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Experimental determination of the wall effect for spheres falling axially in cylindrical vessels

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Abstract

Experimental data of the drag exerted by the walls of a cylindrical vessel on a sphere falling axially down it through a liquid are given for Reynolds numbers, based on the diameter of the sphere, between 0·05 *and* 20000. *Existing wall-correction formulae are examined in the light of the new data, the conclusion being that the Francis* (1933) *and Munroe* (1888) *equations are the most reliable in the laminar- and turbulent-flow regions respectively. Graphs show the correction to be applied in the intermediate-flow region.*

1. Introduction

The object of the work was to make reliable measurements of the terminal velocities of spheres falling axially in cylindrical vessels, under conditions extending from purely-laminar to fully-turbulent flow, and to determine the correction to be applied for the retarding effect of the walls on the falling spheres. Available literature relating to the wall effect is both numerous and of high quality (Newton 1687, Munroe 1888, Ladenburg 1907, Sheppard 1917, Faxen 1921, Lunnon 1928, Schmiedel 1928, Barr 1931, Francis 1933, Fulmer and Williams 1936, Gurel 1951, Mott 1951), but most of it is limited to the laminar-flow region. No wall corrections appear to exist for the intermediate-flow region (Reynolds numbers from 1 to 500) and the few published results in the turbulent region (Newton 1687, Munroe 1888, Lunnon 1928, Mott 1951), are widely scattered. In order to fill this gap, experiments were made with 60 spheres (covering a range of diameters and densities), four cylinders of different diameters, and fifteen Newtonian liquids of various viscosities and densities. The range of flow covered Reynolds numbers from 0·054 to 20000 and the results were used to assess the validity of known wall-correction formulae.

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2. Experimental

An indirect method of measuring the falling speed of the test spheres was employed and a detailed description of the apparatus and its accuracy can be found elsewhere (Fidleris and Whitmore 1959). Essentially it consisted of a copper reservoir, in which the liquid was brought to the required temperature, and a jacketed vertical glass cylinder, in which the time of fall of the test spheres over a known distance was measured. The temperature of the water in the jackets of the copper reservoir and the experimental tube was carefully controlled by pumping it through coils immersed in a constant-temperature bath. When equilibrium was reached the temperature of the liquid and the jacketing water could be maintained to within $\pm 0 \cdot 05$ deg c of the required temperature, which for most measurements was chosen to be $20 \cdot 0°$ c.

The detection of the test spheres was based on the measurement of the change of impedance of coils, wound coaxially with the experimental column and spaced ($30 \pm 0 \cdot 01$ cm) apart, when a metallic test sphere passed through them. The time of fall of the test spheres was obtained from a photographic record of the detection signals. By comparison against a standard signal, the fall time could be determined to within $\pm 0 \cdot 3\%$. The top detection coils were situated 40 cm or more below the surface of the liquid to ensure that a test particle reached its terminal velocity before entering the detection system, which was sensitive enough to record the passage of a 0·05 cm diameter steel sphere or a 0·10 cm diameter sphere made of non-ferromagnetic material. The sphere-release mechanisms were designed to ensure that the test particles fell axially down the cylinders, which were precision-bore glass tubes 100 cm in length and 1·50 cm, 2·00 cm, 2·50 cm and 3·00 cm in internal diameter. The accuracy and constancy of tube diameters was in all cases better than $\pm 0 \cdot 1\%$.

The test spheres were made of metal or metal-coated plastic and, with the exception of four cases, were spherical to within a diameter variation of less than $\pm 0 \cdot 15\%$. Their details, including the reliability of the density determinations, are summarized in Table 1.

Table 1. *Test spheres*

No.	*Material*	*Density (g/ml.)*	*Diameter of smallest sphere (cm)*	*Diameter of largest sphere (cm)*
1	Iron-coated plastic	1·275 ± 0·002	0·495	0·554
2	Magnesium	1·800 ± 0·002	0·256	1·018
3	Magnesium–aluminium alloy	2·570 ± 0·005	0·256	0·763
4	Duralumin	2·805 ± 0·005	0·256	1·018
5	Steel	7·72 ± 0·02	0·0793	2·540
6	Lead	11·25 ± 0·03	0·251	0·374
7	Tantalum	16·60 ± 0·04	0·191	0·509
8	Gold	19·36 ± 0·02	0·100	1·000

Details of the liquids which were used in the experiments are given in Table 2. The density of each liquid was determined with a pycnometer to within an accuracy of 0·05% and checked before each experiment with a standardized hydrometer. Their viscosities were originally determined with standard capillary instruments (as described in B.S. No. 188: 1937) and checked before each experiment with a Ferranti rotating-cylinder viscometer. The viscosity was considered to be the least reliable of all the measurements, possessing an error of $\pm 1\%$.

