

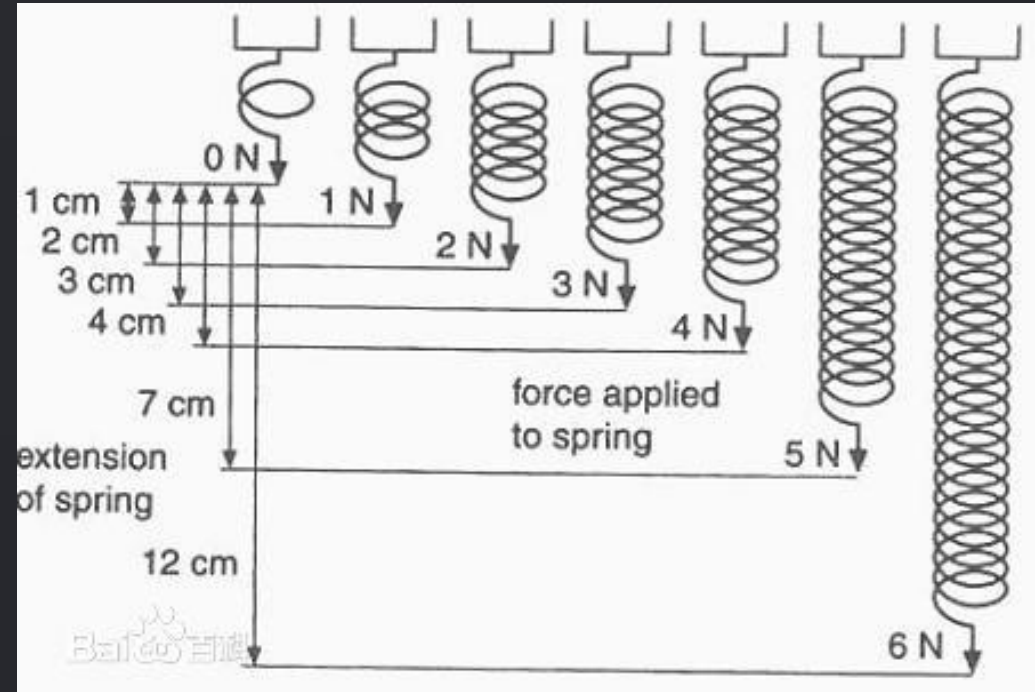
非线性物理实验探究

——结合python仿真模拟

马雨枫 陈子涵 物理学

什么是非线性物理？

- 非线性：一个系统输出与输入并不成正比！
比如：胡克定律



本次实验的研究

- 混沌电路

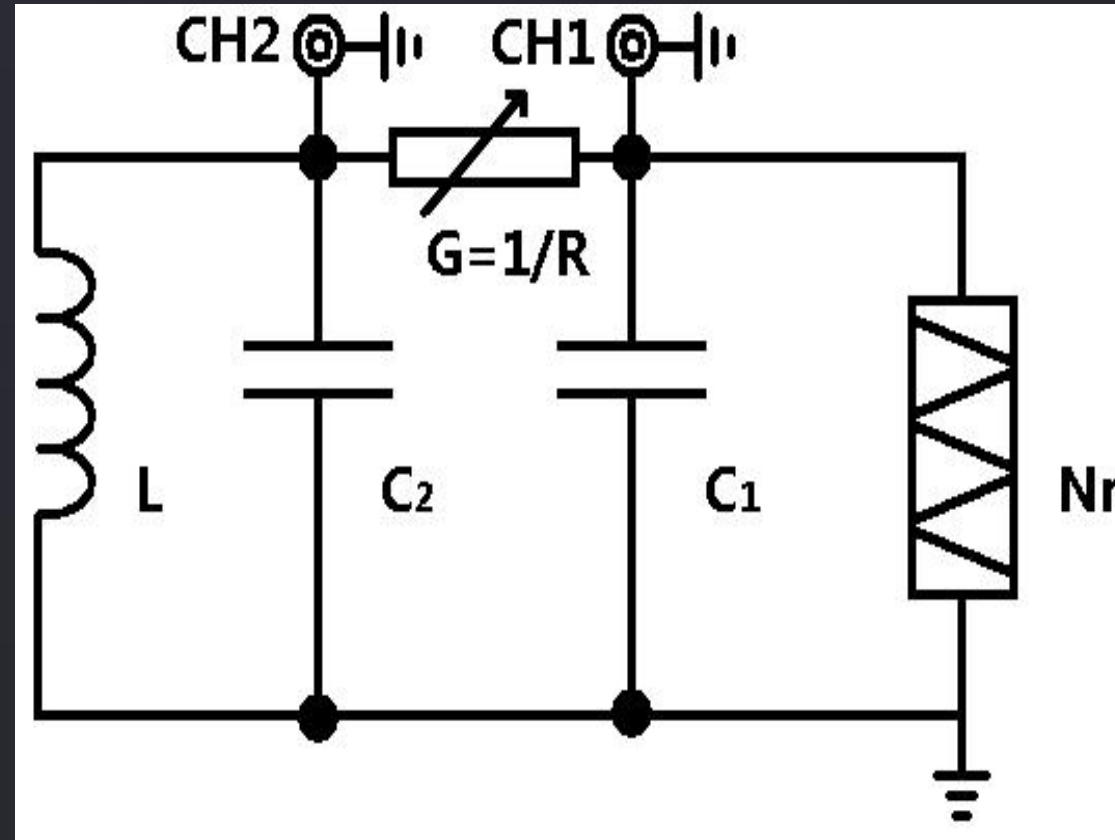
包含蔡氏电路，以及简单细胞神经网络的同步（cellular neural network）

同步混沌以及电路仿真模拟

- 分形探讨

含有许多分形图片的gallery

什么是蔡氏电路



非线性电路满足的方程

- 电路满足方程：

$$\begin{cases} C_1 \frac{dU_1}{dt} = G(U_2 - U_1) - g(U_1) \\ C_2 \frac{dU_2}{dt} = G(U_1 - U_2) + I_L \\ L \frac{dI_L}{dt} = -U_2 \end{cases}$$

- 非线性电阻与电压之间的关系

$$g(U) = G_b U + \frac{G_b - G_a}{2} (|U - E| - |U + E|)$$

- 无量纲化：

$$\begin{cases} \frac{dx}{dt} = a(y - x - f(x)) \\ \frac{dy}{dt} = x - y + z \\ \frac{dz}{dt} = -by \end{cases}$$

$$f(x) = cx + 0.5(d - c)(|x + 1| - |x - 1|)$$

简化方程形式

- 将 $[x,y,z]$ 装置向量看作是新的变化元 X ，则我们可以得到一个简化后的方程

$$\begin{cases} \dot{\mathbf{X}}(t) = \mathbf{A}\mathbf{X}(t) + \mathbf{b} \\ \mathbf{X}(0) = \mathbf{X}_0 \end{cases}$$

- 怎么解这个方程？

不动点+齐次微分方程解的叠加！

不动点 $\mathbf{X}_Q = -\mathbf{A}^{-1}\mathbf{b}$

考虑到非线性负阻具有三个区间，因而所对应的不动点也应当有三个

齐次微分方程解

- 可以相应求得A矩阵的三个本征值 λ_1 、 λ_2 、 λ_3
- 倘若这三个本征值皆为实数：

$$x(t) = c_1 e^{\lambda_1 t} \vec{\xi}_1 + c_2 e^{\lambda_2 t} \vec{\xi}_2 + c_3 e^{\lambda_3 t} \vec{\xi}_3$$

- 倘若A有一个实本征值 γ 和一对共轭的复本征值 $\sigma \pm i\omega$ （符合实验参数）

$$\begin{cases} \mathbf{x}(t) = \mathbf{x}_r(t) + \mathbf{x}_c(t) \\ \mathbf{x}_r(t) = c_r e^{\gamma t} \vec{\xi}_\gamma \\ \mathbf{x}_c(t) = 2c_c e^{\sigma t} [\cos(\omega t + \phi_c) \vec{\eta}_r - \sin(\omega t + \phi_c) \vec{\eta}_i] \end{cases}$$

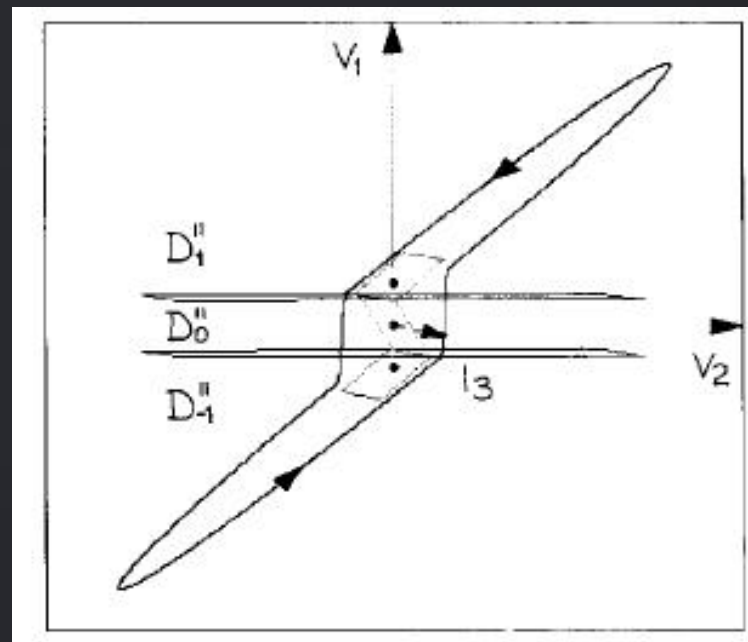
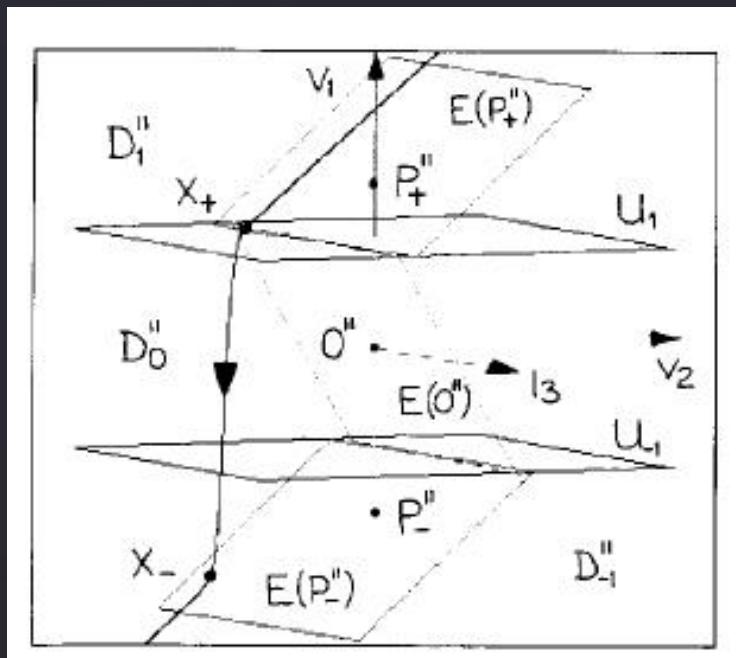
复平面、实平面

稳定解与混沌解条件

- 1. $\sigma < 0, \gamma < 0$
- 2. $\sigma > 0, \gamma < 0$
- 3. $\sigma < 0, \gamma > 0$
- 4. $\sigma > 0, \gamma > 0$

