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Zusammenfassung

In dieser Arbeit werden Ausdrücke für das Verschiebungsfeld in einer elastischen Kugel angegeben, deren Oberfläche a) elastisch gelagert, b) eingespannt, c) frei ist. Die Verschiebungen werden durch diejenigen Verschiebungen und Spannungen ausgedrückt, die in einem unendlichen Körper unter derselben Verteilung von Singularitäten entstehen.

(Received: Mai 19, 1960.)

On the Wall Effect Correction of the Stokes Drag Formula for Axially Symmetric Bodies Moving Inside a Cylindrical Tube[1])

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It is well known that the drag on a body moving in an incompressible viscous fluid may be determined from Stokes equations for small Reynolds numbers. When the flow field is infinite, formulae for computing the drag on bodies of various shapes are available [1, 2][3]). Recently, a class of solutions of Stokes equations for axially symmetric bodies moving in an infinite flow field were given by PAYNE and PELL [3]. Due to the nature of the Stokes equations, however, the effect of the wall of the containing vessel is generally not negligible [4]. This is clear from the fact that flow conditions specified at infinity often play vital roles to the solutions of Stokes equations.

The motion of a sphere in a vertical tube filled with stationary fluid was first studied by LADENBURG [5]. Assuming that the radius of the sphere is much

[1]) Research done under U. S. Air Force Office of Scientific Research Contract Nr. AF 49 (638) – 521.

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[3]) Numbers in brackets refer to References, page 13.

smaller than the radius of the tube, he obtained the following formula for the drag on the moving sphere:

$$D = 6 \pi \mu a U \left(1 + L \frac{a}{R}\right), \tag{1}$$

where a and R are, respectively, the radii of the sphere and the tube; μ is the viscosity of the fluid; U the velocity of the sphere; and L a constant approximately equal to 2·104 [2].

The above formula was obtained from an approximate solution of the Stokes equations. It gives the drag on the sphere to the order of approximation $O(a/R)$. Under the same conditions, we shall show that the drag on any axially symmetric body moving inside a tube of radius R is given by the formula:

$$D = D_0 \left(1 + \frac{\varkappa D_0}{2 \pi^2 \mu U R}\right) + O\left(\frac{a^2}{R^2}\right), \tag{2}$$

where D_0 is the Stokes drag for the body moving in an infinite domain of fluids, and $\varkappa$ is a constant approximately equal to 2·203. The second term inside the bracket of (2) accounts for the wall effect correction; for the sphere, $D_0 = 6 \pi \mu U a$ and then (2) reduces to LADENBURG's formula (1).

Our problem consists in finding the drag force on an axially symmetric body moving with constant velocity U inside a tube filled with viscous fluid. The tube is assumed infinitely long, and the body is moving rectilinearly along the centerline of the tube so that an axially symmetric flow field, stationary at infinity, is maintained. We choose a cylindrical coordinate, taking the centerline of the tube as the z-axis and the radial distance from this axis as the radius r. The origin of the coordinate system is chosen to coincide, at a certain moment of time, with a point on the centerline of the body; the exact location of this point does not have to be specified. The flow velocity $\boldsymbol{q}(r, z)$ and the pressure $p(r, z)$ are given at this moment by the differential equations [5]:

$$\mu \nabla^2 \boldsymbol{q} - \nabla p = 0 \tag{3a}$$

$$\operatorname{div} \boldsymbol{q} = 0 , \tag{3b}$$

and the boundary conditions

$$\boldsymbol{q} = - U \boldsymbol{i} \qquad \text{at body}, \tag{4a}$$

$$\boldsymbol{q} = 0 \qquad \text{for } r = R , \tag{4b}$$

and

$$\boldsymbol{q} \to 0 \qquad \text{when } z \to \pm \infty , \tag{4c}$$

where $\boldsymbol{i}$ is a unit vector pointing in the direction of the positive z-axis.

