



非线性物理—混沌

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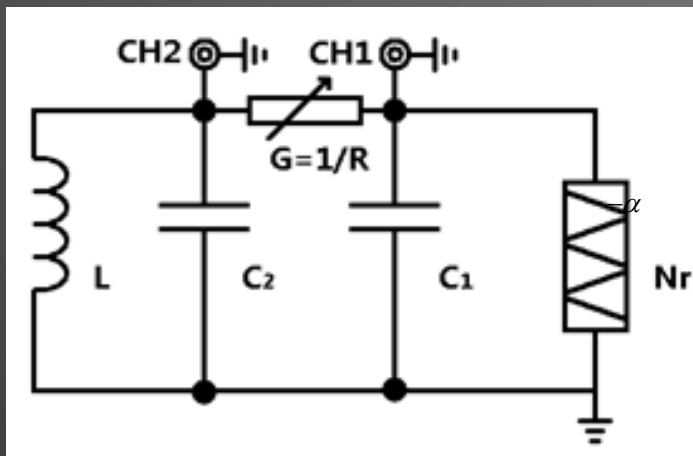
- 混沌理论和蔡氏电路简介
- 混沌现象的观察
- 非线性负阻的 $I-V$ 特性
- 数值模拟

混沌理论

- 非线性科学六个主要研究邻域之一
- 指确定性动力学系统因对初值敏感而表现出的不可预测的、类似随机性的运动
- 著名表述：蝴蝶效应
- 通向混沌的途径：倍周期分岔道路、准周期道路、阵发混沌道路
- 意义：用简单的模型获得明确的非周期结果

在气象、航空及航天等领域的研究里有重大的作用。

蔡氏电路



满足：

- 有一个非线性元件
- 有一个用于耗散能量的电阻
- 有三个存储能量的元件

$$\begin{cases} C_1 \frac{dU_1}{dt} = G(U_2 - U_1) - g(U_1) \\ C_2 \frac{dU_2}{dt} = G(U_1 - U_2) + I_L \\ L \frac{dI_L}{dt} = -U_2 \\ g(U_1) = G_b U + \frac{G_b - G_a}{2} (|U - E| - |U + E|) \end{cases}$$

可化简为

$$\dot{X}(t) = \frac{d}{dt} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -\alpha & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \alpha \begin{pmatrix} f(x) \\ 0 \\ 0 \end{pmatrix}$$

其中

$$f(x) = \begin{cases} bx + a - b & x \geq 1 \\ ax & -1 \leq x \leq 1 \\ bx - a + b & x \leq -1 \end{cases}$$

蔡氏电路的不动点

$\dot{X}(t)=0$ 的解即为相空间的不动点 X_0

$$\begin{cases} x + f(x) = 0 \\ x + z = 0 \\ y = 0 \end{cases} \longrightarrow X_0 = \begin{cases} (k, 0, -k) & x \geq 1 \\ (0, 0, 0) & -1 < x < 1 \\ (-k, 0, k) & x \leq -1 \end{cases} \quad k = \frac{b-a}{b+1} = \frac{G_b - G_a}{G_b + G}$$

判断不动点的稳定性,

$$J = \begin{pmatrix} -\alpha(g+1) & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{pmatrix} \quad g = \begin{cases} a & -1 < x < 1 \\ b & |x| \geq 1 \end{cases}$$

$$|\lambda I - J| = \lambda^3 + (1 + \alpha + g\alpha)\lambda^2 + (\beta + g\alpha)\lambda + \alpha\beta(1 + g)$$

根据Routh三阶特征方程的判据, 若 $\frac{\alpha^2 g^2 + \beta}{\alpha g(1 + \alpha)} < g$, 则当 $-g < G < -\frac{\alpha^2 g^2 + \beta}{\alpha g(1 + \alpha)}$ 时, 稳定