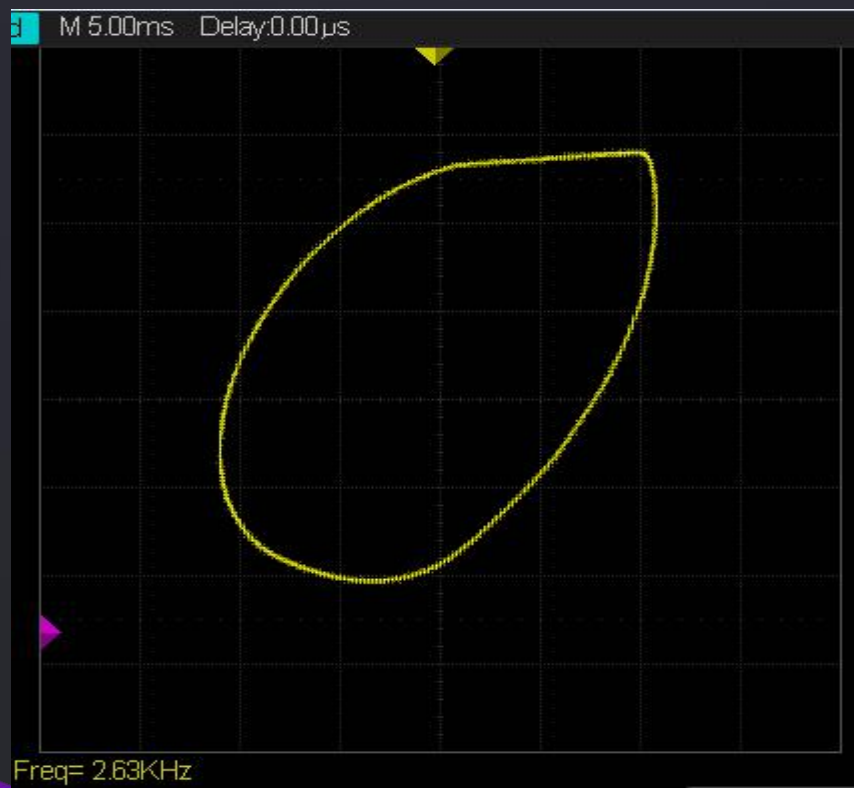
如何观测混沌现象？

- 利用 x, y 的二维相图就可以得到：

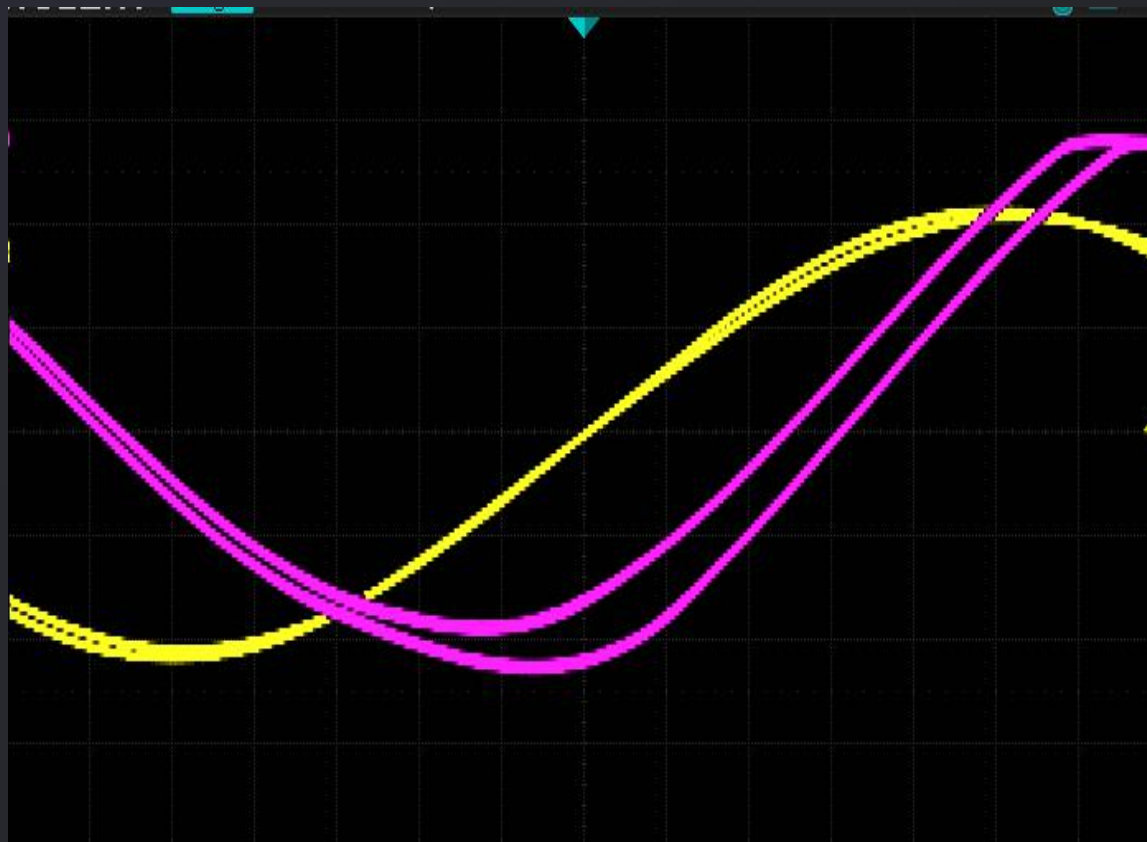
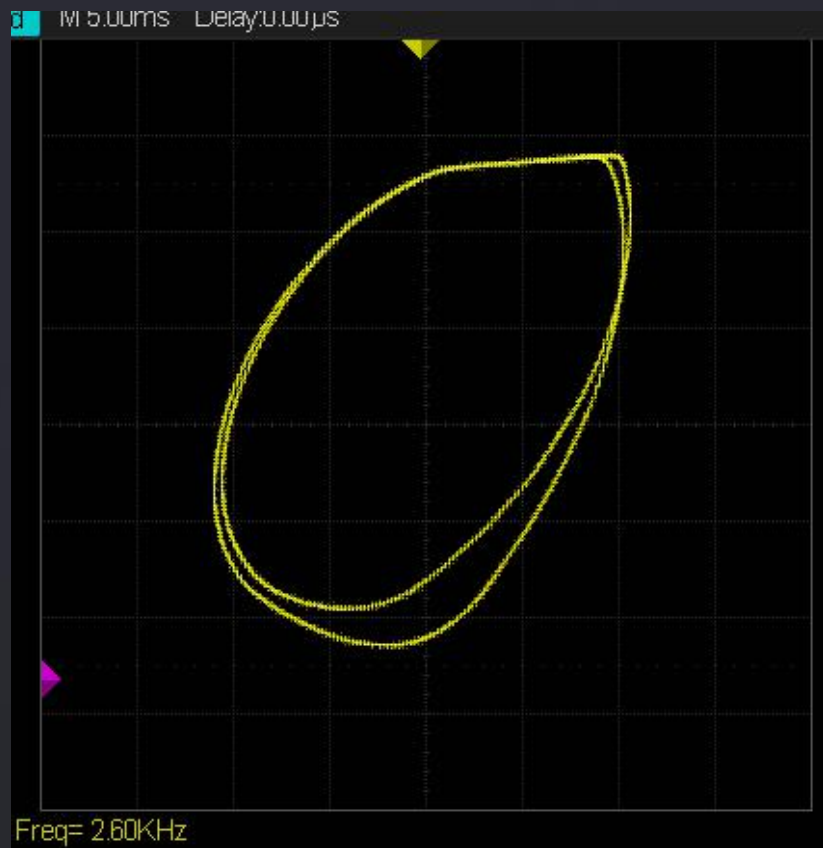


实验观察得到的混沌相图

- 单周期



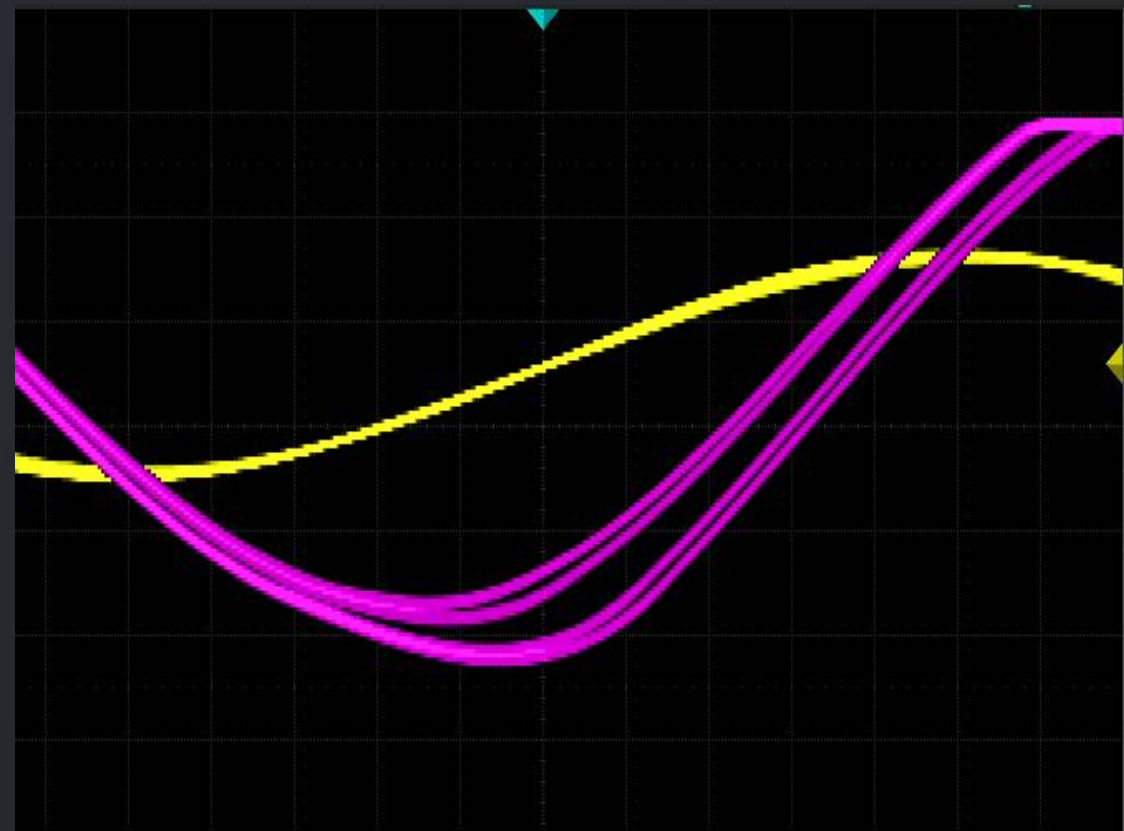
双周期



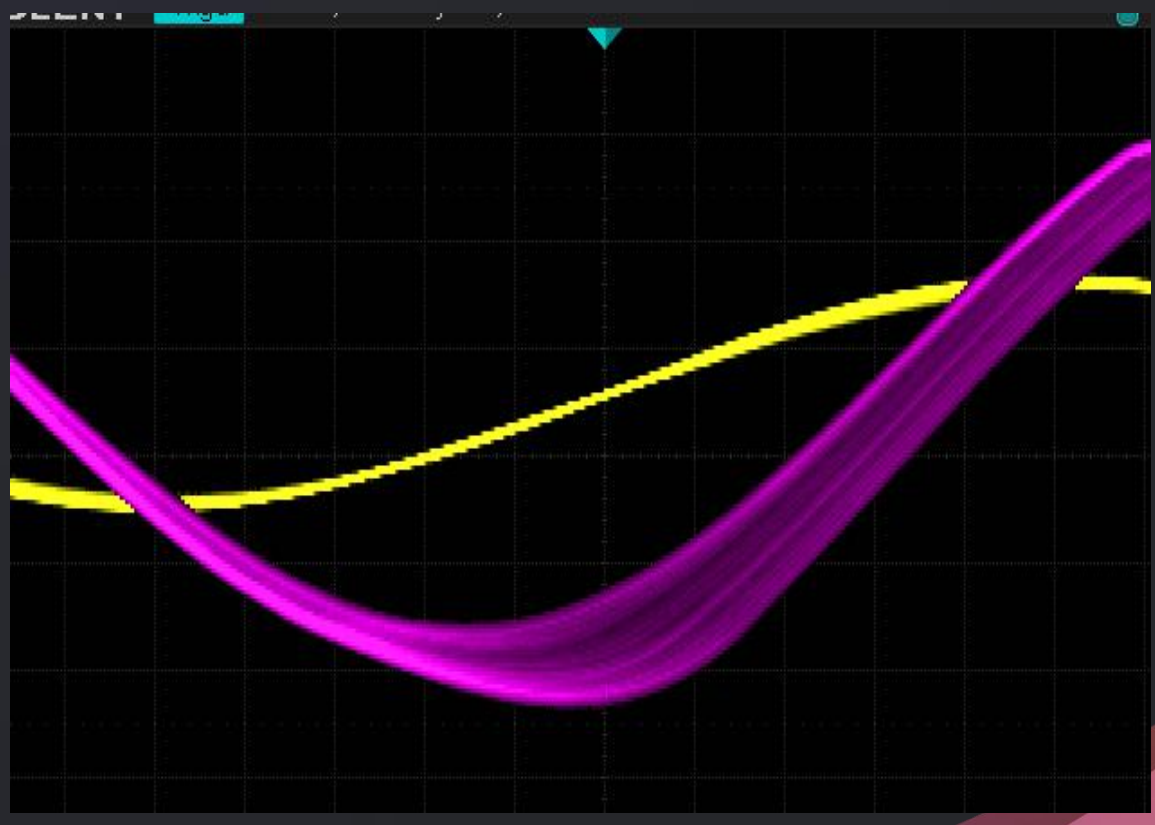
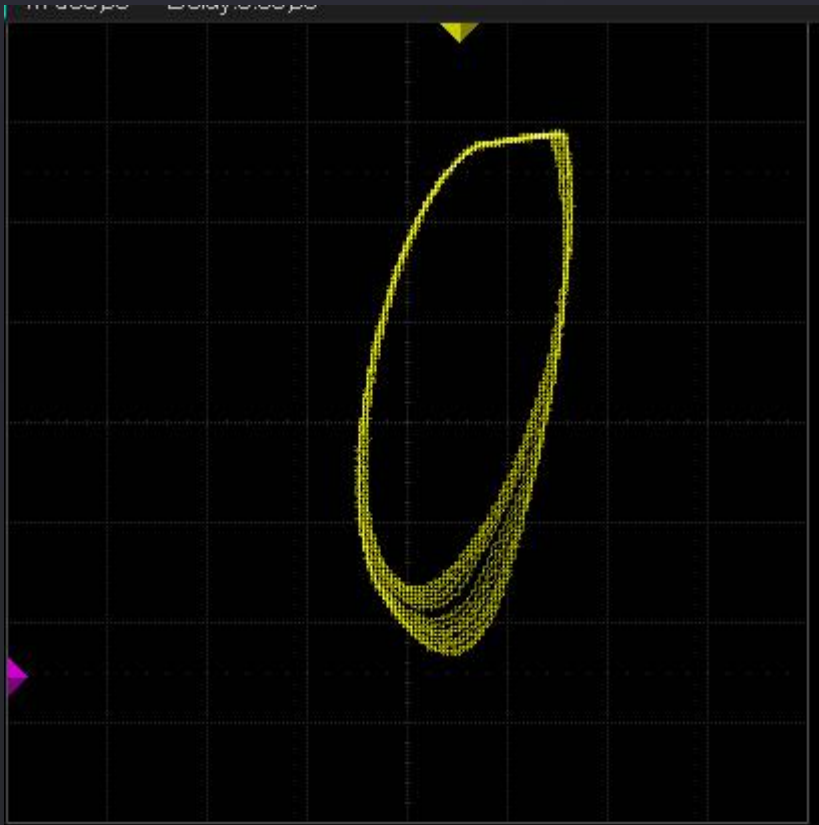
三周期



