

Simulation and Optimization of Tesla Valves

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ABSTRACT

Tesla valve is a no-moving parts valve. It has many advantages over conventional check-valves in microscale. However, there has been very few literatures discussing its optimization. We present a comprehensive method to optimize the valve. Complete design optimization parameters of the valve are identified for the first time. A method to construct the geometry of the valve is also proposed. Numerical method is used to study steady flow 2-D models of different valves and to derive formulas for optimum geometrical parameters. The optimum angle α and the optimum straight segment L are inversely proportional and proportional to the Reynolds number, respectively. This work allows systematic designing of Tesla valves. Depending on the required flow range, an optimum design can be proposed.

Keywords: Tesla valve, valveless, no-moving parts valves, optimization.

1 INTRODUCTION

Tesla valve is a no-moving parts (NMP) valve. NMP valves are fluid channels with direction-dependant flow resistances. In microscale, they have distinct advantages over moving-parts valves. They are reliable and easy to fabricate. Moreover, they are not sensitive to particulates or contaminants [1].

The most important parameter for evaluating the performance of a NMP valve is the diodicity D_i , which is the ratio between pressure drops in the reverse flow direction Δp_r and the forward direction Δp_f at the same flow rate [1].

$$D_i = \frac{\Delta p_r}{\Delta p_f} \quad (1)$$

The Tesla valve concept in macro scale was invented in 1920 [2]. Forster et al. in 1995 applied it into micropump [1]. It achieved a diodicity of 1.2. The valves can be easily fabricated on silicon using DRIE or SU-8. Figure 2 shows a Tesla valve that we realized in SU-8 on a Silicon substrate. It is under testing.

Although introduced quite early, up to now, there has been no systematic method to optimize the steady flow performance of Tesla valve. There is no literature fully identifying design optimization parameters or the

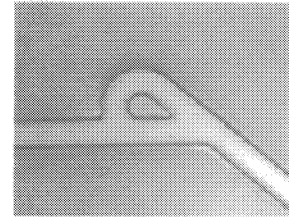


Figure 1: A Tesla valve fabricated in SU-8

method to construct the geometry of the valve. To the best of the authors' knowledge, there were only two papers presenting Tesla valve's optimization. Bardell et al. used electrical analogy to establish a linear model of the micropump-Tesla valve system and to optimize its dynamic performance [3]. Ref [4] reported the numerical analysis of the influence of the geometry to the valve's performance. However, it considered only one parameter—the internal wall length. No clear conclusion could be derived.

A systematic design optimization method is needed to successfully apply Tesla valves into microfluidics. This paper proposes a comprehensive approach to optimize the steady flow performance of a Tesla valve.

- First, the geometry of the valve is analyzed to identify design optimization parameters.
- Second, the design parameters are combined to form different valve configurations.
- Third, numerical software (ANSYS 6.0) is used to simulate steady flow 2-D models of the valve configurations. The diodicity at different configurations and under various flow rates are obtained and compared to find the optimum parameters.

2 METHODOLOGY

2.1 Design Parameters

According to Figure 2, the Tesla valve's geometry is fully determined by: the channel width W , the channel depth D , the entry and exit L_1 and L_2 , the angle α , the straight segment L , and the radius R of the inner curve.

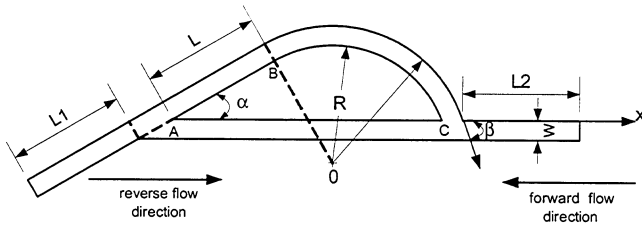


Figure 2: Geometry of a typical Tesla valve

Based on six parameters mentioned above, we propose the method to construct the Tesla valve as follows:

- *Construct the inner wall.* Choose arbitrarily a starting point, called A. From A, draw a straight line segment AB with length L and angle α . From B, draw circle (O) (center O, radius R) tangent to L . Circle (O) intersects the X axis at C. The close polyline ABC forms the inner wall of the valve.
- *Construct the outer wall.* Draw the offset boundary to the polyline ABC, at the distant W (the width of channel). Add the two entrance and exit sections L_1 and L_2 to obtain the full valve structure.

The inner curve radius R has a minimum value R_{min} , because the circle (O) needs to intersect the X axis. R_{min} is calculated by Equation(2) (when (O) is tangent to X axis).

$$R_{min} = L \tan \frac{\alpha}{2} \quad (2)$$

2.2 Numerical Models

We use CFD FLOTTRAN (ANSYS 6.0) to solve steady flow 2-D models of the valves. The working fluid is water (density $\rho = 1000 \text{ kg/m}^3$ and viscosity $\mu = 4.6 \times 10^{-4}$). For a specific valve configuration (α , L , R), constant flow rates are alternatively applied to forward and reverse directions. The pressure drops across the valve in both directions are obtained for every flow rate. Diodicity is then calculated by using Equation(1).