If the radius of the tube, R, is very large compared with the characteristic length a of the body, an approximate solution of (3) which gives the drag formula (2) may be obtained in the following manner. We consider the flow field in two regions: an *inner* region which is immediately adjacent to the body and an *outer* region which is far away from the body. The velocity $\boldsymbol{q}$ and the pressure p are treated in these two regions by the following successive approximations:

Inner solutions:

$$\boldsymbol{q} \sim \boldsymbol{h}_0 + \boldsymbol{h}_1 + \cdots, \tag{5a}$$

$$p \sim \overline{p}_0 + \overline{p}_1 + \cdots. \tag{5b}$$

Outer solutions:

$$\boldsymbol{q} \sim \boldsymbol{g}_0 + \boldsymbol{g}_1 + \cdots, \tag{6a}$$

$$p \sim \tilde{p}_0 + \tilde{p}_1 + \cdots. \tag{6b}$$

The first order *inner* solution $\boldsymbol{h}_0$ and $\overline{p}_0$ are obtained by letting $R \to \infty$. The differential equations and the boundary conditions for determining $\boldsymbol{h}_0$ and $\overline{p}_0$ are then

$$\mu \nabla^2 \boldsymbol{h}_0 - \nabla \overline{p}_0 = 0, \tag{7a}$$

$$\operatorname{div} \boldsymbol{h}_0 = 0, \tag{7b}$$

and

$$\boldsymbol{h}_0 = - U \boldsymbol{i} \qquad \text{at body}, \tag{8a}$$

$$\boldsymbol{h}_0 \to 0 \qquad \text{at infinity}, \tag{8b}$$

and the solution is simply that for a body moving in an infinite flow field, which is assumed already obtained. The drag force D_0 given by such a solution is also assumed known.

For later reference, the asymptotic expressions for the velocity $\boldsymbol{h}_0$ and the pressure field $\overline{p}_0$, valid for large values of r and z, are given below (cf. equation (4.9) and (4.11) of reference [3]):

$$\boldsymbol{h}_0 \sim - \frac{D_0}{4 \pi \mu} \left(\frac{\boldsymbol{i}}{\varrho} - \nabla \frac{z}{2 \varrho} \right) + O\left(\frac{a^2}{\varrho^2} \right), \tag{9a}$$

$$\overline{p}_0 \sim - \frac{D_0 z}{4 \pi \varrho^3} + O\left(\frac{a^3}{\varrho^3} \right), \qquad (\varrho^2 = r^2 + z^2). \tag{9b}$$

The above expressions hold true for any shape of bodies with characteristic length a. The terms displayed satisfy the Stokes equations and give description to the velocity and pressure fields induced by applying a concentrated force of strength $- D_0 \boldsymbol{i}$ on the fluid at the origin. Such solutions are usually referred to as the fundamental solutions of the Stokes equations [4].

The *outer* solutions are valid approximations of the exact solutions in a region far from the body; for large tube radius R, this includes the region near the wall. In determining the outer solutions using the Stokes equations, the non-slip condition $\boldsymbol{q} = 0$ at the wall is used. Near the body the outer solutions are not expected to be valid, hence the boundary condition at the body may not be used. In the place of the later, we introduce the *matching conditions* existing between the *inner* and the *outer* solutions. Loosely speaking, this condition states that the leading terms of the outer solutions when expanded into a series for small values of ϱ must agree with the leading terms of the inner solutions when expanded into a series for large values of ϱ. When such *matching* is possible, an additional condition is available for the determination of the solutions.