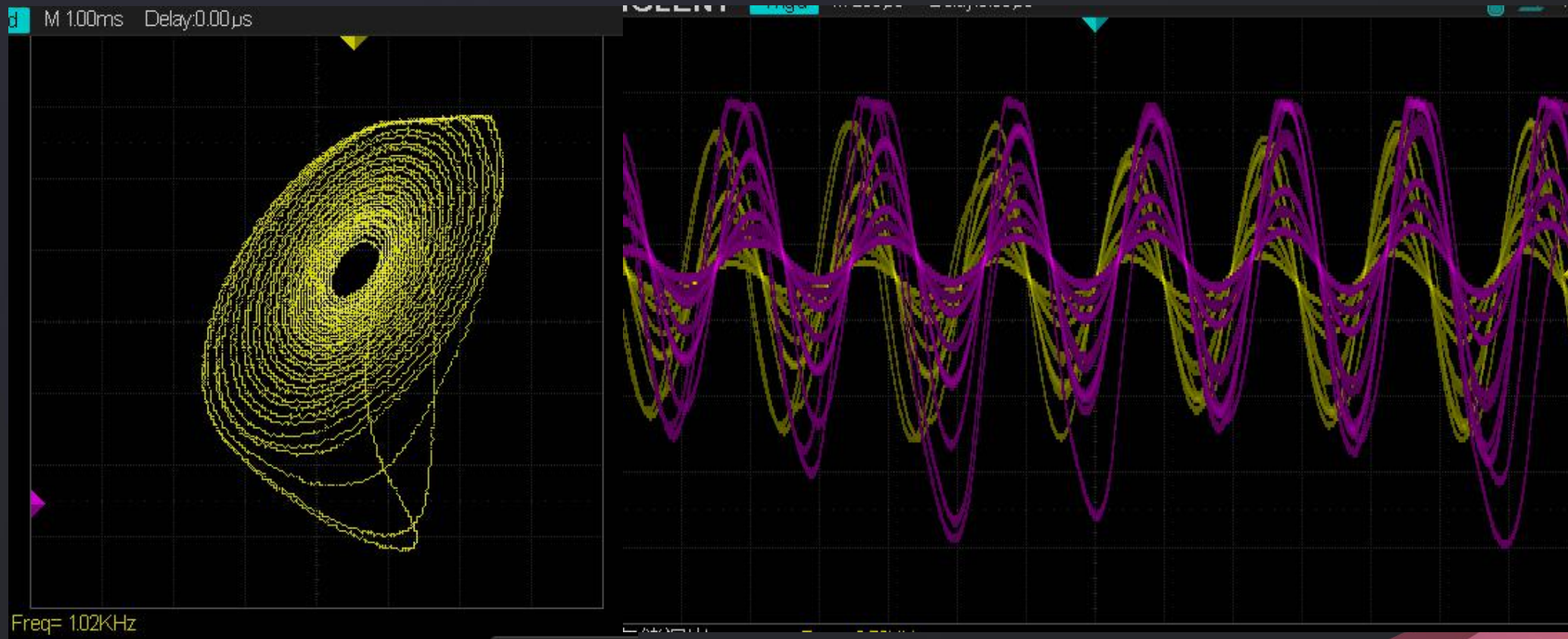
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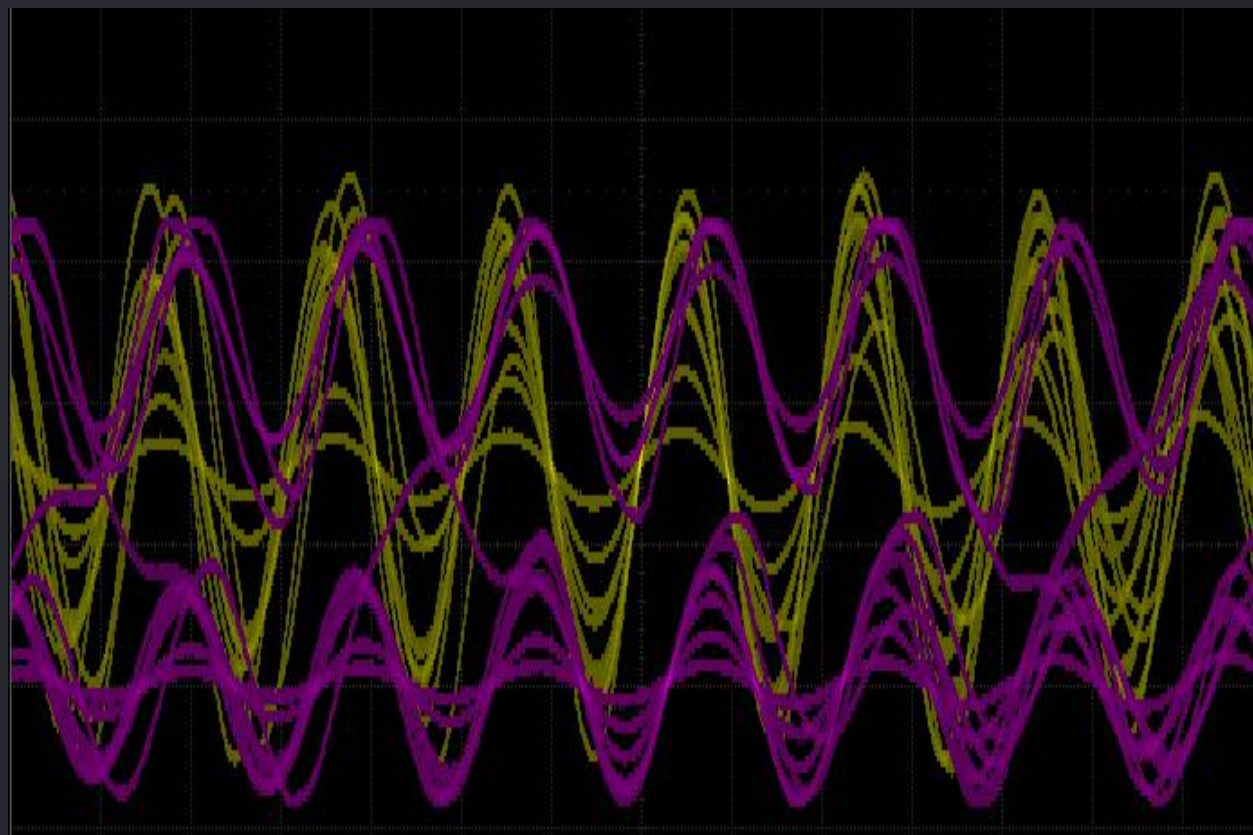
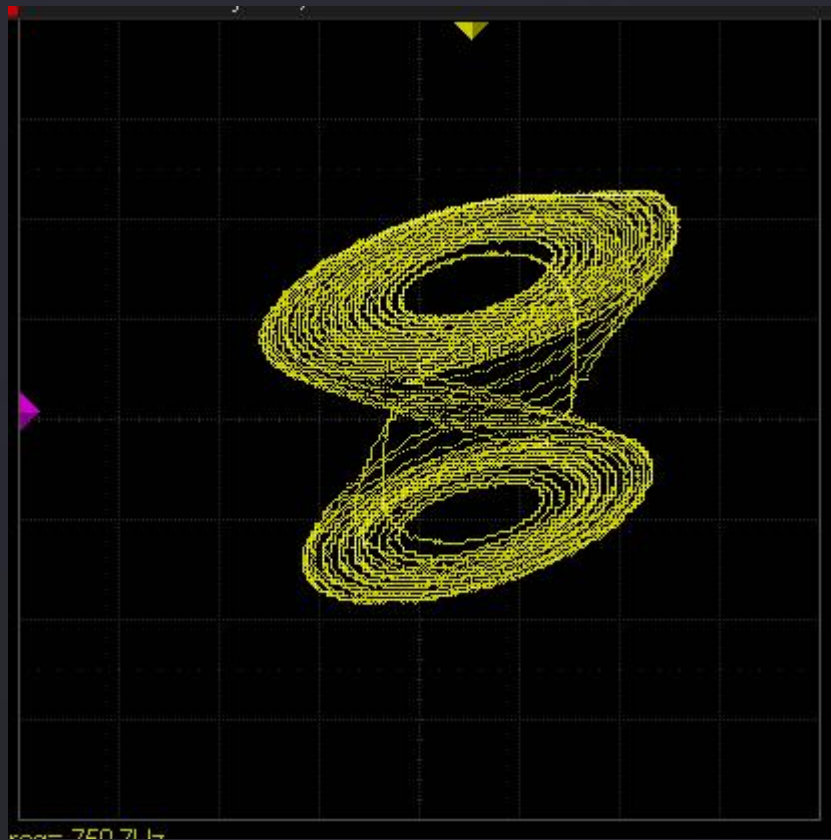
阵发混沌



