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The Free Motion of a Sphere in a Rotating Fluid

By **H.-G. Moll**

Summary: The free motion of a sphere in a fluid with solid body rotation is considered. The equations of motion of the sphere are formulated and a theoretical model for the surface force is introduced. From theory it follows that after commencement of the motion the horizontal part of the trajectory of the sphere is a logarithmic spiral. This result is verified by experiment. It was also found experimentally that the drag of the sphere is considerably higher than that of a sphere moving in a non-rotating fluid. The flow around the sphere is asymmetrical thus producing a dynamic lift during the motion. This points to the existence of a Taylor-column moving with the sphere. The results make it possible to compute not only the forces acting on the surface but also the trajectory of the sphere.

Übersicht: Es wird die freie Bewegung einer Kugel in einem starr rotierenden Fluid behandelt. Die Bewegungsgleichungen für die Kugel werden aufgestellt, ein Ansatz für die auf die Kugel wirkende Oberflächenkraft wird eingeführt. Aus der Theorie ergibt sich, daß nach einem Anlaufvorgang der horizontale Anteil der Kugelbahnkurve einer logarithmischen Spirale entspricht. Das wird experimentell bestätigt. Außerdem ergeben die Experimente, daß der Widerstand der Kugel beträchtlich höher ist als er von Kugelbewegungen in nichtrotierenden Fluiden her bekannt ist und daß die Kugel unsymmetrisch umströmt wird und so einen dynamischen Auftrieb erfährt. Dieses deutet darauf hin, daß Taylor-Säulen die Kugelbewegung beeinflussen. Die gemessenen Ergebnisse ermöglichen es, sowohl die Oberflächenkräfte als auch die horizontale Bahnkurve der Kugel zu berechnen.

1. Introduction

Commercial centrifuges, used to clean sewage etc., show rather poor results with respect to efficiency and flow rate. To analyse this deficiency an idea of Wille was adapted who considered the formation of Taylor-columns as a possible reason. Taylor-columns appear during the motion of a body through a fluid in solid body rotation. To the knowledge of the author this problem has not been studied yet.

The difficulties in solving this problem analytically are well demonstrated by various papers dealing with the straight steady motion of a sphere in a rotating fluid, a problem which is considerably easier to handle than the one at hand. From the two papers by Stewartson [1], [2] it appears that even this less complicated problem can not be solved completely. In his second paper Stewartson found for instance that the flow around the Taylor-column is identical with the potential flow around a cylinder. This result, however, contradicts experimental observations.

Because of the apparently extreme difficulties connected with an analytical attempt to solve the problem, it was considered a more promising way to undertake an experimental investigation. In the following sections 2. and 3. equations are developed for later evaluations of the experiments.

2. The Equations of Motion of the Sphere

In a coordinate-system rotating with the constant angular velocity ω, the external forces acting on the sphere are composed of the buoyancy force A_i, the weight G_i, the surface force S_i due to the reaction of the motion of the sphere, and the surface force Z_i due to the centrifugal field which is independent of the motion of the sphere.

From Newton's law one obtains the following three relations, provided that the x-axis and the axis of rotation of the system are identical, and that the weight and the buoyancy force are acting in x-direction only:

$$M\,g\left(1 - \frac{\varrho_f}{\varrho}\right) - S_x = M\,\ddot{x}\,, \tag{1}$$

$$Z_r - S_r = M\,(\ddot{r} - r\,\dot{\varphi}^2 - 2\,\omega\, r\,\dot{\varphi} - r\,\omega^2)\,, \tag{2}$$

$$Z_\varphi - S_\varphi = M\,(r\,\ddot{\varphi} + 2\,\dot{r}\,\dot{\varphi} + 2\,\omega\,\dot{r})\,, \tag{3}$$

where $\dot{x}$ is the axial velocity, $\dot{r}$ the radial velocity and $r\,\dot{\varphi}$ the azimuthal velocity of the sphere. M is the mass of the sphere; g is the gravitational acceleration and ϱ the density. The index f stands for fluid. In the following we shall only consider the horizontal motion. The boundary conditions for equations (2) and (3) are

$$t = 0: \qquad r = r_0\,, \qquad \varphi = 0\,, \qquad \dot{r} = \dot{\varphi} = 0 \tag{4}$$

where r_0 is the initial radial location of the sphere.

The integration of the pressure which is the consequence of the rotation of the fluid yields:

$$Z_r = -\frac{4}{3}\pi\, a^3\, \varrho_f\, \omega^2\, r \qquad \text{and} \qquad Z_\varphi = 0 \tag{5}$$

where a is the radius of the sphere.

