivalent to that of a varying gravity. On the other hand, the gravity determines the drop shape, such as the thickness h_0 through eq. (4.3) and the plane radius R_0 (note the drop volume $\cong \pi R_0{}^2 h_0 =$ const.). Vibratory changes of h_0 and R_0 affect the frequency of the free

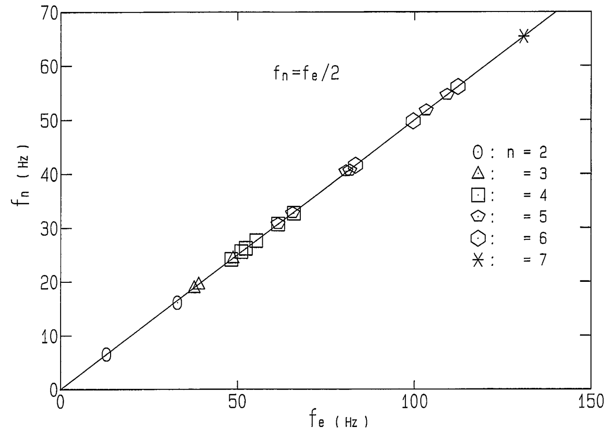

Fig. 3. Relation of the measured frequency f_n of stationary vibration and the frequency f_e of the oscillating plate.

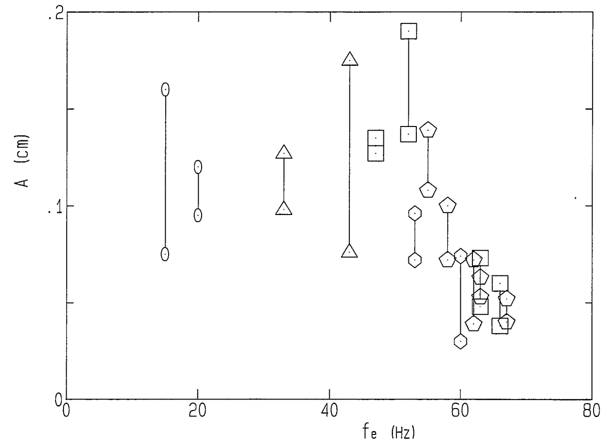

Fig. 4. Range of the amplitude A of plate oscillation (peak to peak) for occurrence of large amplitude drop vibration.

vibration. Therefore, the oscillation of the plate will lead to a modulation of the vibration frequency, hence causes a parametric excitation.

For the vertical oscillation of the plate with frequency f_e and amplitude $A/2$ the gravitational acceleration is modified effectively to

$$\tilde{g} = g + \frac{1}{2}A\omega_e^2 \cos\omega_e t, \qquad \omega_e = 2\pi f_e. \tag{4.4}$$

Here, we assume that $\varepsilon \equiv \frac{1}{2}A\omega_e^2/g \ll 1$. The modulation of gravity causes those of thickness h_0 ($\propto g^{-1/2}$) and the radius $R_0(\propto g^{1/4})$. Since $H \ll 1$ and $GH \gg 1$, the second factor in eq. (4.1) gives a minor effect, and the gravity modulation affects mainly through the factor R_0^{-3}. Then, leaving the first order terms of ε, we have the following modulated frequency:

$$\tilde{\omega}_n^2 = \omega_n^2 + \gamma\cos\omega_e t, \tag{4.5}$$

where

$$\gamma = \frac{d\omega_n^2}{dg}\cdot\frac{1}{2}A\omega_e^2 = -\frac{3}{4}\frac{\omega_n^2}{g}\cdot\frac{1}{2}A\omega_e^2. \tag{4.6}$$

$$\left(R_0 \propto h_0^{-1/2} \propto g^{1/4}, \omega_n^2 \cong \frac{n(n^2-1)}{\rho g R_0^3}\right)$$

This result suggests the following equation as a model of the present drop vibration:

$$\frac{d^2u}{dt^2} + (\omega_n^2 + \gamma\cos\omega_e t)u = 0. \tag{4.7}$$

where u is the horizontal displacement of the drop periphery. In this equation ω_n^2 is determined from eq. (4.1), if the mode number n is given, while γ and ω_e are quantities controlled experimentally.