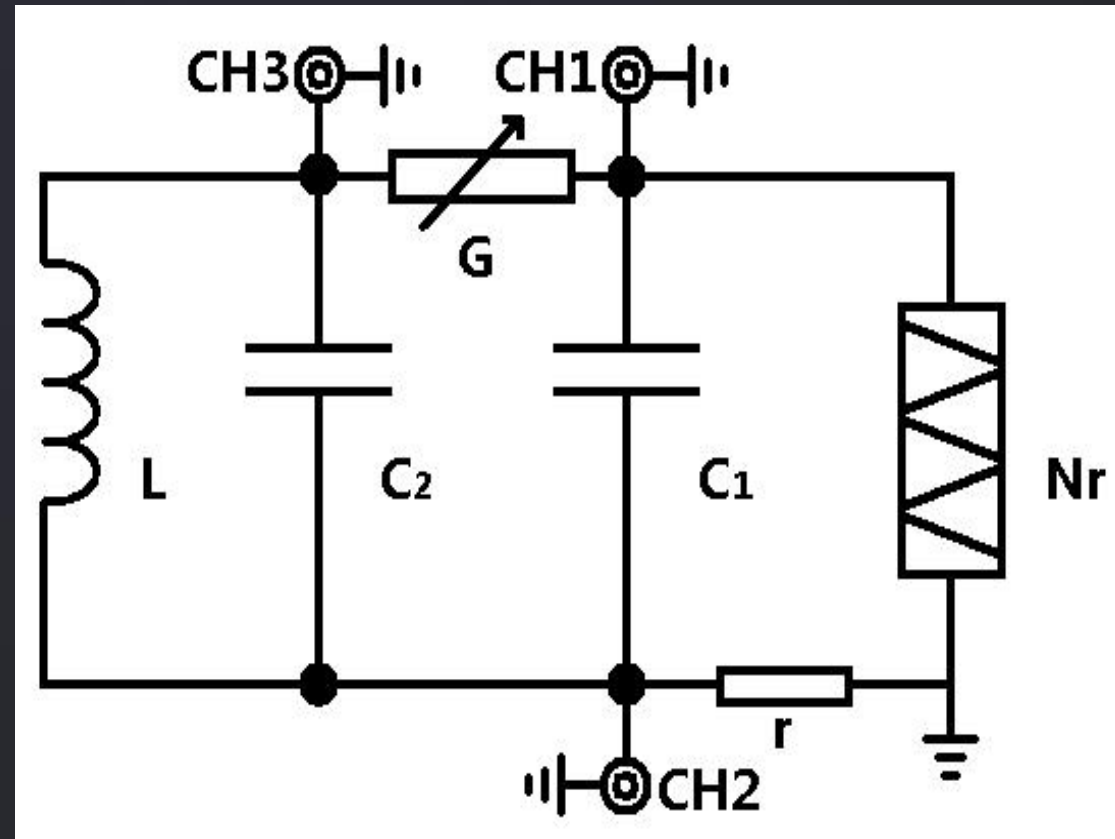
单吸引子



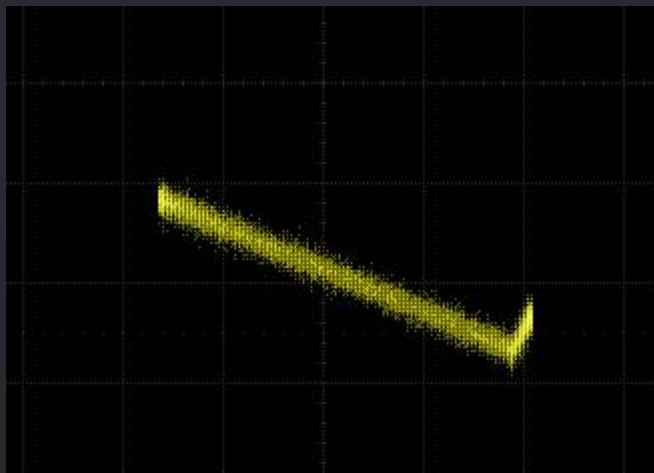
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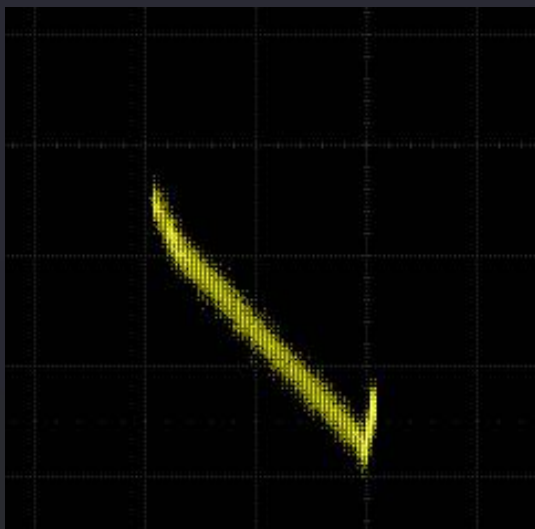
蔡氏电路充当信号发生器研究I-V特性



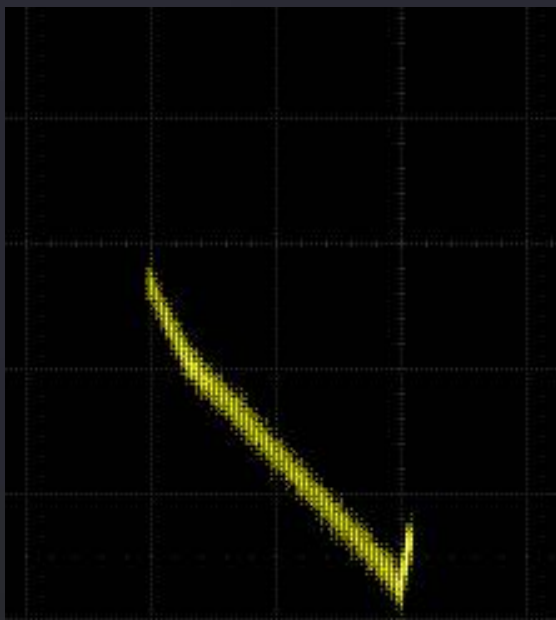
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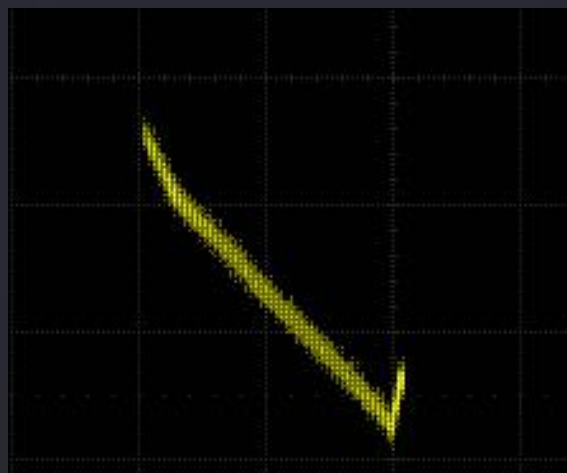
双周期



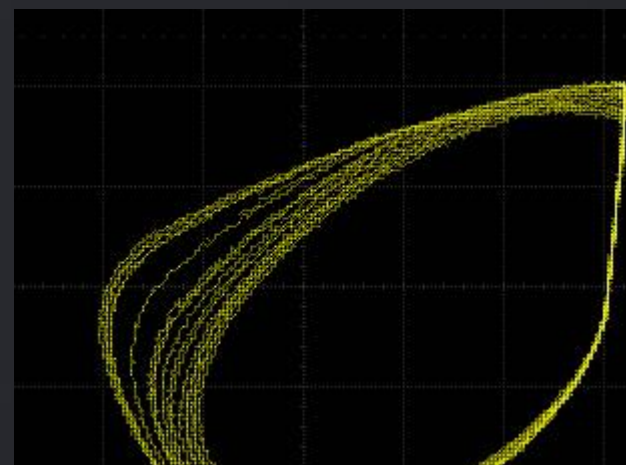
三周期



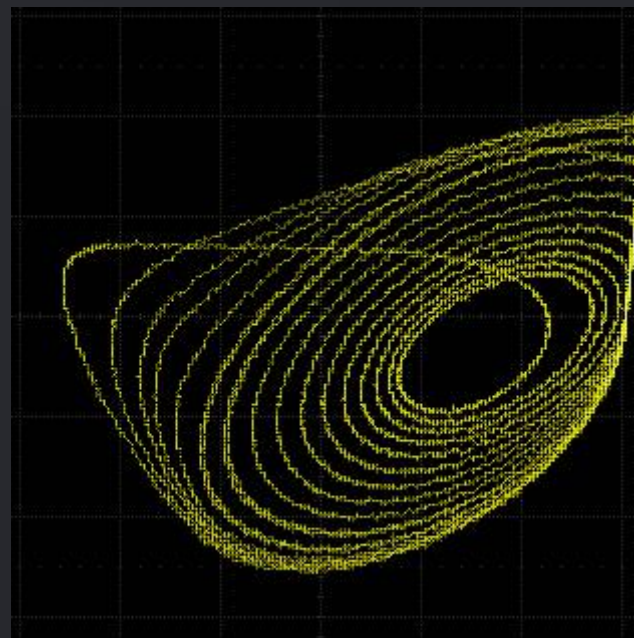
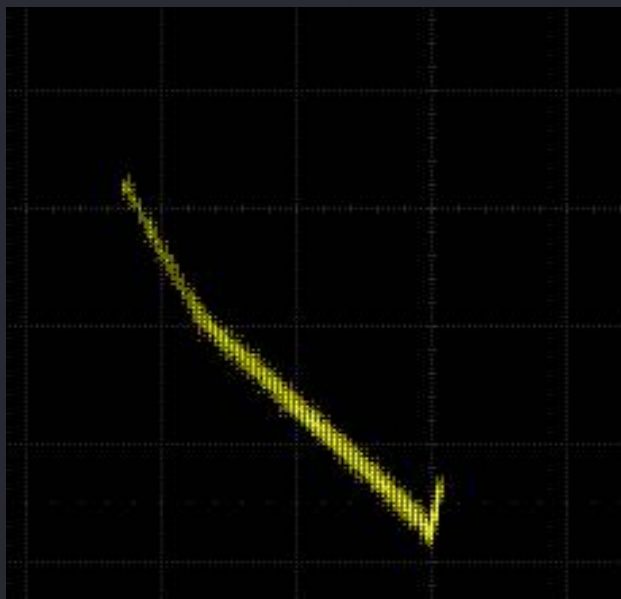
四周期



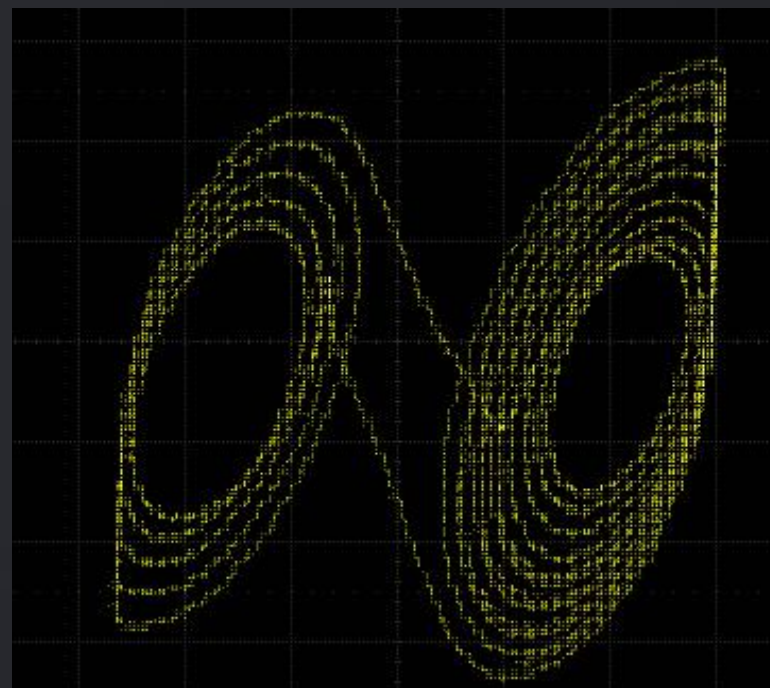
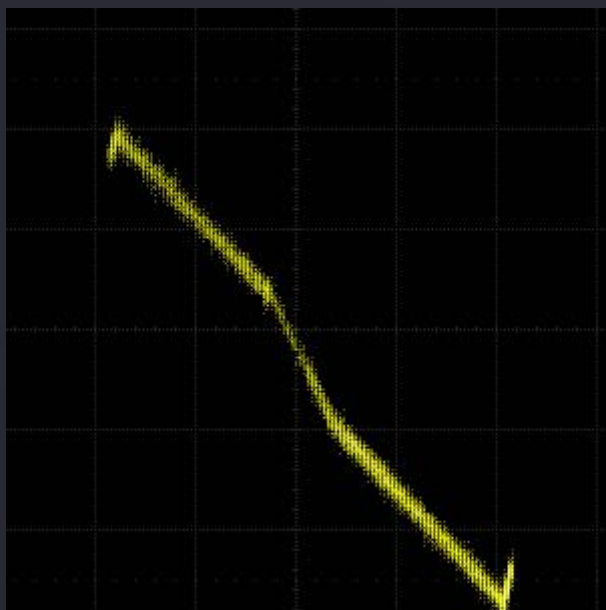
阵发混沌



单吸引子



双吸引子



I-V曲线与相图的解释

- 结合动画演示

python数值模拟与动画展示

- 数值模拟方法Runge-Kutta方法

$$y_{n+1} = y_n + \frac{h}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$
$$t_{n+1} = t_n + h$$

for $n = 0, 1, 2, 3, \dots$, using

$$k_1 = f(t_n, y_n),$$
$$k_2 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1\right),$$
$$k_3 = f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2\right),$$
$$k_4 = f(t_n + h, y_n + hk_3).$$

不要慌!!!