$\frac{\alpha^2 g^2 + \beta}{\alpha g(1 + \alpha)} > g$, 不稳定

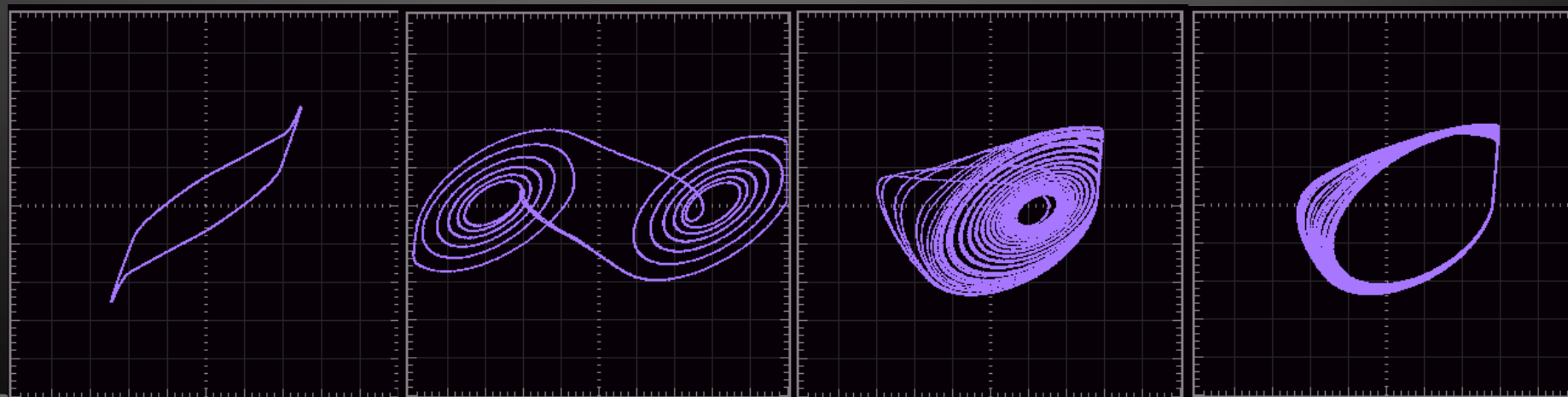
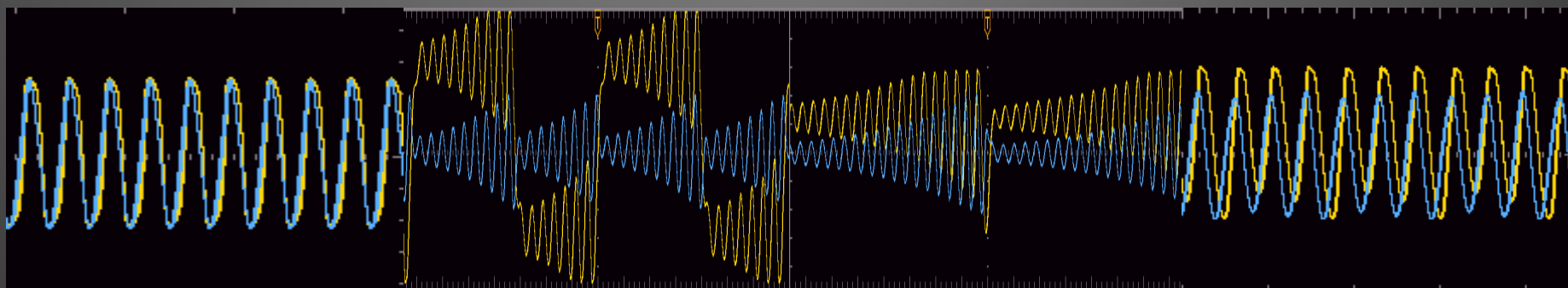
混沌现象的观察 — G调制（逐渐增大可变电阻R）