Equation (4.7) is reduced to the Mathieu equation

$$\frac{d^2u}{d\tau^2} + (a + 2q\cos 2\tau)u = 0, \tag{4.8}$$

by transformations

$$\omega_e t = 2\tau,\ a = \frac{4\omega_n^2}{\omega_e^2},\ q = \frac{2\gamma}{\omega_e^2} = -\frac{3\omega_n^2 A}{4g}. \tag{4.9}$$

Solution $u(t)$ of eq. (4.8) grows or decays with time depending on the values of a and q. In Fig. 5 regions of these parameters for stable (decaying) vibration are indicated by shadows. The symbols in the figure are normalized experimental values (same as those in Fig. 3). Most of these data are located inside the unstable (growing) region, and supports the present model. But, some data with $n = 2$ lie out of this region. A possible explanation of this discrepancy is given in the next section.

Order estimation of the parameter ε is given here. From the values of $A \sim 0.05$ cm, $\omega_e^2 \sim \omega_n^2 \sim 10^4$ s^{-1}, $g \sim 10^3$ cm/s^2, we have $\varepsilon \sim O(1)$, contradicts to the assumption $\varepsilon \ll 1$. Therefore, the above analysis should be taken as giving a qualitative understanding.

§5. Conclusions and Discussion

The results of the present experiment and model analysis, lead to the following conclusions.

(i) In order to excite the drop vibration effectively the frequency f_e of the forcing plate oscillation should have a special value, i.e. $\omega_e \cong 2\omega_n$, and the amplitude of forcing should be within a certain range.

(ii) This drop vibration can be understood at least qualitatively in terms of a parametric excitation due to an effective variation of gravity.

Some comments are given here on these conclusions. First, the relation $f_e \cong 2f_n$ has been already pointed out in the past studies,[1-3] while the dependence on the forcing amplitude is a new result.

Second, there is a discrepancy in the range of q between the experiment and the model analysis. The two data with $n = 2$ are below the boundary in Fig. 5. This discrepancy will be explained in the following way. The formula (4.1) is valid for small amplitude vibration, but the mode with $n = 2$ had rather large amplitude. Therefore, the model analysis in §4 can not be applied to the $n = 2$ mode. Next, some data with $n \geq 3$ lie far above the lower branch of the theoretical boundary. This fact may suggest that there was a physical effect not included in the above analysis. A possible candidate for it is the viscous effect acting within the liquid drop, which is expected to shift up the lower branch. On the other hand, the behavior of the drop near the upper branch of the theoretical boundary is difficult to predict, because the

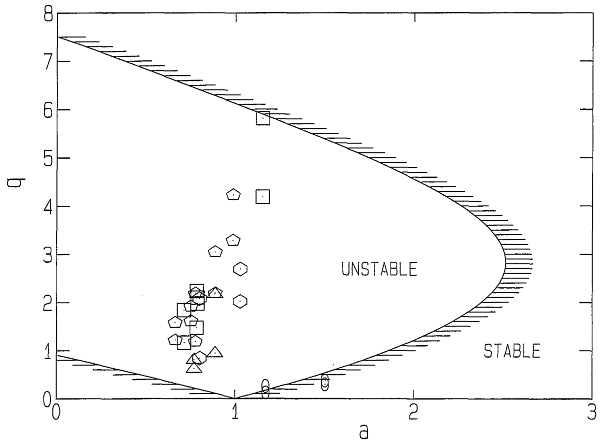

Fig. 5. Stable (decaying) and unstable (increasing) regions of the solution of eq. (4.8). Normalized experimental data are also plotted. The theoretical curve is correct up to the seventh order expansion with respect to q.

vibration amplitude is too large to analyse by the use of the linear equation (4.7).

Third, in this work we focused on the horizontal motion of the drop. In the experiment, however, the drop surface was also moving vertically although very weakly. Wavy motion of a liquid surface in a vertically oscillating container is well known with the name of Farady resonance. Examination of this effect will be also necessary for deeper understanding of this phenomenon.

Lastly, a few shortcomings in the present experiment are pointed out. Recognition of large amplitude vibration was made with the naked eyes, because the distinction of vibrating and non-vibrating states was clear. However, a quantitative measurement of the vibration amplitude is necessary to improve the data. Measurement of the contact angle had a considerable error and there is a room for improvement. These shortcomings will be considered in future studies.

Acknowledgement

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