The remaining unknowns in the equations (2) and (3) are now only S_r and S_φ. The only theoretical way to obtain their values is to solve the Navier-Stokes-equations for a rotating system under the conditions at hand. This, however, seems to be difficult and it appeared to be a better way to construct a model for S_r and S_φ and to determine the unknowns in the model by experiment:

$$S_r = C_s\, l\, \dot{r}\,, \tag{6}$$

$$S_\varphi = C_s\, m\, r\, \dot{\varphi}\,. \tag{7}$$

For the steady motion of the sphere in a non-rotating fluid Abraham [3] was able to find a law for the drag which is an extension of the well known Stokes' law [4]. Using $C_s = 6\,\pi\, a\, \varrho_f\, \nu$ from Stokes he got for the range $Re \leq 2500$:

$$l = m = (1 + 0{,}156\, Re^{0{,}5})^2\,. \tag{8}$$

Here Re is the Reynolds number, defined as

$$Re = \frac{a\, c}{\nu}\,, \tag{9}$$

with ν the kinematic viscosity and c the velocity of the sphere.

Since a general analytical solution is not possible we shall introduce a restriction: The coefficients l and m are considered to be constant; they denote the dissimilarity between the motion of a sphere calculated by Stokes and the motion considered here. In addition the horizontal and the vertical motion of the sphere are then completely separated.

Equations (2), (3) and (4) contain six adjustable quantities: a, ν, ω, ϱ, ϱ_f, r_0. This yields three parameters determining the motion:

$$P_1 = \frac{\nu}{a^2\,\omega} = \text{Ekman number} = Ek\,, \qquad P_2 = \frac{\varrho}{\varrho_f}\,, \qquad P_3 = \frac{r_0}{a}\,.$$

With

$$C_1 = \frac{9}{2}\frac{\varrho_f}{\varrho} Ek \qquad \text{and} \qquad C_2 = \frac{1}{2}\left(\frac{\varrho_f}{\varrho} - 1\right)$$

the equations (2), (3) and (4) take the nondimensional form:

$$\ddot{r} - r\,\dot{\varphi}^2 + C_1\, l\, \dot{r} - 2\, r\,(\dot{\varphi} - C_2) = 0\,, \tag{10}$$

$$r\,\ddot{\varphi} + 2\,\dot{r}\,\dot{\varphi} + C_1\, m\, r\,\dot{\varphi} + 2\,\dot{r} = 0\,, \tag{11}$$

$$t = 0: \qquad r = 1\,, \qquad \varphi = 0\,, \qquad \dot{r} = \dot{\varphi} = 0\,. \tag{12}$$

The dependent variables were normalized by r_0 and ω. This is the reason why P_3 does not occur explicitly in C_1, C_2 nor in the three equations.

The system of equations (10)—(12) can easily be solved by numerical computation, for example with the Runge-Kutta-procedure.

An analytical solution is possible for the commencement of the motion. But this phase is not very interesting, since it only concerns a very small part of the motion to be considered.

The nondimensional surface force due to the reaction of the motion of the sphere has the components: $\zeta_r = S_r/M\, r_0\, \omega^2 = C_1\, l\, \dot{r}$ and $\zeta_\varphi = S_\varphi/M\, r_0\, \omega^2 = C_1\, m\, r\, \dot{\varphi}$.

3. The Quasistationary Phase

Considering the motion of a sphere in a non-rotating fluid, one can obtain the quantities describing the steady phase by making the acceleration term zero. For the motion of a sphere in a rotating fluid this cannot be done, since the force responsible for the motion is no longer constant. To make this force a "constant" equation (10) has to be devided by r. Introducing the abbreviation

$$\dot{F} = \frac{\dot{r}}{r} \tag{13}$$

(10) and (11) yield:

$$\ddot{F} + \dot{F}^2 - \dot{\varphi}^2 + C_1\, l\, \dot{F} - 2\,(\dot{\varphi} - C_2) = 0\,, \tag{14}$$

$$\ddot{\varphi} + 2\,\dot{F}\,\dot{\varphi} + C_1\, m\, \dot{\varphi} + 2\,\dot{F} = 0\,. \tag{15}$$

It is assumed that now the procedure described for the non-rotating fluid can be applied for a rotating fluid as well: The motion of the sphere after its commencement in a rotating fluid is determined by $\ddot{F} = \ddot{\varphi} = 0$. This means that $\dot{F}_A =$ constant and $\dot{\varphi}_A =$ constant (A for "asymptotic"), which justifies the term "quasistationary".

With (13) we obtain for this phase:

$$r_A = r_{A\,0}\, e^{\dot{F}_A t}\,, \tag{16}$$

$$\varphi_A = \dot{\varphi}_A\, t + \varphi_{A\,0}\,. \tag{17}$$

It must be mentioned, that the point $r_{A\,0}/\varphi_{A\,0}$ is only a virtual origin of the motion, i.e. the point where the motion would have started, if no commencement of motion would be present.