Table 2. *Test liquids*

No.	Liquid	Density at 20° C (g/ml.) minimum	Density at 20° C (g/ml.) maximum	Viscosity at 20° C (cP) minimum	Viscosity at 20° C (cP) maximum
1	Water	0·998		1·005	
2	Olive oil	0·912		78·3	
3	Aqueous solution of glycerol	1·219	1·274	116	225
4	Aqueous solution of glycerol and lead nitrate	1·186	1·187	1·23	27·2
5	Aqueous solution of glycerol and *n*-propyl alcohol	1·055	1·186	40·2	86·4

3. Results

In all, some 3000 velocity determinations were made and the results can be found in detail elsewhere (Fidleris 1958). A typical example of the variation of the terminal velocity with the diameter of the test sphere and the cylinder is shown in Fig. 1, each point representing the average value of three

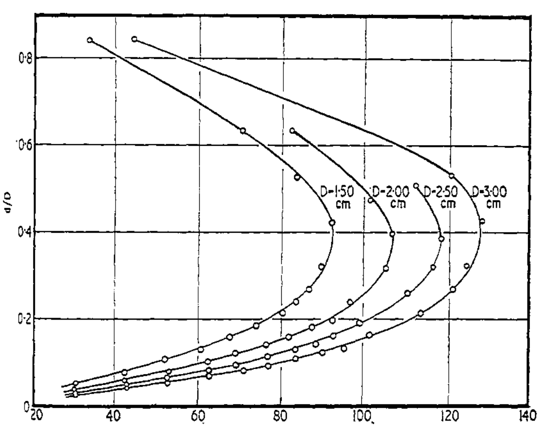

Fig. 1. Terminal velocity of steel spheres falling through water in tubes of various diameters.

or more velocity determinations. In order to summarize all experimental results, use was made of the logarithmic plot of the two non-dimensional quantities:

Reynolds number, $$(Re) = d\rho v/\eta \tag{1}$$

and

resistance coefficient $$\psi = \frac{4g(s-\rho)d}{3\rho v^2} \tag{2}$$

where s = density of test sphere, ρ = density of liquid, g = acceleration due to gravity, d = sphere diameter, v = terminal velocity, η = coefficient of viscosity.

Fig. 2 contains the experimental results for a number of d/D ratios, where D is the diameter of the cylinder. They cover Reynolds numbers (Re) ranging from 0·05 to 20 000 and diameter ratios d/D from 0 to 0·60. It will be seen that for any given d/D ratio all experimental results lie along a smooth curve which broadly follows the $(\log \psi, \log (Re))$ curve for spheres falling through infinite fluid as determined by other workers (Wieselsberger 1922, Liebster 1927, Lunnon

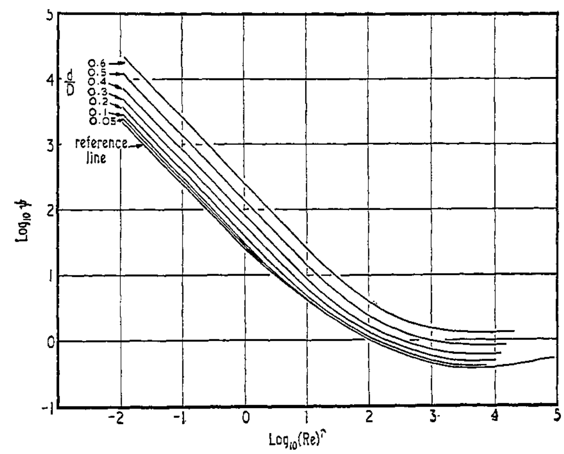

Fig. 2. Resistance coefficient–Reynolds number curves for spheres falling in cylindrical containers.

1928, Goldstein 1938, Möller 1938, Davies 1945). The scatter of the experimental points is such that 95% are within $\pm 3\%$ of the mean line.

4. Discussion of results

From Fig. 2 it is clear that the retarding effect of the wall on a falling sphere diminishes on transition from laminar to turbulent flow and decreases with increasing Reynolds number to an approximately constant value. Any correction to the falling speed of a sphere to allow for wall interference must, therefore, be a function of Reynolds number and in this paper a graphical method of arriving at the correction is suggested.