$R = -1.94 \text{ k}\Omega$

$R = 1.94 - 2.02 \text{ k}\Omega$

$R = 2.02 - 2.04 \text{ k}\Omega$

$R = +2.04 \text{ k}\Omega$



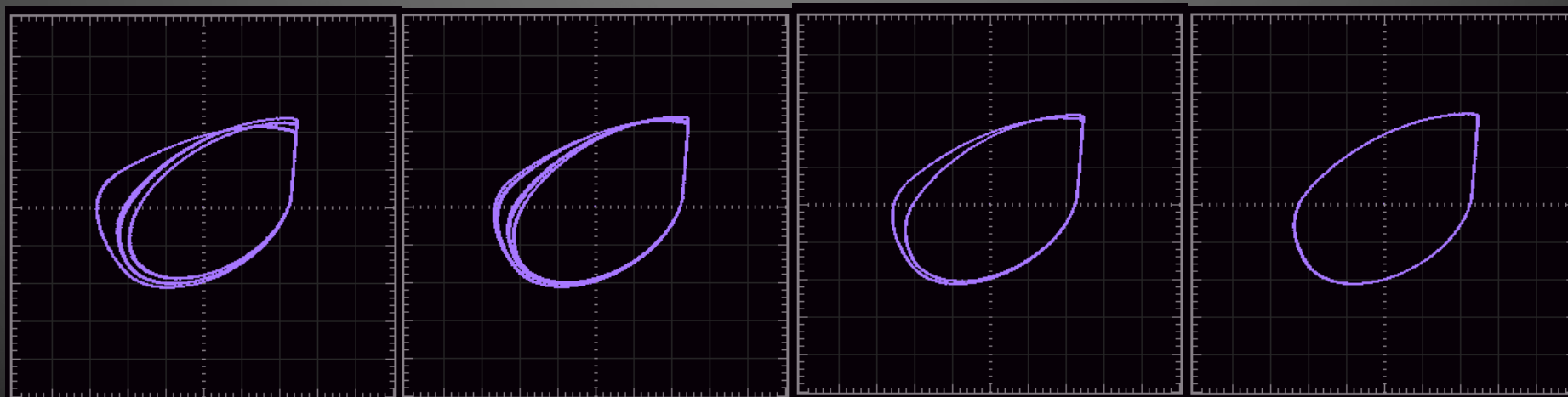
初始状态

双吸引子

奇异吸引子

阵发混沌

$R=+2.04 \text{ k}\Omega$



三周期

四周期

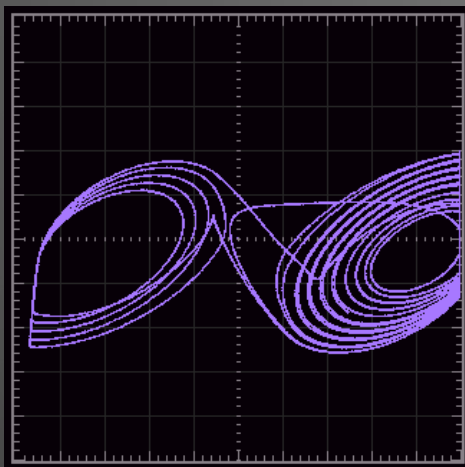
双周期

单周期

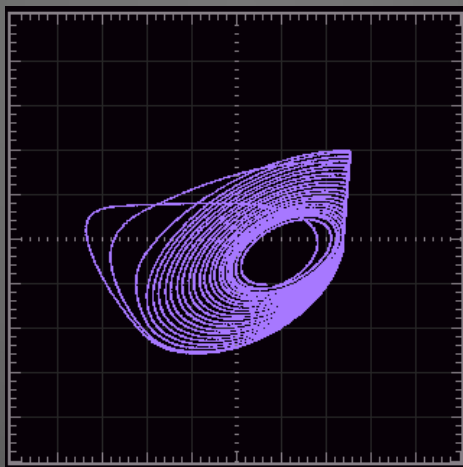