Using the derived equations it is now easy to predict the motion of the sphere with or without consideration of the drag resistance: The horizontal part of the free motion of a sphere in a rotating fluid can be divided into a phase of commencement of motion and a quasistationary phase, in which the trajectory of the sphere is described by a logarithmic spiral. By reducing the drag resistance this spiral approaches a circle. At the same time the region of commencement of motion will be elongated. If the drag resistance is zero, the quasistationary phase, which then would be a circular motion, will not be reached, since the commencement of motion cannot be damped.

4. Experimental Determination of $\dot{F}_A$, $\dot{\varphi}_A$, r_{A0} and φ_{A0}

The apparatus which is shown in Fig. 1 consists of a rotating stand, a mirror, a rotoscope and a Bolex-filmcamera. This is completed by a mechanical revolution counter, a frequency counter, a clock and a Robot-photocamera.

The rotation stand consists of a cylindrical plexiglass-container which rotates around its axis of symmetry with a constant angular velocity. In this container a plexiglass-tube is mounted and filled with up to 15 spheres. The spheres are elevated upward by a pressure-system until they are leaving the tube.

The motion of the sphere having left the above mentioned tube is registered continuously by the filmcamera. At the same time the readings of the instruments are registered by the photocamera. This camera is triggering a light bulb which is mounted on the top of the mirror, so that the instant when an instrument reading is taken can be seen from the film. The rotoscope is connected with the driving unit of the rotation stand by a flexible shaft. It consists mainly of a dove prism which suppresses the rotation of the image of the plexiglass-container. The design of the rotoscope was stimulated by Hide and Ibbetson [5], who used this term.

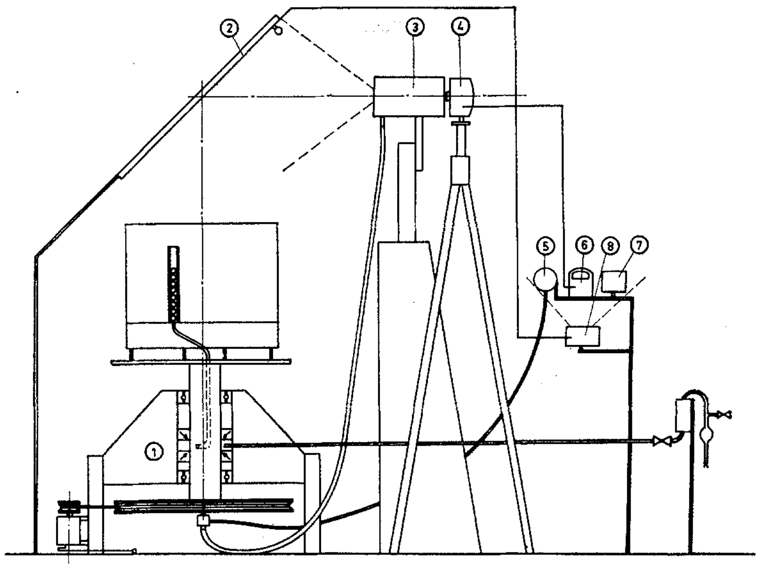

Fig. 1. The apparatus.

1 Rotation stand, 2 Mirror, 3 Rotoscope, 4 Bolex-filmcamera, 5 Revolution counter, 6 Frequency counter, 7 Clock, 8 Robot-photocamera

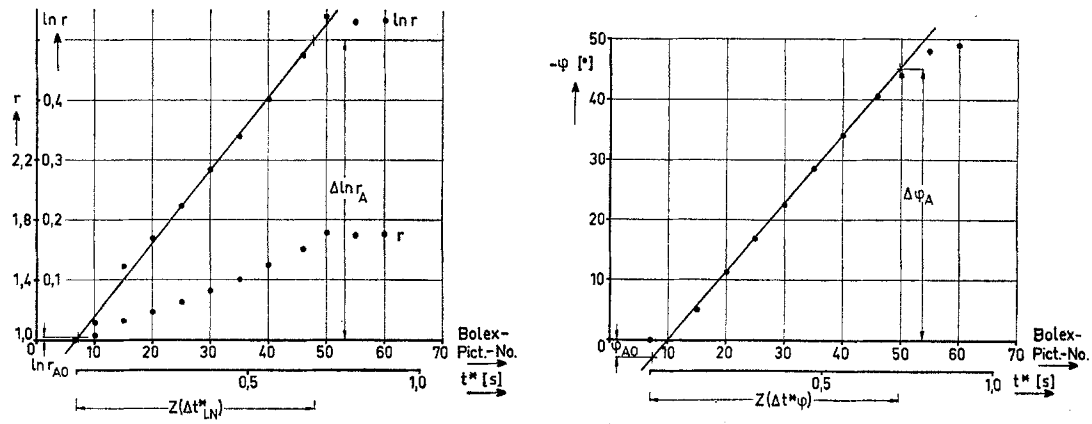

Fig. 2. Example for the evaluation of the experiments

The experiments were carried out with the following parameter variations:

$$\varrho/\varrho_f = 1.023 \text{ to } 1.436$$
$$Ek = 0.004 \text{ to } 0.009$$
$$r_0/a = 15.75 \text{ to } 39.37\,.$$

From the locations of the sphere in the individual frame of the film the temporal variation of its radial and azimuthal coordinates were evaluated. Fig. 2 shows an example of the evaluation. In addition the dependence of ln r is shown. It must be mentioned that the points in Fig. 2 are levelling off where the sphere reaches the edge of the container. For every experiment diagrams like those in Fig. 2 were drawn. From these the four unknowns of (16) and (17) can be obtained: From ln $r_{A\,0}$ one obtains $r_{A\,0}$; $\varphi_{A\,0}$ is taken directly from Fig. 2. The value of $\dot{F}_A$ is the ascent of the ln r-curve, $\dot{\varphi}$ is the ascent of the φ-curve.

Fig. 2 shows a linear time dependence of the measured values of $\ln r$ and φ which is in agreement with the theoretical prediction. Thus $\dot{F}_A = d(\ln r_A)/dt$ and $\dot{\varphi}_A = d\varphi_A/dt$ are truly constant and the existence of the quasistationary phase is confirmed.

It is usually accepted that the drag coefficients l and m are equal, i.e. $x = l/m = 1$. This implies symmetrical flow around the sphere. If this assumption is not valid, a simple model for x which in general depends on all three parameters would be $x =$ constant. Then the equations of motion yield the following relation between $\dot{F}_A$ and $\dot{\varphi}_A$:

$$\dot{F}_A = \sqrt{\frac{(\dot{\varphi}_A^2 + 2\,\dot{\varphi}_A - 2\,C_2)\,\dot{\varphi}_A}{\dot{\varphi}_A - 2\,x\,(\dot{\varphi}_A + 1)}}\,. \tag{18}$$

The lines $x =$ constant in the $\dot{F}_A - \dot{\varphi}_A$-diagram depend only on the ratio of ϱ/ϱ_f. Since ϱ was kept constant for all experiments, ϱ/ϱ_f was varied by using different fluids: Methanol ($\varrho/\varrho_f = 1.4356$), water ($\varrho/\varrho_f = 1.1404$) and saltwater ($\varrho/\varrho_f = 1.0225$).

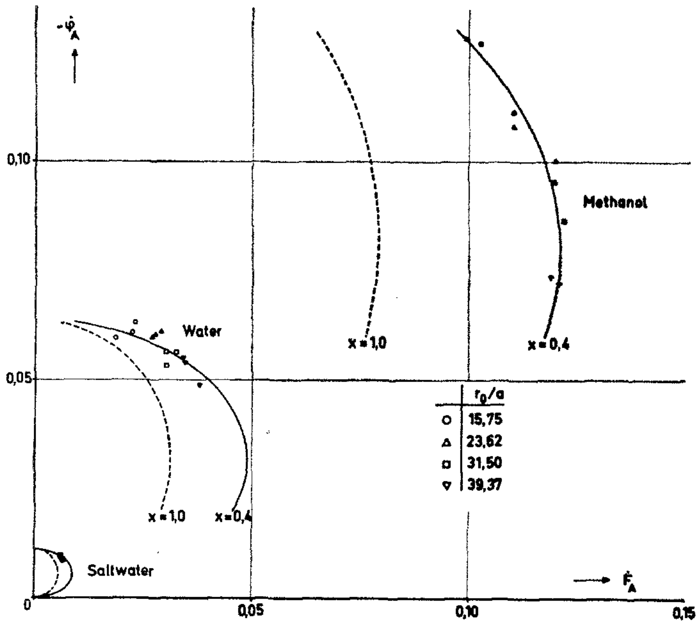

Fig. 3. Equation (18) for $x = 0.4$ and $x = 1.0$

In Fig. 3 the lines $x = 1$ for these different fluids are drawn. The points obtained from the experiments do not, however, coinside with these lines. They are well represented by lines for which $x = 0.4$. This fact indicates that the flow around the sphere is not symmetrical. An explanation for this remarkable fact is given later.