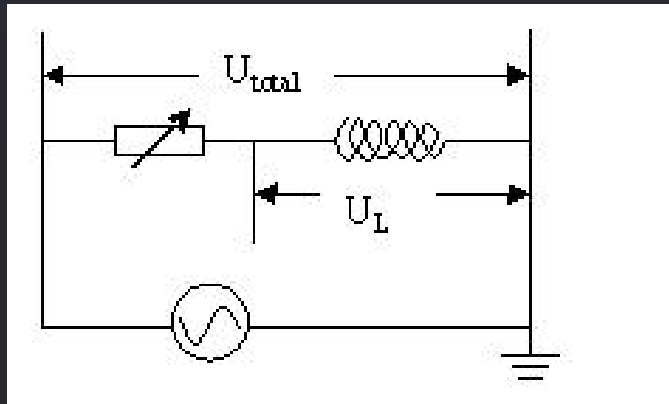
编程可以迅速将算法实现

不过就解微分方程而言，通常采用各个语言数值计算包中的特别方法。

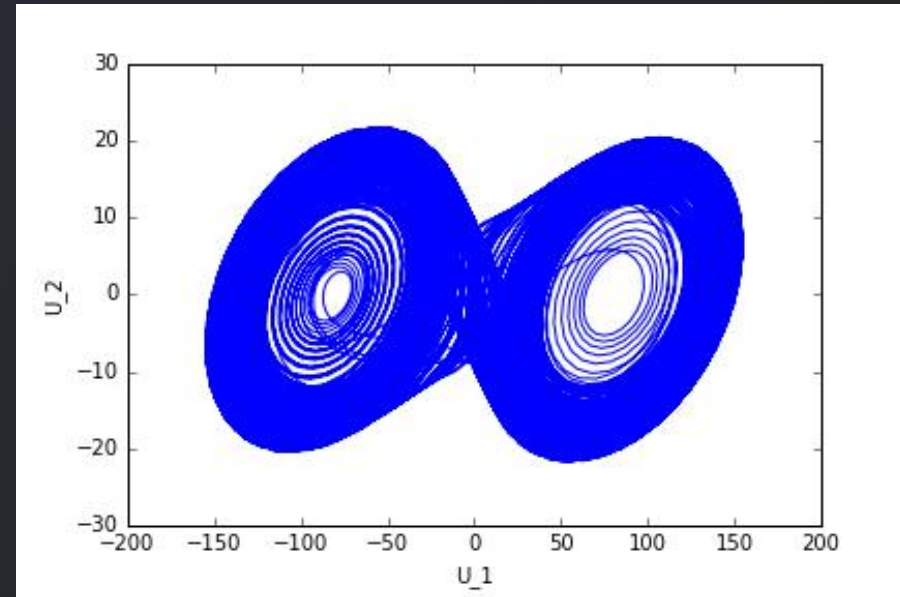
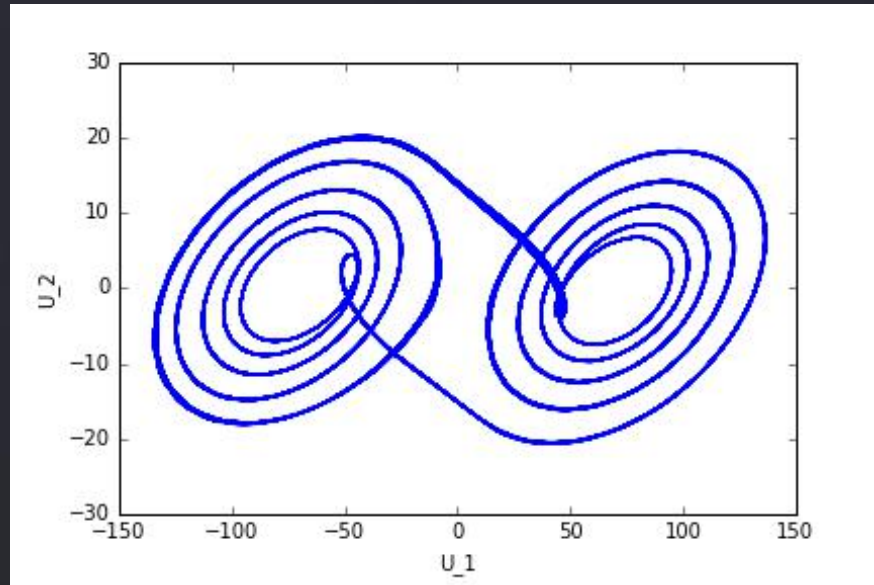
元件测量

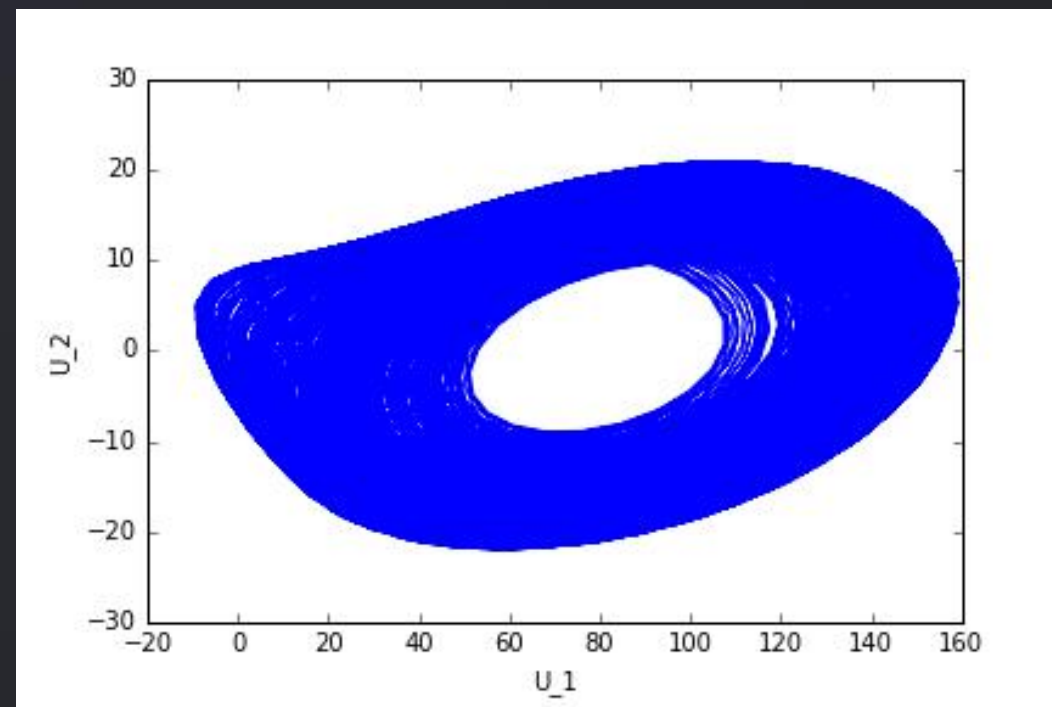
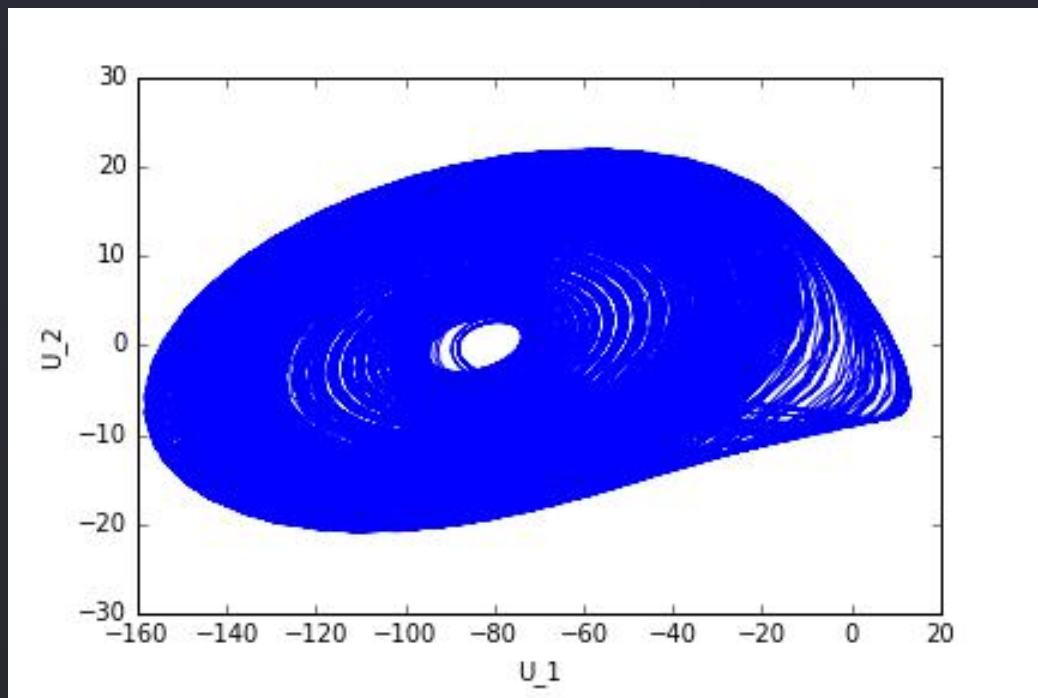
- $C_2=0.0966 \mu F$
- $C_1=10.194nF$
- 电感 $L=22.2mH$

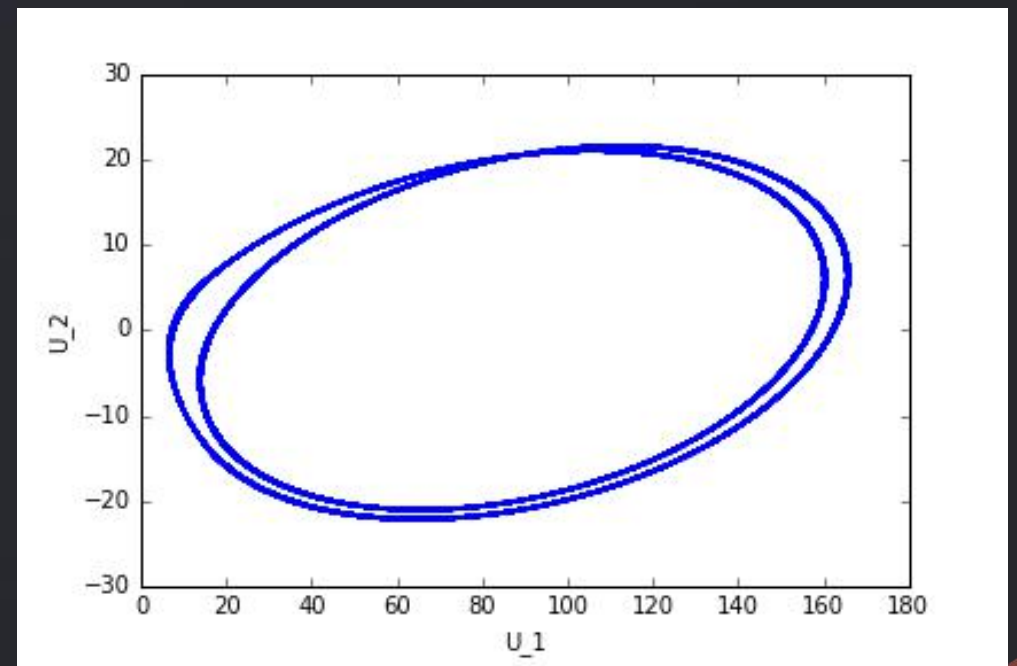
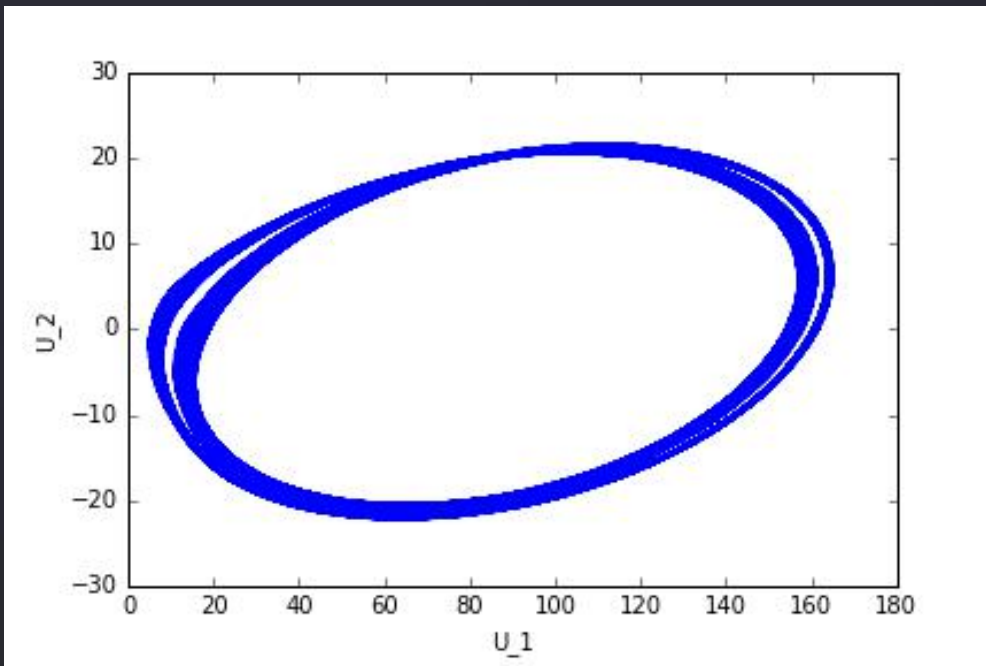
将上述测量值代入微分方程中即可求解理论下的情况。