The first step is to relate the velocities of a sphere, falling through a fluid in cylinders of different diameters, to its terminal velocity in an infinite extent of the fluid. The Reynolds number may then be expressed as $(Re) = c_1 v$ and the resistance coefficient as $\psi = c_2/v^2$, where c_1 and c_2 are constants and depend on the properties of the sphere and the liquid used.

If the results are plotted logarithmically as in Fig. 2, an alteration in velocity from v_1 to v_2 due to a change in the diameter of the containing vessel is represented by a movement of the original point along a line of slope -2 for a distance a, equal to $\sqrt{5}\,\log_{10}(v_1/v_2)$ in log-scale units. If the first point corresponds to a sphere falling in infinite medium, its velocity is v_∞. In a vessel of diameter D, the velocity is reduced by wall interference to a value v and $v_\infty/v = 10^{a/\sqrt{5}}$.

Conversely, if the test is made in a vessel of diameter D, it is only necessary to draw a line of slope -2 through the corresponding point on the (log ψ, log (Re)) curve to cut the curve for infinite medium (subsequently referred to as the reference line). The point of intersection represents the fall of the sphere in infinite fluid.

This procedure was applied to the curves plotted in Fig. 2. Because the present experimental investigations could not be extended to very large vessels, the results obtained by Wieselsberger (1922), Liebster (1927), Lunnon (1928), Möller (1938) and Davies (1945) were used to plot the reference line in Fig. 2. The points used in its construction are given in Table 3 and correspond to those quoted by Goldstein (1938). Fig. 3 shows the factors v_∞/v by which readings taken in a cylindrical vessel must be multiplied to give the terminal velocity in infinite medium. In Fig. 4 the corresponding v/v_∞ factors are shown for converting a sphere velocity, as calculated in infinite medium, to its falling velocity in a cylindrical vessel of given d/D ratio. The two corrections coincide in the laminar region and nearly so under full turbulence, but they differ significantly in the intermediate-flow region. This is because the Reynolds number is calculated from the uncorrected velocity and the slope of the log ψ–log (Re) curve alters appreciably in the intermediate-flow region.

Table 3. *Co-ordinates of the 'reference line' for spheres* ((log ψ, log (Re)) curve for infinite fluid)

(Re)	ψ	(Re)	ψ
0·01	2400	600	0·525
0·1	244	1000	0·455
0·4	63·0	2000	0·394
1·0	26·2	4000	0·380
4·0	8·20	7000	0·382
10	4·27	10 000	0·393
30	2·03	20 000	0·430
60	1·36	40 000	0·472
100	1·05	60 000	0·510
300	0·660		

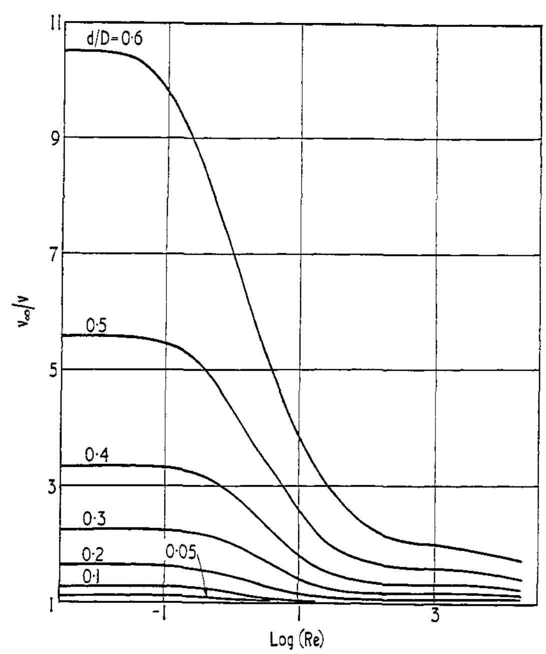

Fig. 3. Correction of fall velocity in a cylindrical vessel to that in infinite fluid.

5. Comparison with existing wall-correction formulae

The retarding effect of the walls of a cylindrical vessel on a sphere falling axially down it can be expressed in the form of a multiplying factor for correcting the falling velocity in an infinite medium v_∞ to the velocity v in a vessel possessing a particular d/D ratio.

An inspection of Fig. 2 suggests that the logarithmic (ψ, (Re)) curves are straight and parallel at Reynolds numbers less than unity, and very nearly so between Reynolds numbers of 1000 and 10 000. Thus under fully laminar or fully turbulent conditions, wall-effect equations which involve only d/D and not Reynolds number should be possible.