By plotting $\dot{F}_A$ and $\dot{\varphi}_A$ against Ek no correlation could be found. The diagrams $\dot{F}_A$ and $\dot{\varphi}_A$ over r_0/a are plotted in Figs. 4 and 5. Here $\dot{\varphi}_A$ seems to be a linear function of r_0/a:

$$\dot{\varphi}_A = M(\varrho/\varrho_f)\,\frac{r_0}{a} + N(\varrho/\varrho_f)\,. \tag{19}$$

The straight lines which can be drawn accordingly intersect with the $\dot{\varphi}_A$-axis at those points, which correspond to the values from (18) for $\dot{F}_A = 0$.

The values of N and M in (19) are now evaluated. One finds from (18):

$$N = -1 + \sqrt{1 + 2\,C_2} \tag{20}$$

whereas M can be approximated by

$$M = 0.0017\left(\frac{\varrho}{\varrho_f} - 1\right) + \left(\frac{\varrho}{\varrho_f} - 1\right)^{7.74}. \tag{21}$$

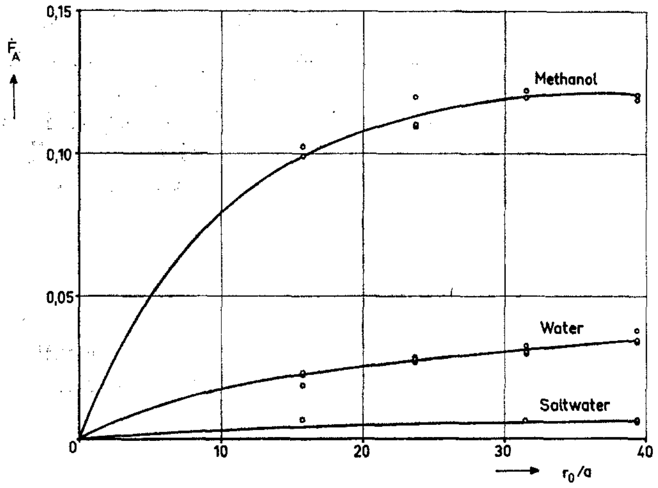

Fig. 4. $\dot{F}_A$ as a function of ϱ/ϱ_f and r_0/a

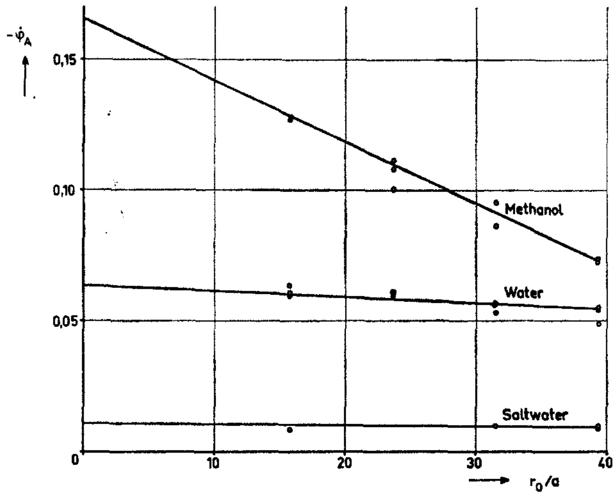

Fig. 5. $\dot{\varphi}_A$ as a function of ϱ/ϱ_f and r_0/a

Since a relation between $\dot{F}_A$ and $\dot{\varphi}_A$ is given by (18) and $x = 0.4$ is ascertained, no equation for $\dot{F}_A$ — like (19) — is necessary. The curves in Fig. 4 are computed from (18) with $x = 0.4$ and (19), (20) and (21).

For evaluating (16) and (17) it is still necessary to determine the coordinates of the virtual origins. These coordinates are independent of Ek analogous to what has been ascertained for $\dot{F}_A$ and $\dot{\varphi}_A$. They are both linear in r_0/a.

If we take the model

$$\varphi_{A\,0}[^\circ] = R(\varrho/\varrho_f)\,\frac{r_0}{a} + T(\varrho/\varrho_f) \tag{22}$$

we can approximate this initial coordinate by

$$R = -\,0.3294\left(\frac{\varrho}{\varrho_f} - 1\right)^{0.8169} + 0.0159\left(\frac{\varrho}{\varrho_f} - 1\right)^{1.6338} \tag{23}$$

and

$$T = 34.3390\left(\frac{\varrho}{\varrho_f} - 1\right)^{0.8688} - 27.3915\left(\frac{\varrho}{\varrho_f} - 1\right)^{1.7376}. \tag{24}$$

With

$$r_{A\,0} - 1 = U\,\varphi_{A\,0}[^\circ] + V\,, \tag{25}$$

we get the approximations

$$U = 0.0031\left(\frac{\varrho}{\varrho_f} - 1\right)^{-0.7075} + 0.0147\,, \tag{26}$$

$$V = -\,0.2262\left(\frac{\varrho}{\varrho_f} - 1\right)^{0.6956} - 0.1655\left(\frac{\varrho}{\varrho_f} - 1\right)^{1.3912}. \tag{27}$$

The initial points in the $r_{A\,0} - \varphi_{A\,0}$-plane are given in Fig. 6.