The first order *outer* solutions $\boldsymbol{g}_0$ and $\tilde{p}_0$ are thus solutions of the Stokes equations which satisfy the boundary condition $\boldsymbol{g}_0 = 0$ at the wall and which reduce to equation (9) for small values of ϱ. By direct calculation one can show that the proper solutions for $\boldsymbol{g}_0$ and $\tilde{p}_0$ are given by the equations

$$\mu \nabla^2 \boldsymbol{g}_0 - \nabla \tilde{p}_0 = D_0 \, \delta(\boldsymbol{r}) \, \boldsymbol{i} \,, \tag{10a}$$

$$\operatorname{div} \boldsymbol{g}_0 = 0 \,, \tag{10b}$$

and the boundary conditions

$$\boldsymbol{g}_0 = 0 \qquad \text{for } r = R \,, \tag{11a}$$

$$\boldsymbol{g}_0 \to 0 \qquad \text{when } z \to \pm \infty \,, \tag{11b}$$

where $\delta(\boldsymbol{r})$ is the δ-function which is zero when $\boldsymbol{r} \neq 0$ and is ∞ when $\boldsymbol{r} = 0$ and its integral over any volume containing the origin is equal to unity. From equation (10a) one sees that the boundary condition at the body (4a) is expressed by introducing a forcing term $D_0 \, \delta(\boldsymbol{r}) \, \boldsymbol{i}$ in the momentum equation. This is equivalent to replacing the body by its retarding force on the fluid.

The solution of $\boldsymbol{g}_0$ may be divided into two parts: $\boldsymbol{g}_0 = \boldsymbol{g}_0^{(1)} + \boldsymbol{g}_0^{(2)}$. The first part, $\boldsymbol{g}_0^{(1)}$, consists of terms which are singular at $\boldsymbol{r} = 0$. These terms arise due to the singular forcing function $D_0 \, \delta(\boldsymbol{r}) \, \boldsymbol{i}$. We shall show by explicit calculation that this part is identical to the asymptotic expressions of $\boldsymbol{h}_0$ and $\overline{p}_0$ displayed in (9). The second part, $\boldsymbol{g}_0^{(2)}$, contains terms which are regular at $\boldsymbol{r} = 0$ and arises due to the non-slip boundary condition $\boldsymbol{g}_0 = 0$ at the wall. The second flow field is therefore induced by the *reflection* of the wall. Since the flow is axially symmetric, the induced velocity $\boldsymbol{g}_0^{(2)}$ has only the z-component at the point $\boldsymbol{r} = 0$. For small values of ϱ, one finds (cf. eq. (31) below)

$$\boldsymbol{g}_0^{(2)} = u' \, \boldsymbol{i} + O\left(\frac{a^2}{R^2}\right). \tag{12}$$

Next, we consider the second order inner solutions $\boldsymbol{h}_1$ and p_1. The requirements for $\boldsymbol{h}_1$ and $\overline{p_1}$ are then that they should match with $\boldsymbol{g}_0^{(2)}$ which is the unmatched part between the first order *inner* and *outer* solutions. Thus

$$\mu \nabla^2 \boldsymbol{h}_1 - \nabla \overline{p_1} = 0 , \tag{13a}$$

$$\operatorname{div} \boldsymbol{h}_1 = 0 , \tag{13b}$$

with the boundary conditions

$$\boldsymbol{h}_1 = 0 \qquad \text{at body} , \tag{14a}$$

$$\boldsymbol{h}_1 \to u' \, \boldsymbol{i} \qquad \text{at infinity} . \tag{14b}$$

The second condition, (14b), is the *matching condition* mentioned above. By considering equations (13) and (14) in conjunction with equation (7) and (8), one sees easily that the sum $\boldsymbol{h}_0 + \boldsymbol{h}_1$ is the solution of the Stokes equations which correspond to the body moving with a relative velocity $U + u'$. The effect of the wall, to the order $O(a/R)$, is therefore an apparent increase of the velocity of the body in the ratio $1:(1 + u'/U)$. There is an equal amount of increase in the drag force if the momentum integral of the velocity field is computed. The drag force on the body in the presence of the wall is then equal to

$$D = D_0 \left(1 + \frac{u'}{U}\right) + O\left(\frac{a^2}{R^2}\right). \tag{15}$$

This basic formula to be used below was also used by LADENBURG in obtaining (1) based on a different argument. The above formulation follows essentially the singular perturbation procedure developed in [6]. In a formal application of the theory one should first introduce the small parameter $\varepsilon = a/R$ and then consider the various limits of the flow quantities $\boldsymbol{q}$ and p corresponding to different orders of the space variables r and z with respect to ε. This systematic procedure leads to an *inner* and an *outer* expansion similar, respectively, to (5) and (6). Such formal development, while leading to justification to many arguments given above, is too lengthy to be used in this short paper.