The former region has been extensively investigated by other workers (Ladenburg 1907, Sheppard 1917, Faxen 1921, Schmiedel 1928, Barr 1931, Francis 1933, Fulmer and Williams 1936, Gurel 1951) and a number of theoretical and empirical wall-effect equations have been proposed. Of them the three most widely used are those by Ladenburg (1907)

$$v/v_\infty = 1/\{1 + 2\cdot 1(d/D)\} \tag{3}$$

which in turn is the first approximation of a theoretical equation derived by Faxen (1921), which for low Reynolds numbers may be expressed as

$$v/v_\infty = 1\text{-}2\cdot 104(d/D) + 2\cdot 09(d/D)^3 - 0\cdot 95(d/D)^5. \tag{4}$$

Both of them have been shown (Barr 1931) to be limited in their range of application to small values of d/D, i.e. below 0·2 in the case of Eqn (4) and below 0·1 in the case of Eqn (3).

The empirical equation obtained by Francis (1933)

$$\frac{v}{v_\infty} = \left\{\frac{1 - (d/D)}{1 - 0\cdot 475(d/D)}\right\}^4 \tag{5}$$

was found (Gurel 1951) to be satisfactory for d/D ratios of less than 0·9.

In Table 4, Eqns (3), (4) and (5) are compared with the experimental results of Fig. 4, at a Reynolds number of 0·1.

Table 4. *Comparison of wall-correction equations in the laminar region of flow* (v/v_∞)

d/D	*Experimental from Fig. 4*	*Ladenburg from Eqn (3)*	*Faxen from Eqn (4)*	*Francis from Eqn (5)*
0·05	0·894	0·905	0·895	0·898
0·10	0·788	0·826	0·792	0·797
0·20	0·610	0·704	0·596	0·611
0·30	0·446	0·613	0·423	0·444
0·40	0·300	0·543	0·283	0·301
0·50	0·180	0·488	0·179	0·185
0·60	0·094	0·442	0·115	0·098

It will be seen that the agreement is good in the case of the Francis equation and rather poorer using the Faxen equation.

In the turbulent region, at least five wall-effect equations have been proposed, but there is little published experimental evidence against which to test them. Newton (see

Table 5. *Experimental values of v/v_∞ in the turbulent region of flow*

d/D	$(Re) = 100$ *from Fig. 3*	$(Re) = 100$ *from Fig. 4*	$(Re) = 1000$ *from Fig. 3*	$(Re) = 1000$ *from Fig. 4*	$(Re) = 3000$ *from Fig. 3*	$(Re) = 3000$ *from Fig. 4*	$(Re) = 10000$ *from Fig. 3*	$(Re) = 10000$ *from Fig. 4*
0·1	~1	~1	~1	~1	~1	~1	~1	~1
0·2	0·956	0·948	0·960	0·954	0·962	0·958	0·962	0·962
0·3	0·862	0·856	0·868	0·866	0·876	0·874	0·892	0·892
0·4	0·740	0·720	0·766	0·760	0·782	0·776	0·816	0·800
0·5	0·590	0·560	0·640	0·632	0·654	0·648	0·700	0·676
0·6	0·430	0·390	0·506	0·496	0·530	0·518	0·562	0·546

Table 6. *Comparison of wall-correction equations in the turbulent flow region* (v/v_∞)

d/D	*Experimental at (Re) = 3000 averages from Table 5*	*Newton from Eqn (6)*	*Munroe from Eqn (7)*	*Lunnon from Eqn (8)*	*Mott from Eqn (9)*	*Mott from Eqn (10)*
0·1	—	0·987	0·968	0·977	0·982	—
0·2	0·960	0·950	0·911	0·954	0·933	—
0·3	0·875	0·889	0·836	0·931	0·860	—
0·4	0·779	0·806	0·747	0·908	0·776	—
0·5	0·651	0·702	0·646	—	0·690	0·500
0·6	0·524	0·543	0·535	—	0·607	0·325

Barr 1931) made some calculations of the wall-effects of large spheres and obtained the correction term

$$\frac{v}{v_\infty} = \frac{(D^2 - d^2)\sqrt{\{D^2 - (d^2/2)\}}}{D^3}. \qquad (6)$$