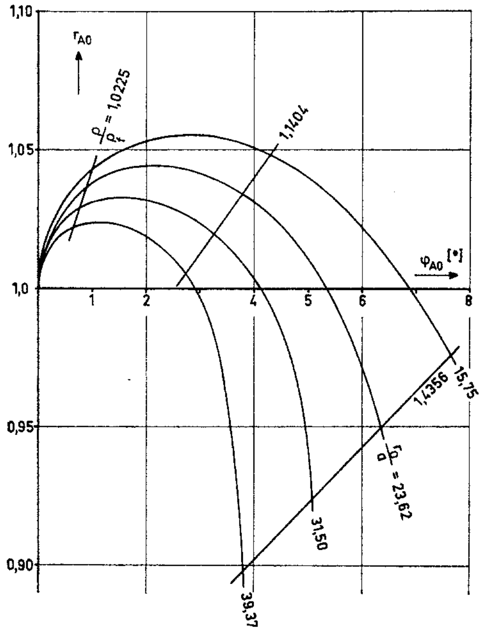

Fig. 6. The coordinates of the virtual origins

5. Determination of the Drag and the Lift

It was mentioned above that the flow around the sphere is not symmetrical. This means that in the horizontal plane a drag component as well as a lift component is present. Using $x = 0.4$ and with $\tan\alpha = \dot{F}_A/\dot{\varphi}_A$ which is constant for every run of the sphere we obtain for the drag (see Fig. 7):

$$D = (1 + 1.5\cos^2\alpha)\;C_1\,l\,c \tag{28}$$

and for the lift

$$L = 1.5\sin\alpha\cos\alpha\;C_1\,l\,c\,. \tag{29}$$

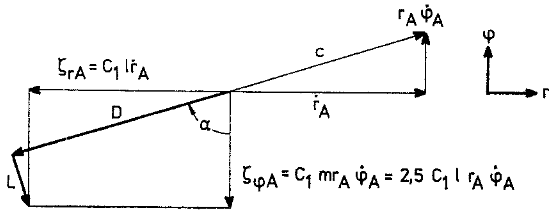

Fig. 7. Schematic illustrating the forces D and L

In these equations c is the velocity of the sphere as defined in equation (9), but now in a non-dimensional form and only the horizontal component: $c = \sqrt{\dot{F}_A^2 + \dot{\varphi}_A^2\,r_A}$. The value of l which appears in these relations is

$$l = \frac{\dot{\varphi}_A^2 - \dot{F}_A^2 + 2\,\dot{\varphi}_A - 2\,C_2}{C_1\,\dot{F}_A}\,. \tag{30}$$

6. Comparison of the Drag of a Sphere in a Non-rotating and a Rotating Fluid

In a non-rotating fluid the drag-force during the steady phase is — according to the law of Abraham —:

$$W_N = S_x = M\,g\left(1 - \frac{\varrho_f}{\varrho}\right) = 6\,\pi\,a\,\varrho_f\,\nu\,(1 + 0{,}156\,Re^{0{,}5})^2\,c\,. \tag{31}$$

Here the velocity of the sphere c is non-dimensionless. According to (28) the drag-force in a rotating fluid is

$$W_R = D\,M\,r_0\,\omega^2 = C_1\,l\,M\,r_0\,\omega^2\,(1 + 1{,}5\cos^2\alpha)\,c \tag{32}$$

where c is dimensionless and equal to $\dot{F}_A\,r_A/\sin\alpha$. This leads to the ratio of the drag-forces:

$$\frac{W_R}{W_N} = \frac{C_1\,l\,\dot{F}_A}{1 - \frac{\varrho_f}{\varrho}}\,\frac{1 + 1{,}5\cos^2\alpha}{\sin\alpha}\,\frac{r_0\,\omega^2}{g}\,r_A = \frac{2}{9}\,C_1\,l\,\frac{\varrho}{\varrho_f}\,Ek^{-1}\,\frac{1 + 1{,}5\cos^2\alpha}{(1 + 0{,}156\,Re^{0{,}5})^2}\,. \tag{33}$$