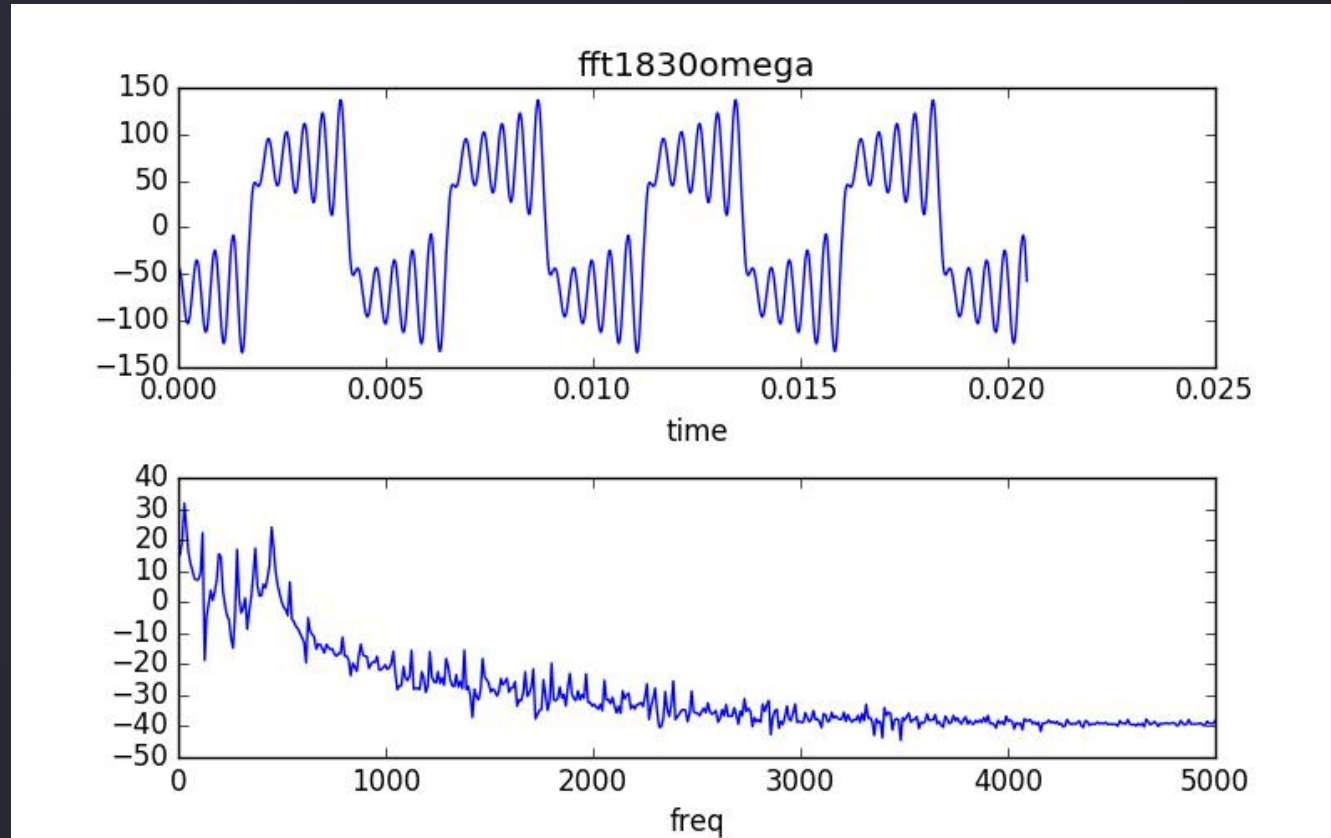
仿真图片+动画以及原理解释

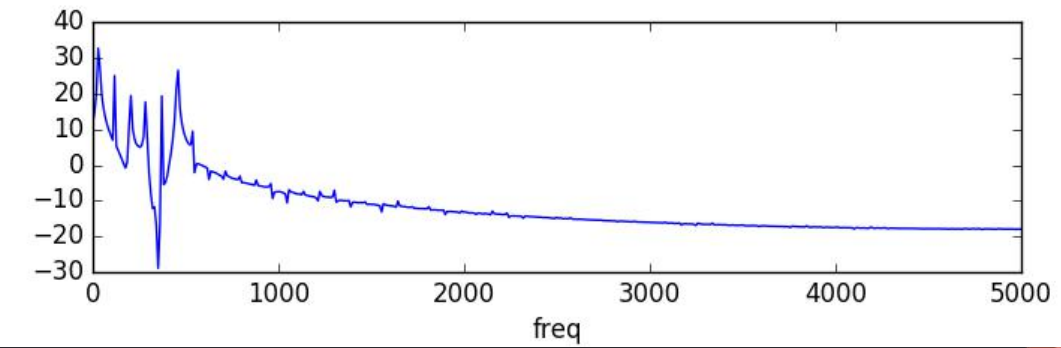
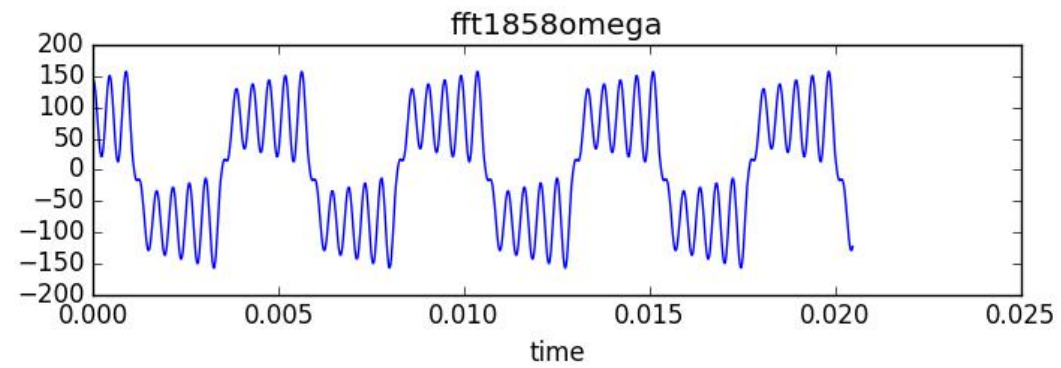
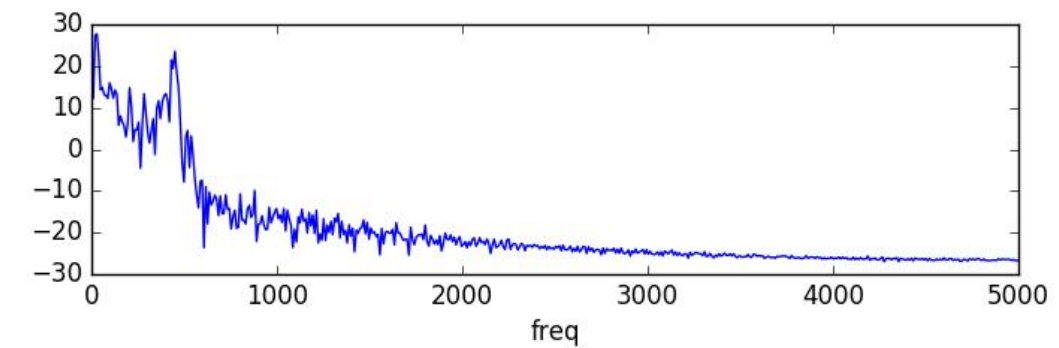
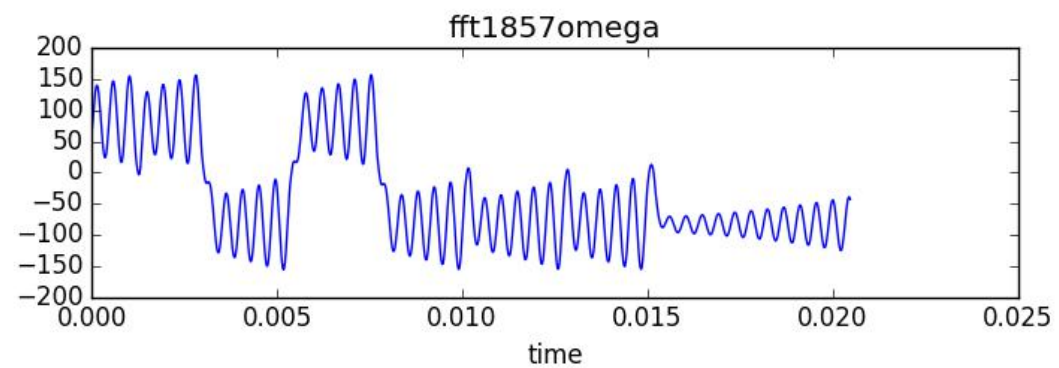