Munroe (1888) suggested the approximate formula

$$v/v_\infty = 1 - (d/D)^{3/2} \qquad (7)$$

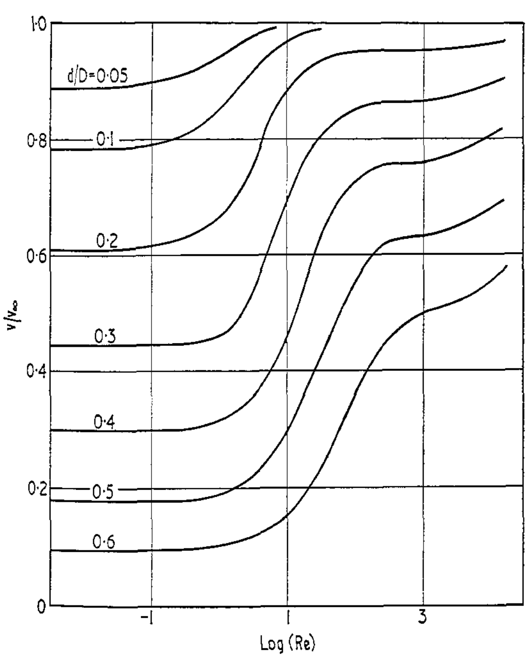

Fig. 4. Correction of fall velocity in infinite fluid to that in a cylindrical vessel.

which he based on measurements of the fall of lead shot in water. He found the wall-effect to be negligible for values of d/D less than 0·1. Lunnon (1928) obtained a series of results with steel spheres falling through air and water in four different tubes, and suggested a relationship

$$v/v_\infty = 1 - 0{\cdot}23(d/D) \qquad (8)$$

which he assumed to hold up to a d/D ratio of 0·3. He did not attempt to derive a correction for higher values of d/D. Mott (1951) used Lunnon's results to give two more formulae. For values of d/D from 0·2 to 0·5 he obtained the equation

$$\frac{v}{v_\infty} = \frac{1}{1 + A(d/D)^2} \qquad (9)$$

where A is a constant varying from 1·8 to 3·2, and for values of d/D from 0·5 to 0·7 the equation

$$\frac{v}{v_\infty} = \frac{1}{1 + (2d/D)^4}. \qquad (10)$$

Mott considered the wall-effects to be negligible below a d/D ratio of 0·15.

In Table 5 the experimental values of v/v_∞, as obtained from Figs 3 and 4, are given for a range of Reynolds numbers and in Table 6 Eqns (6) to (10) inclusive are compared with the averaged results taken from Figs 3 and 4 at a Reynolds number of 3000. There is no close correspondence between any of the equations, although nearly all of them come in places to within 2% of the experimental results. Thus the agreement with Newton's equation (Eqn 6) is within 2% up to a d/D ratio of 0·3, but worsens considerably at higher ratios. The degree of correspondence of any given equation varies to some extent with Reynolds number, although not always in the same way. Munroe's correction (Eqn 7) shows best agreement with experimental results between Reynolds numbers of 1000 and 3000, whereas Newton's (Eqn 6) is most reliable at Reynolds numbers near 10000. The least satisfactory formula is that of Lunnon (Eqn 8).

6. Conclusions

The experimental results show that it is difficult to derive a single relationship to account for the change in the interference effect of the vessel walls on a falling sphere, which occurs with increasing Reynolds number. A correction can, however, be made graphically without difficulty.

It can be concluded that the retarding effect of the wall decreases with increasing Reynolds number, so that for ratios of d/D of 0·05 and 0·10, the wall correction becomes less than 1% for Reynolds numbers exceeding 5 and 30 respectively.

In the laminar-flow region (that is for Reynolds numbers not exceeding approximately $0 \cdot 2$) the Francis equation is the most satisfactory and in the range of d/D ratios from 0 to $0 \cdot 4$ the error does not exceed $\pm 0 \cdot 5\%$. It would therefore appear that in determinations of viscosity from observations of the velocity of a sphere falling axially down a cylinder containing the fluid, corrections for the retarding effect of the wall should be found from the Francis equation.

In the turbulent region the usefulness of available equations is limited, but Munroe's is most satisfactory at Reynolds numbers between 1000 and 3000, where an error of $\pm 2\frac{1}{2}\%$ may be expected for d/D ratios of less than $0 \cdot 6$. Newton's equation increases in reliability when the highest Reynolds numbers reached in the experiments, which were about 10 000, are approached. In this region the error is only $\pm 1\%$.

In the intermediate region Fig. 3 can be used to transpose velocities measured in a cylindrical vessel to the corresponding velocity of fall of a sphere in infinite fluid, the Reynolds number being calculated from the uncorrected fall velocity in each case.

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