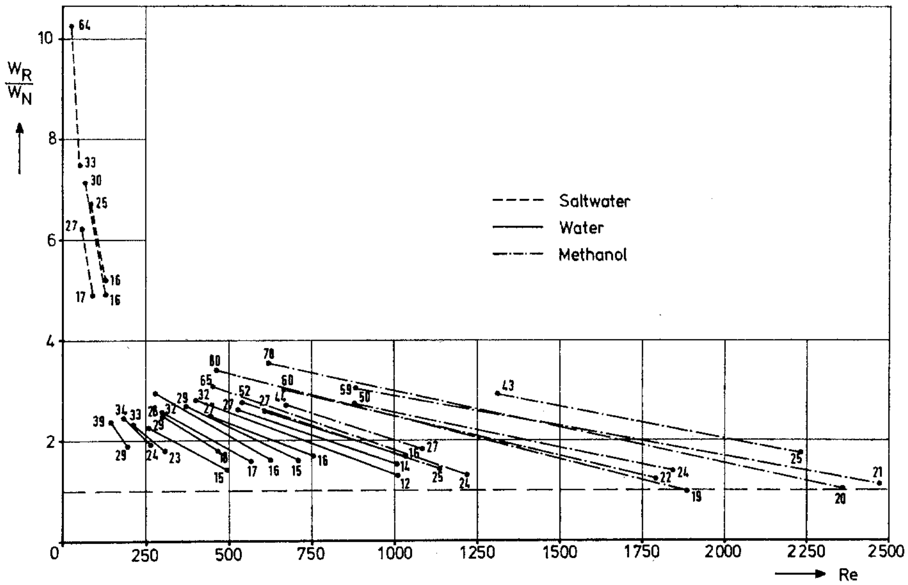

Fig. 8. The ratio of the drag-force W_R in a rotating fluid to that in a non-rotating fluid W_N

The results of (33) — using the values of the experiments — are shown in Fig. 8. In this diagram the beginning and the end of every quasistationary phase is marked by the acceleration number $A\,c$ which is multiplied by 1000. This characteristic number is defined as

$$A\,c = \frac{2\,\dot{F}_A}{Ek\,Re}\,. \tag{34}$$

According to equation (33) the ratio W_R/W_N is not linear with Re. For simplification, however, linearity is shown in Fig. 8. From this picture two substantial results can be taken: (1) The dragforce according to the free motion of a sphere in a rotating fluid is up to more than 10-times greater than that of the motion of a sphere in a non-ratating fluid. (2) The acceleration number is not a suitable parameter.

7. Discussion of Results

The two main results obtained so far are the increased drag of the sphere and the asymmetric flow around the sphere.

The evaluation of the experiments yielded a drag-force which, in a certain range, was partially more than 10-times greater than in the comparable motion of a sphere in a non-rotating fluid. This may be caused by the rotation of the fluid or by the acceleration of the sphere which is also absent in the Abraham-case.

We shall first consider the influence of the acceleration. Buzzard and Neddermann [6] reported of experiments with nylonspheres in air. For acceleration numbers $5.1 \cdot 10^{-4} \leq A\,c \leq 52 \cdot 10^{-4}$ they found no perceptible deviation from the law of the steady motion of a sphere up to $Re = 1500$. No experiments were carried out for higher Reynolds numbers. In addition to these experiments only the results of Lunnon [7] are known which are comparable to the results of the present work, i.e. for $Re \leq 2500$. In the region $0.1 \cdot 10^{-4} \leq A\,c \leq 2 \cdot 10^{-4}$ Lunnon obtained no substantial difference between his results and those known from steadily falling spheres in fluids at rest. Only at $Re \approx 15\,000$ Lunnon found that the drag coefficient had approximately doubled.

Contrary to the experiments of Buzzard and Neddermann and of Lunnon the acceleration acting on the sphere is not constant in the rotating fluid. This is, however, expected to be of negligible influence on the drag coefficient. One may therefore assume that the increase of the drag is caused only by the rotation of the fluid, i.e. the vortex structure of the rotating fluid.