Now, let us find the value of u' by considering (10) and (11). We put

$$\boldsymbol{g}_0 = \frac{D_0}{\mu} \left[\nabla^2 \Phi \, \boldsymbol{i} - \nabla \frac{\partial \Phi}{\partial z} \right], \tag{16a}$$

$$\tilde{p}_0 = - D_0 \nabla^2 \frac{\partial \Phi}{\partial z} . \tag{16b}$$

By inserting these expressions into equations (10), one easily verifies that (10b) is identically satisfied and (10a) becomes

$$\nabla^2 \nabla^2 \Phi = \delta(\boldsymbol{r}) . \tag{17}$$

Furthermore, (11a) and (11b) lead to the following conditions for $\Phi(r, z)$:

$$\frac{\partial \Phi}{\partial r} = \frac{\partial^2 \Phi}{\partial r^2} = 0 \qquad \text{for } r = R \,, \tag{18a}$$

$$\Phi \to 0 \qquad \text{when } z \to \pm \infty \,. \tag{18b}$$

We define the Fourier transform of a function $F(r, z)$ by the pair

$$F(r, \lambda) = \int_{-\infty}^{\infty} e^{-i\lambda z} F(r, z)\, dz \tag{19a}$$

and

$$F(r, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{i\lambda z} \bar{F}(r, \lambda)\, d\lambda \,, \tag{19b}$$

and denote $\nabla^2 \Phi$ by the function $\varphi(r, z)$. After applying the transform to equation (17) there follows

$$\frac{d^2 \bar{\varphi}}{dr^2} + \frac{1}{r} \frac{d\bar{\varphi}}{dr} - \lambda^2 \bar{\varphi} = \delta(r) \,. \tag{20}$$

Here $\delta(r)$ is a δ-function defined by

$$\delta(\boldsymbol{r}) = \delta(r)\, \delta(x) \,. \tag{21}$$

The solution of equation (20) is

$$\bar{\varphi}(r, \lambda) = -\frac{1}{2\pi} K_0(\lambda r) + \frac{A}{2\pi} I_0(\lambda r) \,. \tag{22}$$

The equation for $\bar{\Phi}(r, \lambda)$ is then

$$\frac{d^2 \bar{\Phi}}{dr^2} + \frac{1}{r} \frac{d\bar{\Phi}}{dr} - \lambda^2 \bar{\Phi} = \bar{\varphi}\,(\lambda r) \,, \tag{23}$$

which gives the result

$$\lambda^2 \bar{\Phi}(r, \lambda) = \frac{\lambda r}{4\pi} K_1(\lambda r) + \frac{A}{2\pi} \lambda r\, I_1(\lambda r) + \frac{B}{2\pi} I_0(\lambda r) \,. \tag{24}$$

In (24), A and B are the constants of integration to be determined from boundary conditions (18) and I_0, I_1, K_0 and K_1 are the modified Bessel functions. By direct calculation one finds

$$A(\lambda R) = \frac{I_1(\lambda R)\, K_0(\lambda R) - \frac{\lambda R}{2} [I_0(\lambda R)\, K_0(\lambda R) + I_1(\lambda R)\, K_1(\lambda R)]}{I_0(\lambda R)\, I_1(\lambda R) + \frac{\lambda R}{2} [I_1^2(\lambda R) - I_0^2(\lambda R)]} \tag{25a}$$

and

$$B(\lambda R) = \frac{\frac{\lambda R}{4}}{I_0(\lambda R)\, I_1(\lambda R) + \frac{\lambda R}{2}\,[I_1^2(\lambda R) - I_0^2(\lambda R)]} . \tag{25b}$$