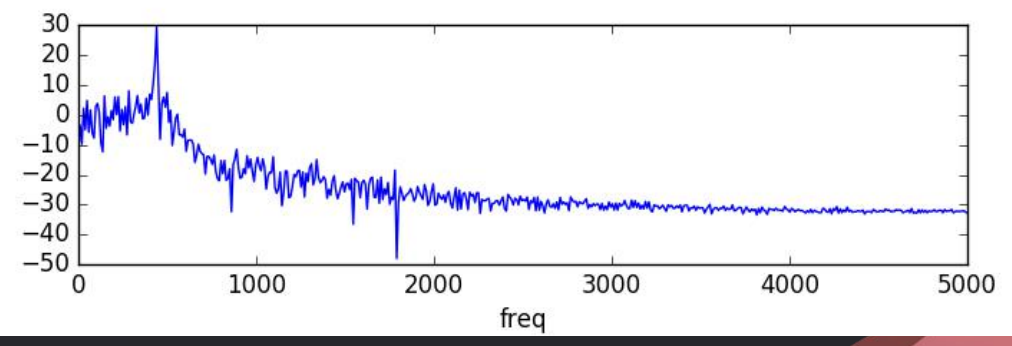
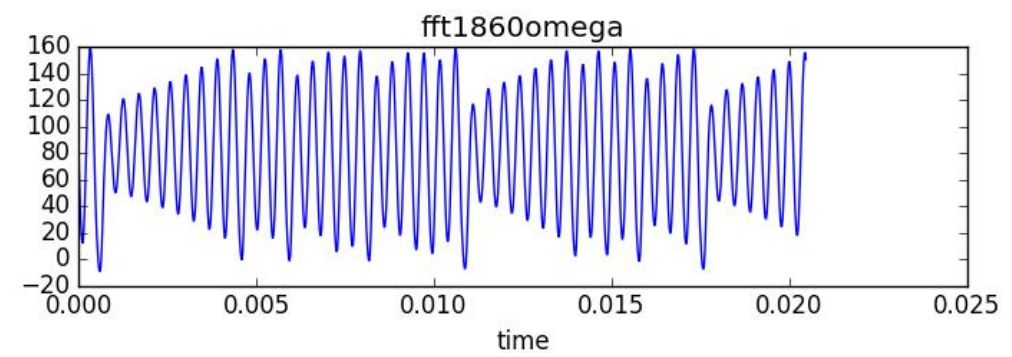
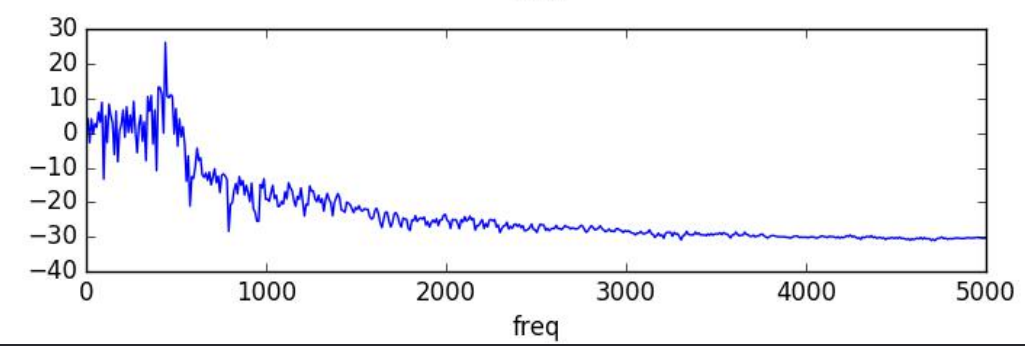
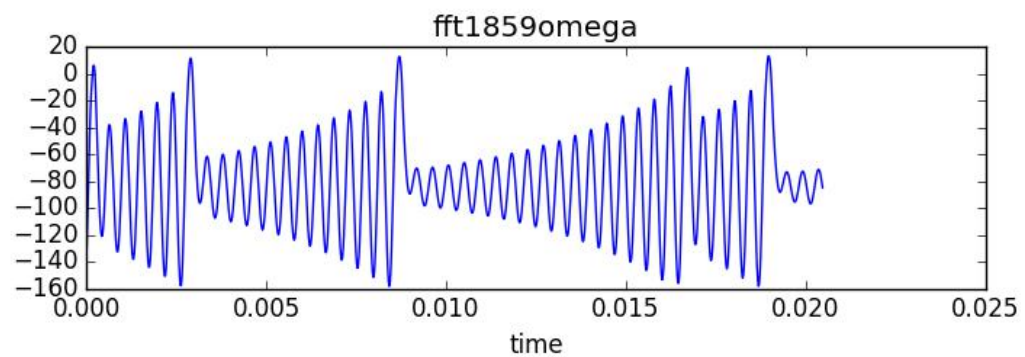


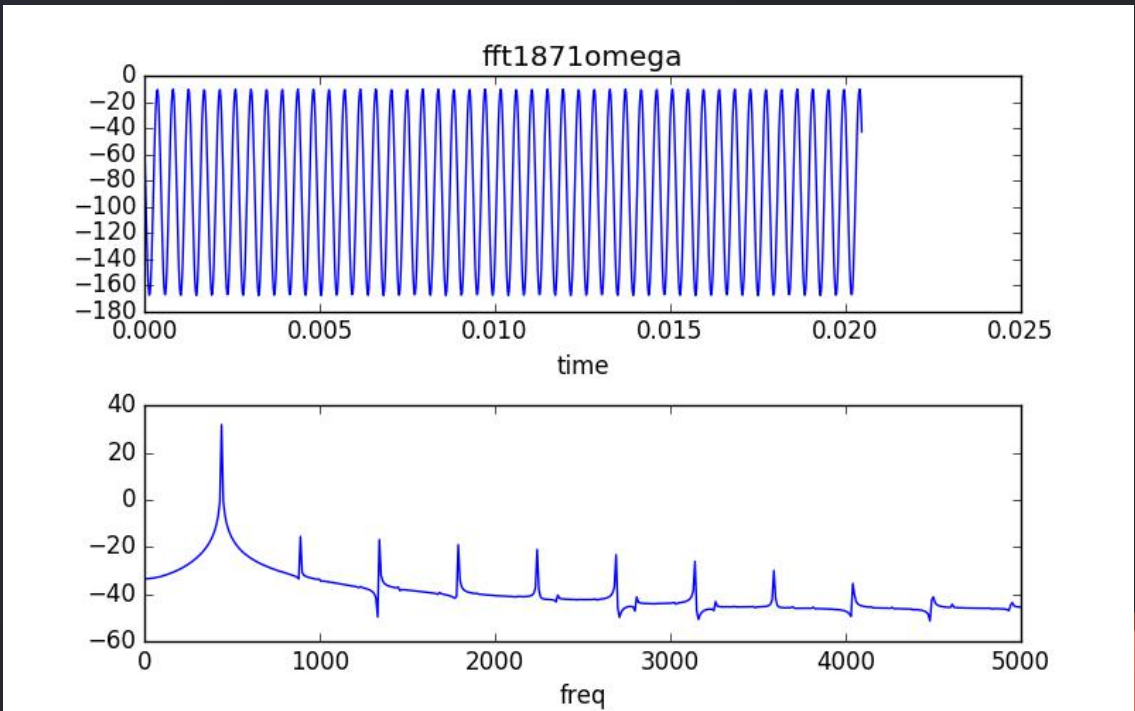
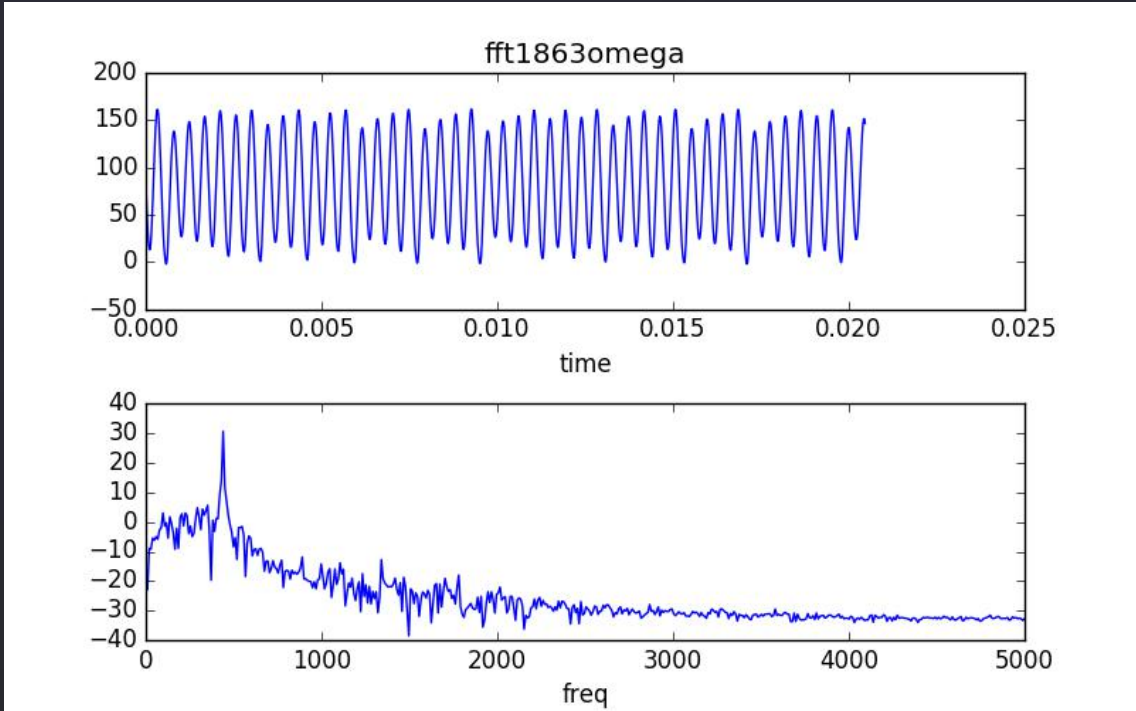


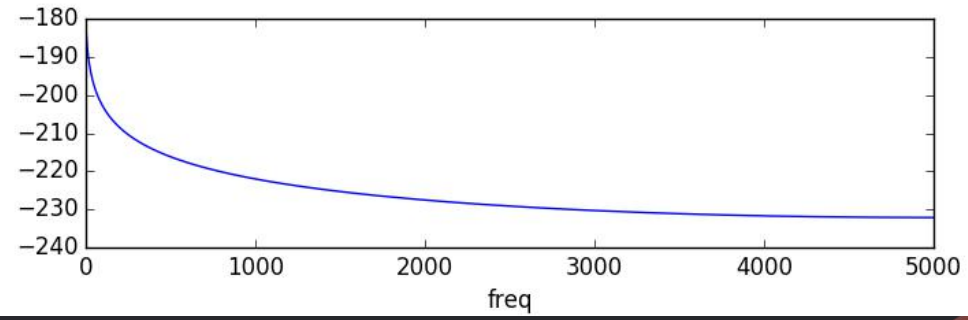
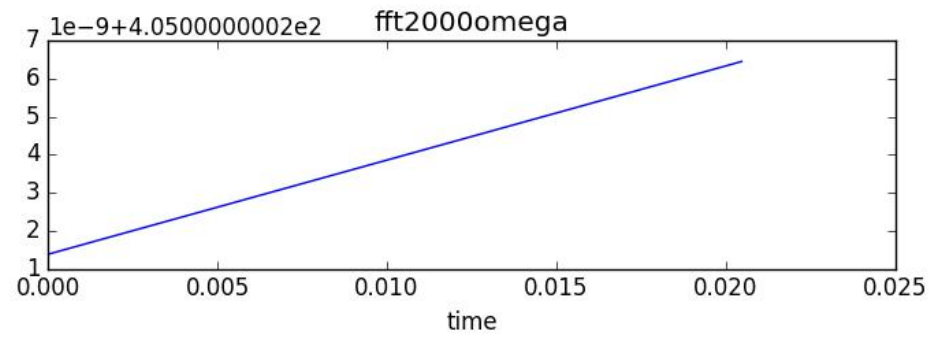
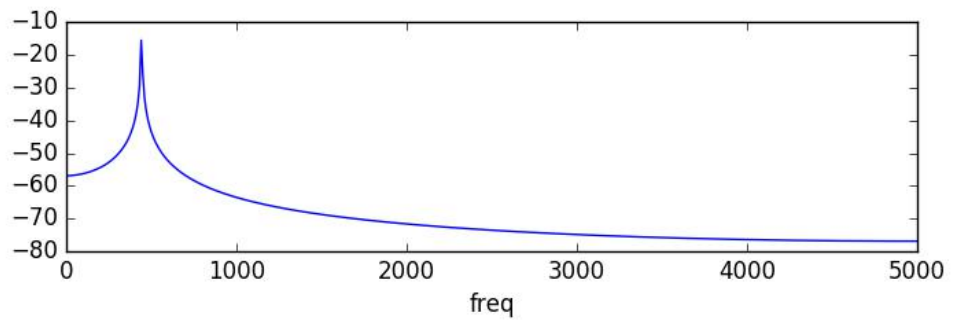
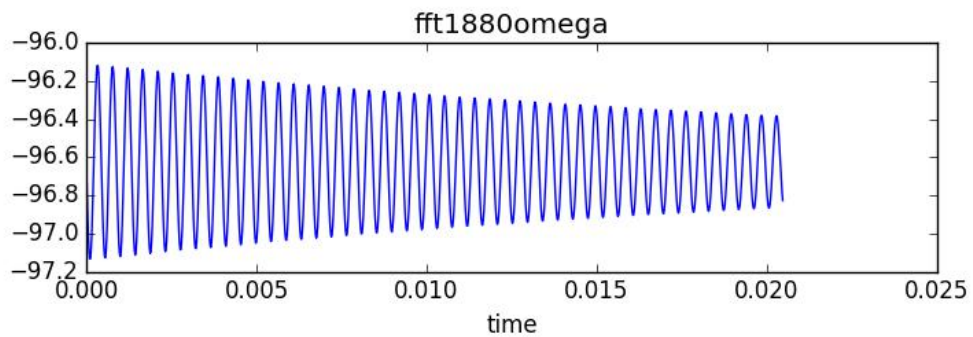
fft频域变换