$R \approx 2.1 \text{ k}\Omega$ 图像消失

混沌现象的观察 — C调制（逐渐减小可调电容）

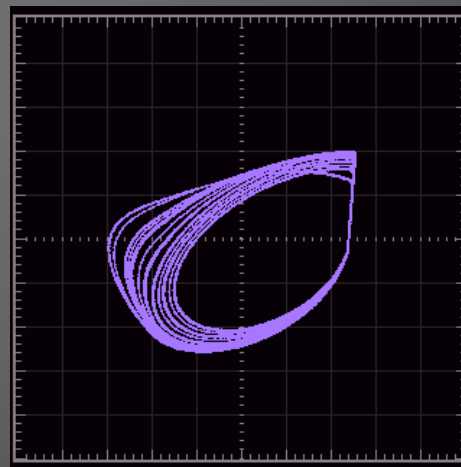
C=10 nF



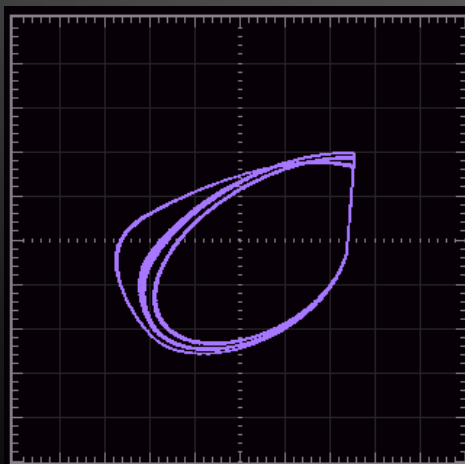
C=9 nF



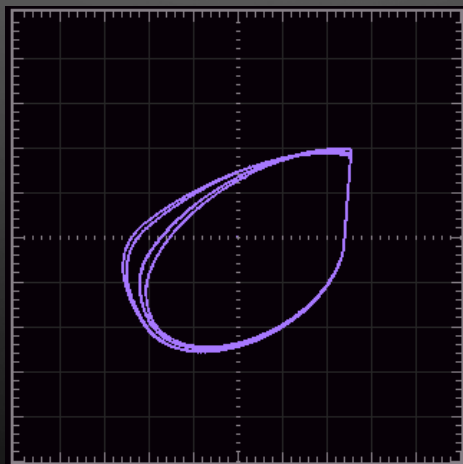
C=8 nF



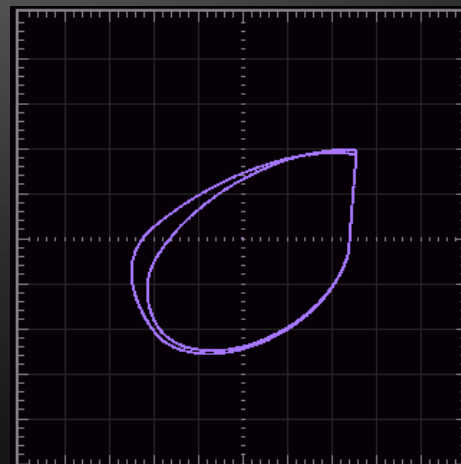
C=4 nF



C=2 nF

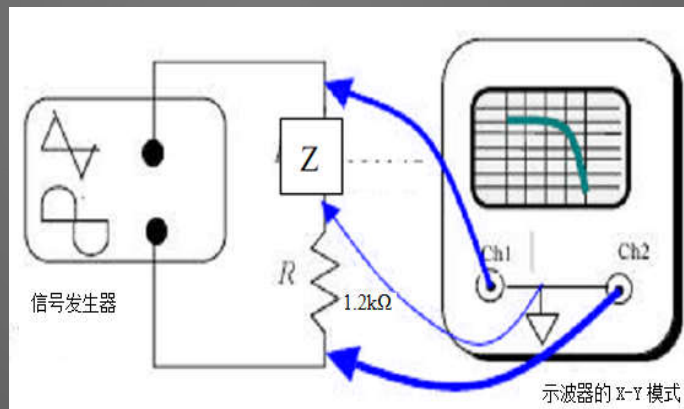