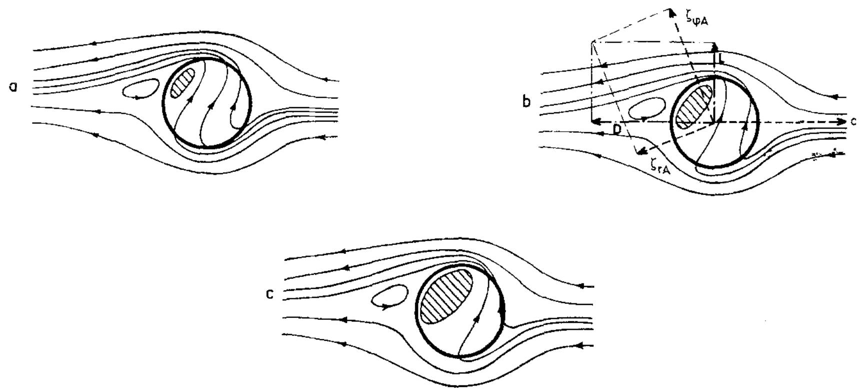

Fig. 9. The flow around a cylinder in a rotating fluid (Hide and Ibbetson [5])

From Fig. 9 (Hide and Ibbetson) it can be gathered that Taylor-columns may be the cause for the existence of the dynamical lift force in the horizontal plane. This picture shows the streaklines about 10 centimeters above a slowly and steadily moving cylinder. The rotation of the system is counterclockwise. A flow perpendicular to the direction of motion through the region right above the cylinder is clearly seen. This implies a force acting on the cylinder in this direction, i.e. a lifting force. The two force components are drawn in Fig. 9b.

If the regions above and below the sphere had a flow pattern like that at the cylinder, the lift force found in the experiments could be explained.

In concluding, it appears possible to interpret the main results of this work as the effect of the vortex structure of the rotating fluid, i.e. the existence of Taylor-columns.

8. An Example

Four different methods were applied to compute the trajectory of the sphere:

Stokes: Equations (10), (11) and (12) are solved with $l = m = 1$.

Abraham: Equations (10), (11) and (12) are solved with l and m according to equation (8).

Reuter: This method is common in chemical engineering. Reuter [8] assumes that the coriolis acceleration can be neclegted, that Stokes' law is valid ($l = 1$) and that $\ddot{r} = \ddot{\varphi} = \dot{\varphi} = 0$. When solving equation (10) with equation (12) the trajectory is a straight line coinciding with the r-axis.

Moll: Equations (16) and (17) are solved by using the results for $\dot{F}_A$, $\dot{\varphi}_A$, $r_{A\,0}$ and $\varphi_{A\,0}$. Only the trajectory of the quasistationary phase is thus computed.

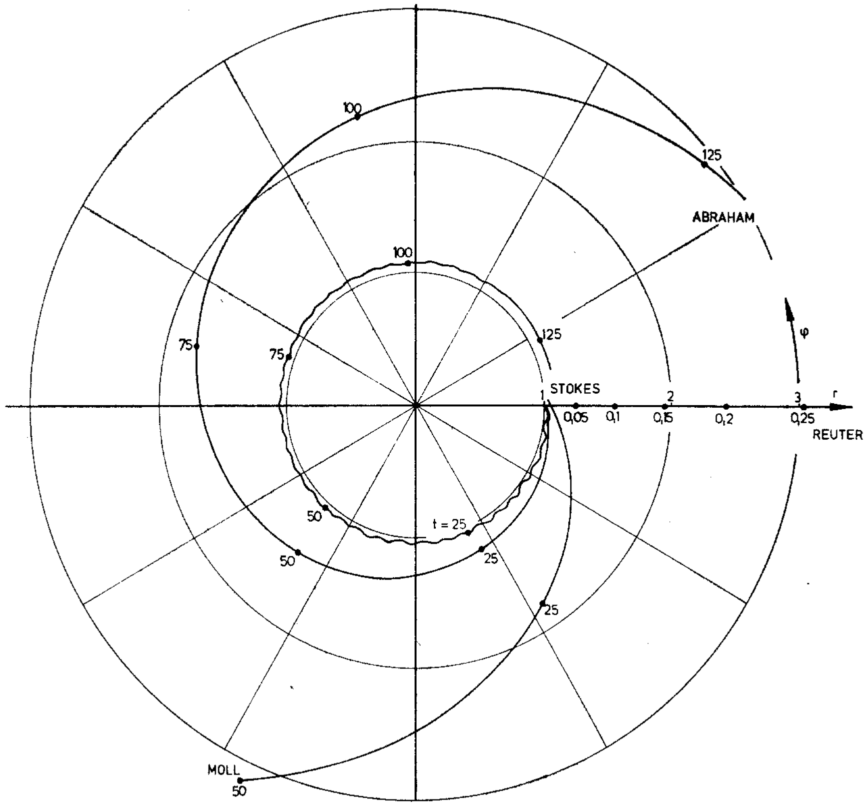

Fig. 10. The horizontal trajectory of a sphere as computed by four different methods

Fig. 10 shows the four different trajectories for the parametercombination $Ek = 0{,}005$, $\varrho/\varrho_f = 1{,}1$, $r_0/a = 30$. It should be mentioned that in ranges of the parameters which have not been studied in the experiments the differences between the four trajectories may be quite different.

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