The z-component of $\boldsymbol{g}_0$, $u(r, z)$, is given by the expression

$$u(r, z) = \frac{D_0}{\mu} \left[\varphi(r, z) - \frac{\partial^2}{\partial z^2}\, \Phi(r, z)\right] . \tag{26}$$

The Fourier transform of $u(r, z)$ is

$$\bar{u}(r, \lambda) = \frac{D_0}{\mu} \,[\bar{\varphi}(r, \lambda) + \lambda^2 \bar{\Phi}(r, \lambda)] \tag{27}$$

$$= \bar{u}^{(1)}(r, \lambda) + \bar{u}^{(2)}(r, \lambda) , \tag{28}$$

where

$$\bar{u}^{(1)}(r, \lambda) = -\frac{D_0}{2 \pi \mu} \left[K_0(\lambda r) - \frac{\lambda r}{2}\, K_1(\lambda r)\right] \tag{29}$$

and

$$\bar{u}^{(2)}(r, \lambda) = \frac{D_0}{2 \pi \mu} \,[A(\lambda r) \big(I_0(\lambda r) + \lambda r\, I_1(\lambda r)\big) + B(\lambda R)\, I_0(\lambda r)] . \tag{30}$$

These two parts of $\bar{u}(r, \lambda)$ correspond, respectively, to the z-component of $\boldsymbol{g}_0^{(1)}$ and $\boldsymbol{g}_0^{(2)}$. The inverse of $\bar{u}^{(2)}(r, \lambda)$ is obtained by using (19b):

$$u^{(2)}(r, z) = \frac{1}{2 \pi R} \int_{-\infty}^{\infty} e^{i \sigma (z/R)}\, \bar{u}\Big(\frac{r}{R}\, \sigma, \sigma\Big)\, d\sigma ; \quad \sigma = \lambda R , \tag{31a}$$

and $u^{(2)}(r, z)$ is regular at the point $r = 0$:

$$u^{(2)}(r, z) = u' + O\Big(\frac{1}{R^2}\Big) , \tag{31b}$$

where u' is the value of $u(r, z)$ at the point $r = 0$. One finds

$$\left.\begin{aligned} u' &= \frac{D_0}{2 \pi \mu} \int_{-\infty}^{\infty} [\bar{\varphi}(0, \lambda) + \lambda^2 \bar{\Phi}(0, \lambda)]\, \mathrm{d}\lambda \\ &= \frac{D_0}{4 \pi^2 \mu} \int_{-\infty}^{\infty} [A(\lambda R) + B(\lambda R)]\, d\lambda = \frac{D_0}{2 \pi^2 \mu R} \int_{0}^{\infty} H(\sigma)\, d\sigma \\ &= \frac{\varkappa D_0}{2 \pi^2 \mu R} , \end{aligned}\right\} \tag{32a}$$

where

$$\left.\begin{aligned}\varkappa = \int_0^\infty H(\sigma)\,d\sigma = \int_0^\infty \frac{I_1(\sigma)\,K_0(\sigma) - \frac{\sigma}{2}\,[I_0(\sigma)\,K_0(\sigma) + I_1(\sigma)\,K_1(\sigma)] + \frac{\sigma}{4}}{I_0(\sigma)\,I_1(\sigma) + \frac{\sigma}{2}\,[I_1^2(\sigma) - I_0^2(\sigma)]} \\ \cong 2{\cdot}203\,.\end{aligned}\right\} \tag{32b}$$

So, finally,

$$D = D_0\left(1 + \frac{u'}{U}\right) = D_0\left(1 + \frac{\varkappa\,D_0}{2\,\pi^2\,\mu\,R\,U}\right). \tag{33}$$

In Table I we list several cases where the value D_0 is given in reference [3].