The models are meshed with tetrahedral elements. Finer grid sizes are used near walls or region with high velocity gradients. The simulation is stopped when the pressure residual reaches 10^{-6} .

L_1 and L_2 are chosen to be $600 \mu\text{m}$ so that the flows are fully developed. Channel depth D is set to $100 \mu\text{m}$ but not accounted for in our 2-D models. The channel width W is kept at $100 \mu\text{m}$ and used as a reference to express other quantity in dimensionless representation. Three parameters α , L , and R are varied to find the optimum configuration.

The applied flow rates Q are $500 \mu\text{l/min}$, $750 \mu\text{l/min}$, $1000 \mu\text{l/min}$, $1250 \mu\text{l/min}$, and $1500 \mu\text{l/min}$. The corresponding Reynolds numbers Re , calculated by Equation(3),

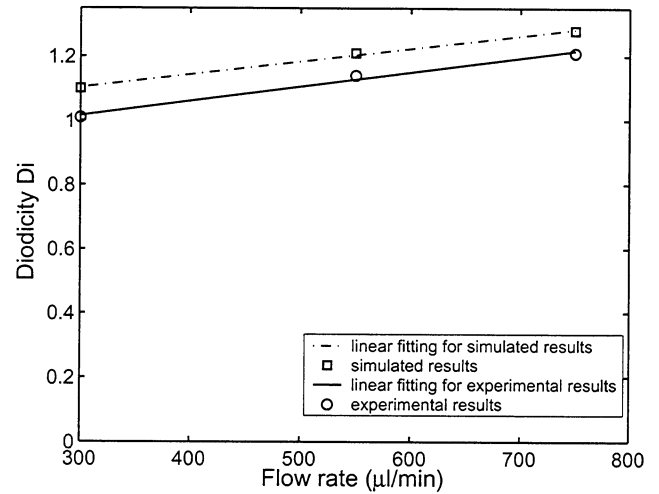


Figure 3: Diodicities of experimental and simulated data vs. flow rates ($\alpha = 45^\circ$, $L = 235 \mu\text{m}$, $R = 228 \mu\text{m}$)

are 181, 272, 362, 453, and 543.

$$Re = \rho \frac{V D_h}{\mu} \quad (3)$$

with ρ is the density, μ is the viscosity; V is the average flow velocity; D_h is the hydraulic diameter of the channel. The largest Reynolds number is 543, much lower than the transition number of 2300, so the flow regime is laminar.

2.3 Validation of Simulation Model

We check the validation of our numerical model by simulate the Tesla valve experiment done by Forster et al. at University of Washington [1]. The simulated diodicities are within 10% of the experimental results, Figure 3. The numerical model is validated and can be used for optimization of Tesla valves.

3 RESULTS AND DISCUSSION

Diodicity D_i is found to be inversely proportional to R , Figure 4. It can be explained as, at fixed values of α and L , the circle with smaller R intersects the X axis at a larger angle β . A larger β reduces the amount of flow entering the curve section in forward flow direction, thus reduces pressure drop. While in reverse flow, a larger β helps the flow in the curve section to block the flow in the straight section more effectively.

The above observation is significant because it reduces our optimization from “finding three variables (α , L , R) so that D_i is maximum” to “finding two variables (α , L , with $R = R_{min}$) so that D_i is maximum”.

Next, we vary α (10° – 80°) and L ($100 \mu\text{m}$ – $600 \mu\text{m}$), with $R=R_{min}$. The calculated diodicity D_i at every combination of α , L , under different flow rates is recorded and compared to find maxima.

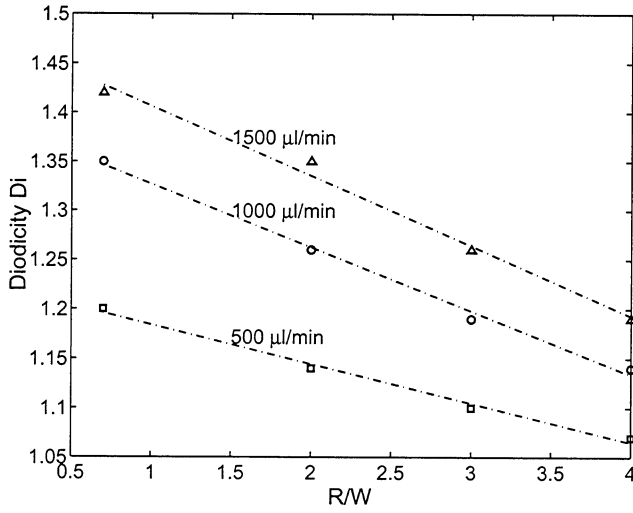


Figure 4: Diodicity vs. R/W ($\alpha = 50^\circ$, $L = 150 \mu\text{m}$)

Figure 5 shows local maxima of D_i versus α . According to the fitting functions, D_i reaches global maxima at 64° , 60° , 57° , 54° , and 52° at flow rates of $500 \mu\text{l}/\text{min}$, $750 \mu\text{l}/\text{min}$, $1000 \mu\text{l}/\text{min}$, $1250 \mu\text{l}/\text{min}$, and $1500 \mu\text{l}/\text{min}$ respectively. From Figure 6, which plots the optimum values of α against Reynolds number Re , we derive the formula for optimum α :

$$\alpha_{opt} = -0.033Re + 69.4 \quad (4)$$

Equation(4) successfully predicts that macroscale Tesla valves (high Reynolds number) have $\alpha < 20^\circ$ [2], while their micro counterparts (under low Reynolds number) have $\alpha > 40^\circ$ [1], [4].