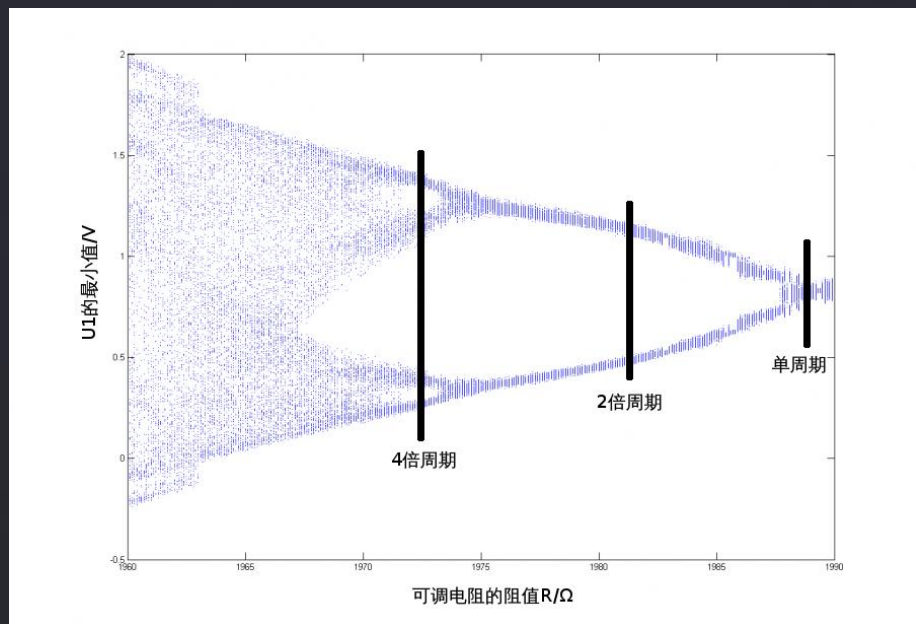




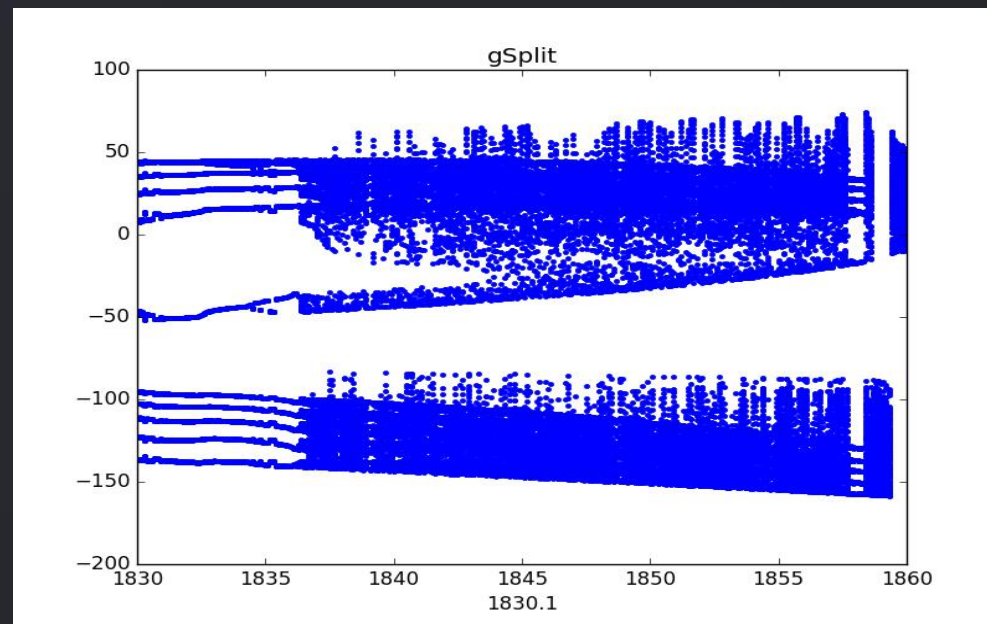


G分叉

理论图像

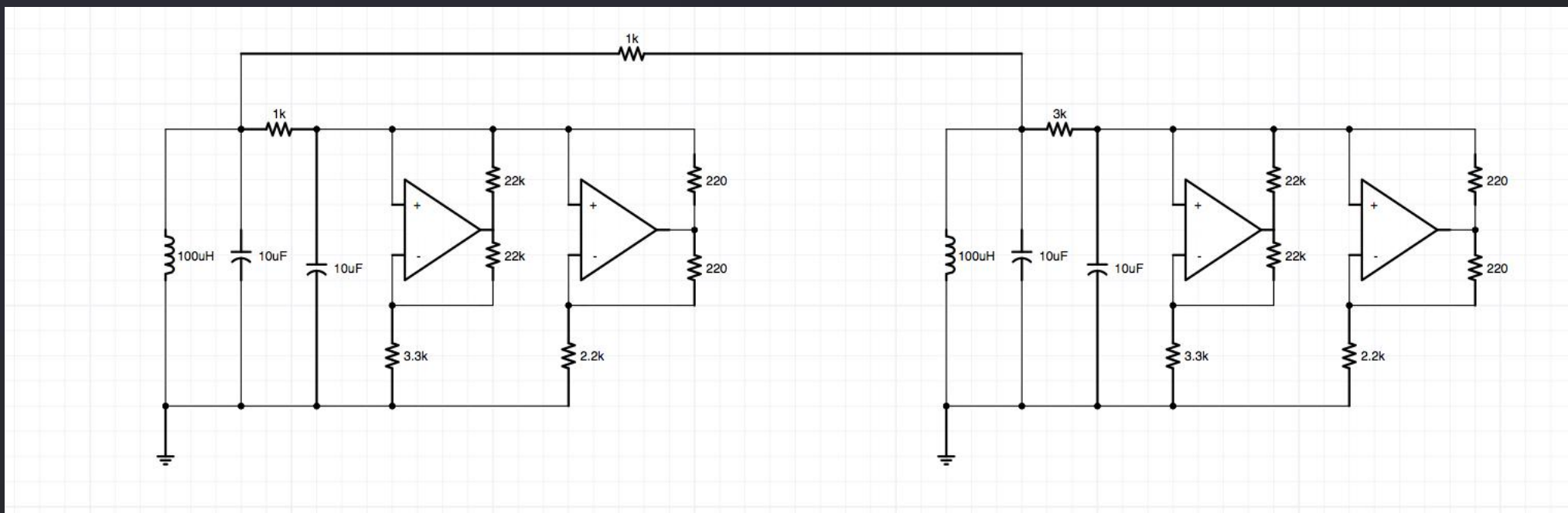


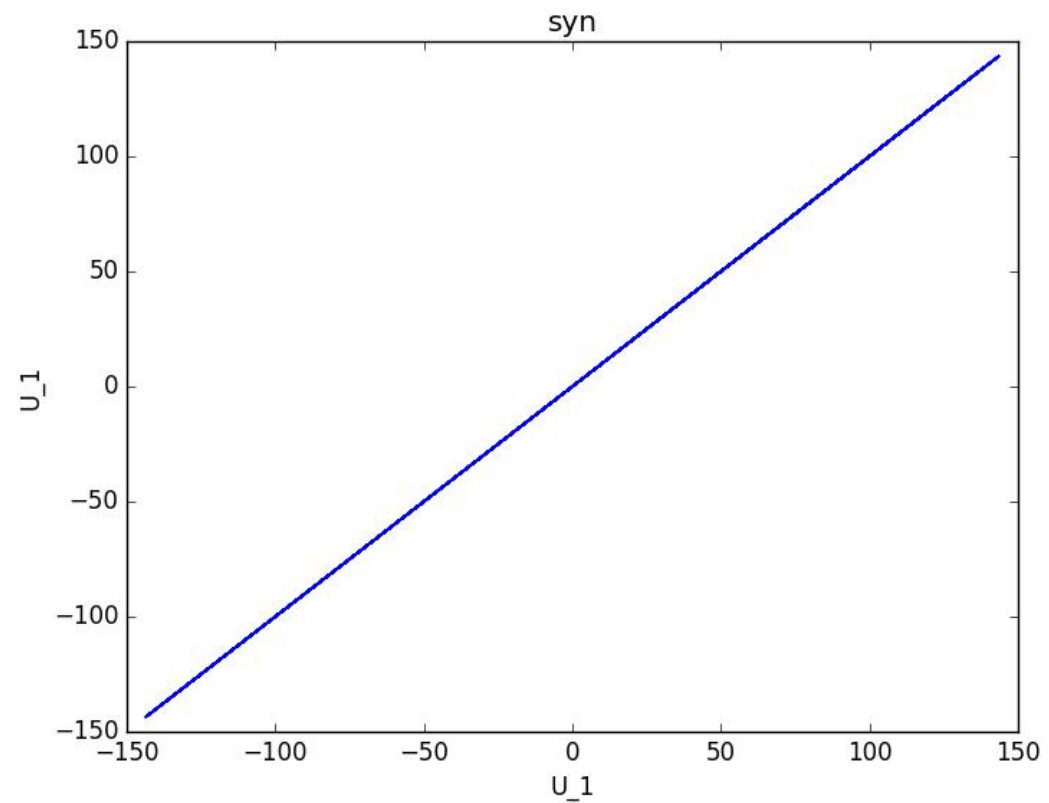
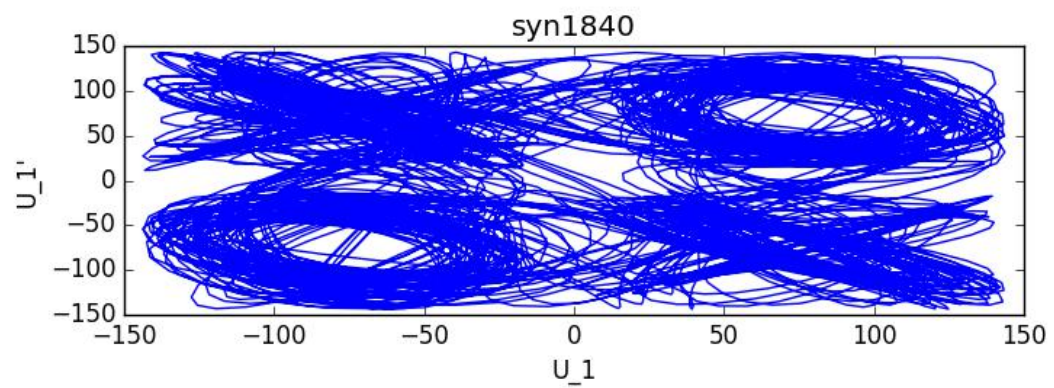
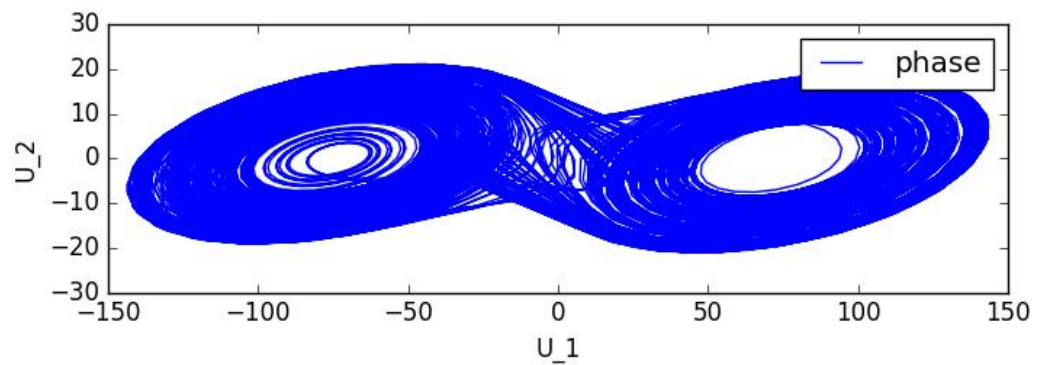
python模拟图像



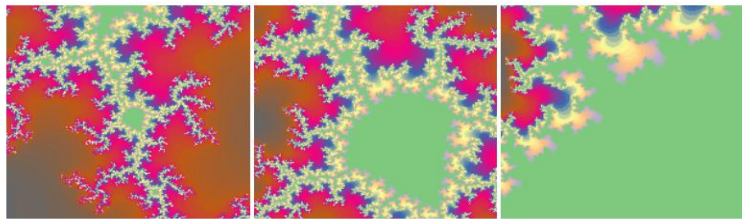
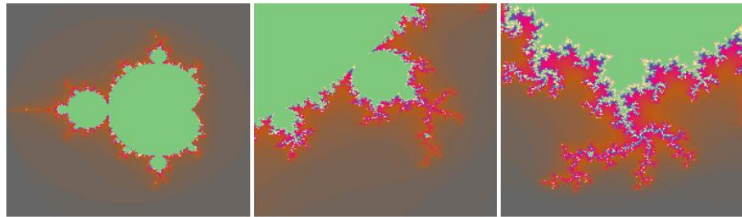


两个蔡氏电路的耦合





分形gallery



参考文献