Table 1

Body	Drag (Infinite Domain)	Drag (Inside Tube of Radius R)
(1) Hemispherical Cup	$17{\cdot}525\,\mu\,a\,U$	$17{\cdot}525\,\mu\,a\,U(1 + 1{\cdot}956\,a/R)$
(2) Flat disc of radius a	$16\,\mu\,a\,U$	$16\,\mu\,a\,U(1 + 1{\cdot}786\,a/R)$
(3) Sphere of radius a	$6\,\pi\,\mu\,a\,U$	$6\,\pi\,\mu\,a\,U(1 + 2{\cdot}104\,a/R)$
(4) Prolate spheroid	$8\,\pi\,\alpha\,\mu\,a\,U$	$8\pi\alpha\mu a\,U\,(1 + 2{\cdot}805\,\alpha\,a/R)$
(5) Oblate spheroid	$8\,\pi\,\beta\,\mu\,a\,U$	$8\pi\beta\mu a\,U(1 + 2{\cdot}805\,\beta\,a/R)$

In the above, a is the radius of the frontal area circle of the body for cases (1)–(3). The cross section of the spheroids, (4), (5), intercepted by the $r - z$ plane, is an ellipse. For the prolate spheroids the foci are at $(\pm\,a, 0)$ and α is equal to the value

$$\alpha = \left[\frac{1}{2}\,(s^2 + 1)\log\frac{s+1}{s-1} - s\right]^{-1}. \tag{34}$$

For the oblate spheroids the foci are at $(0, \pm\,a)$ and β is equal to the value

$$\beta = [(1 - s^2)\cot^{-1} s + s]^{-1}\,. \tag{35}$$

In (34) and (35) s is equal to the ratio of the semi-axis along the z-axis to the focal length a, of the ellipse.

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Zusammenfassung

In dieser Arbeit wird der Einfluss der Wand auf die Bewegung eines rotationssymmetrischen Körpers längs der Achse eines mit zäher Flüssigkeit gefüllten Rohres betrachtet. Der Widerstand des Körpers wird unter der Voraussetzung berechnet, dass die Dimensionen des Körpers gegenüber dem Radius des Rohres klein sind. Wir finden für den Widerstand D eines Körpers, der sich mit der Geschwindigkeit U längs der Achse einer zylindrischen Röhre mit dem inneren Radius R bewegt,

$$D = D_0 \left[1 + \frac{\varkappa D_0}{2 \pi^2 \mu U R}\right],$$

wo D_0 der Widerstand des Körpers in einer sonst den ganzen Raum füllenden Flüssigkeit, $\varkappa$ eine Konstante etwa gleich 2·203, und μ die Viskositätskonstante ist.

(Received: June 10, 1960.)

Die Instabilität der Strömung zwischen zwei rotierenden Zylindern gegenüber Taylor-Wirbeln für beliebige Spaltbreiten

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Einleitung

Diese Arbeit beschäftigt sich mit der zuerst von TAYLOR [1][2]) untersuchten Instabilität der laminaren, inkompressiblen Strömung zwischen zwei rotierenden koaxialen Zylindern. Die Störungen, die von einer gewissen Rotationsgeschwindigkeit des inneren Zylinders an auftreten können, bilden sich bekanntlich in Form eines in Richtung der Zylindererzeugenden periodischen Wirbelmusters aus.

Um die numerischen Auswertungen mit erträglichem Aufwand durchführen zu können, musste TAYLOR, neben anderen Annahmen wie der Kleinheit der betrachteten Störungen, verlangen, dass die Spaltbreite (Differenz der Zylinderradien) klein sei gegenüber den Krümmungsradien der Zylinder.

Die Erweiterung dieser Theorie auf den Fall beliebiger Spaltbreite gelang erstmals CHANDRASEKHAR [2] in neuester Zeit. Die von ihm angewandte Methode zur Lösung des Problems beruht auf einer auch von TAYLOR angesetzten Reihenentwicklung der Störungsamplituden nach Besselfunktionen.

In der vorliegenden Arbeit wird zurLösung desselben Problems ein anderer Weg beschritten, der wesentlich auf der von GÖRTLER [3] und HÄMMERLIN [4]

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[2]) Die Ziffern in eckigen Klammern verweisen auf das Literaturverzeichnis, Seite 29.