With α and R are set at optimum values, L is varied to find maximum D_i . Optimum values of L under different flow rates are plotted in Figure 7. Similarly to α , a formula for optimum L is derived:

$$(L/W)_{opt} = 0.007Re + 0.58 \quad (5)$$

The fact that the optimum values of L and α depending on flow rates (or Reynolds number) leads to an important conclusion, there is no unique optimum Tesla valve configuration for a broad flow range.

4 CONCLUSION AND FUTURE WORKS

This paper identified the design optimization parameters for Tesla valve and studied different valve configurations with numerical software.

We proved, with simulation results, that the valve diodicity is inversely proportional to the radius R , and that optimum configurations (α, L) depend on flow rates. We also deduced the formulas for optimum values of α and L for different flow rates.

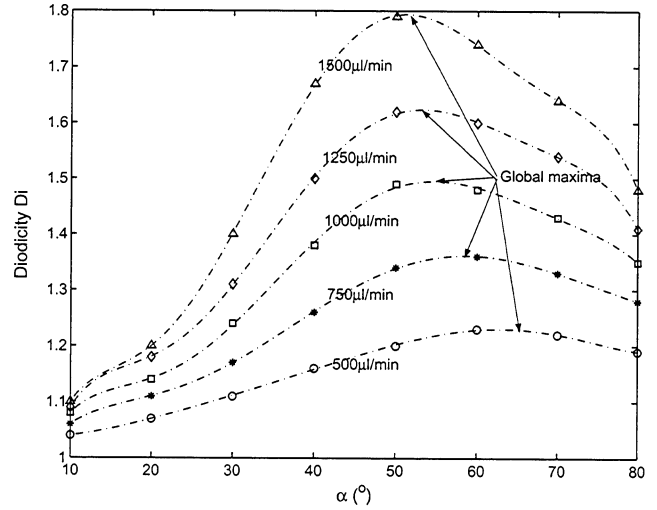


Figure 5: Maxima of diodicity vs. α

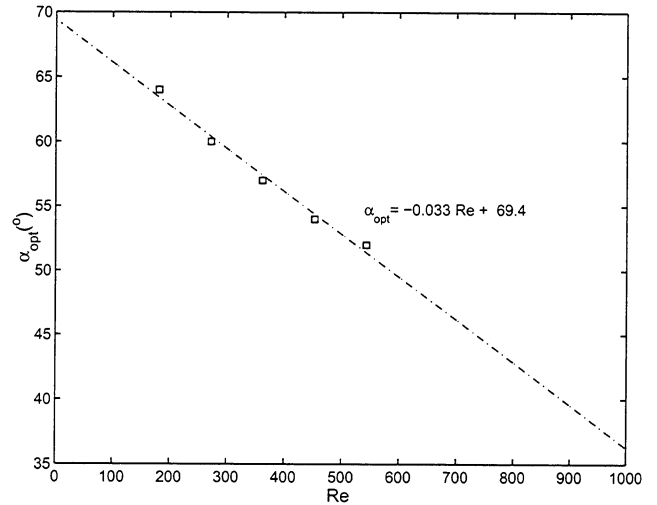


Figure 6: Optimum α vs. Re

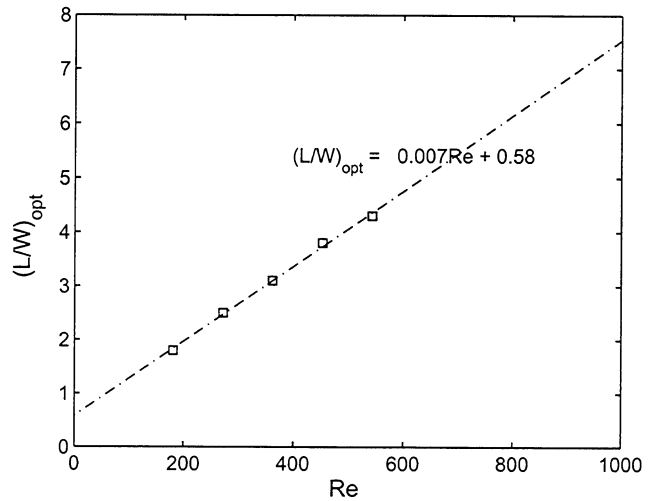


Figure 7: Optimum L/W vs. Re

Tesla valves promise many applications in microfluidics. To unleash their potential, more studies are needed. Numerical analysis at higher flow rates is required to better characterize the Tesla valve over a wider range of flow condition.

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