C=1 nF

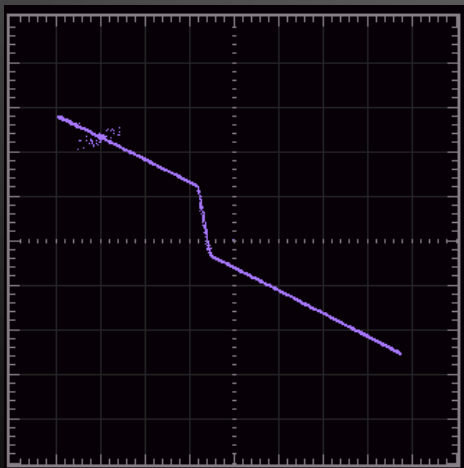


非线性负阻的 I-V 特性

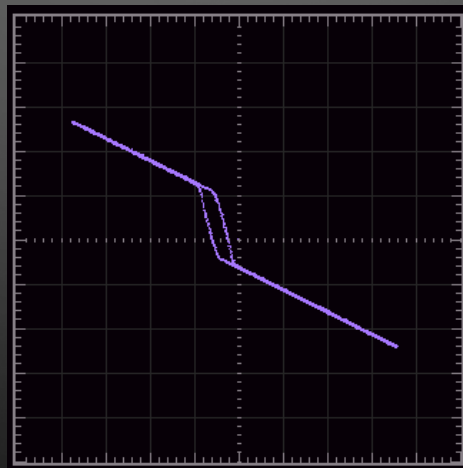
◆ 外部信号扫描



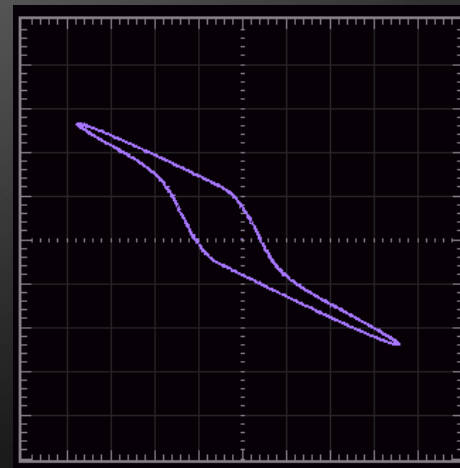
$f=30\text{ Hz}$



$f=0.3\text{ kHz}$

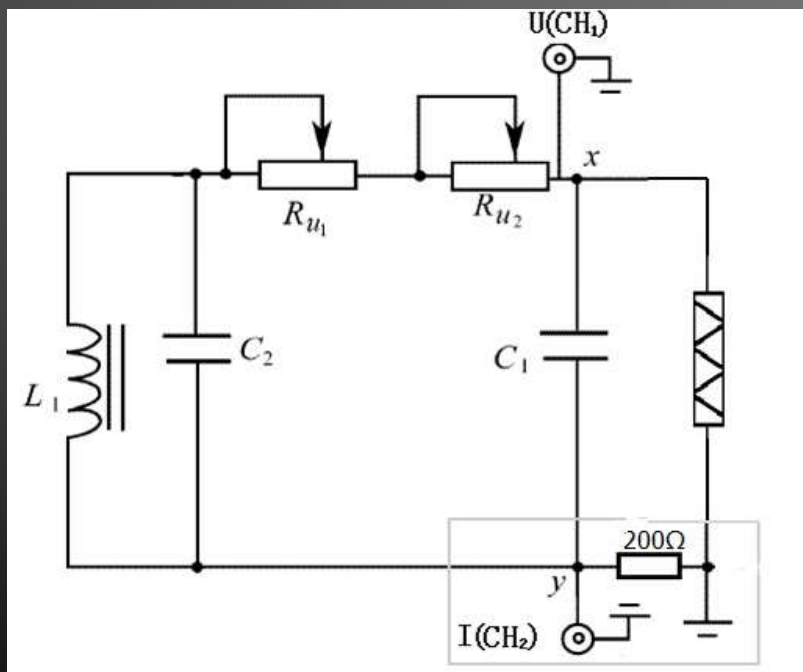


$f=3.3\text{ kHz}$

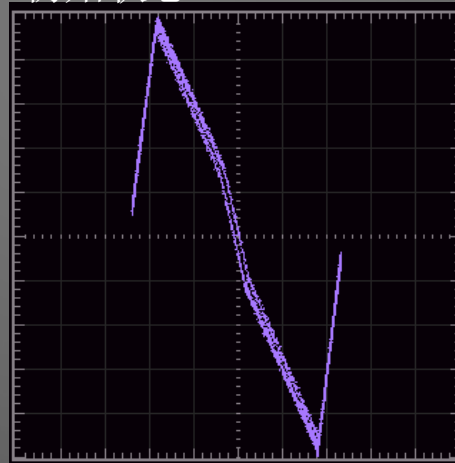


非线性负阻的 I-V 特性

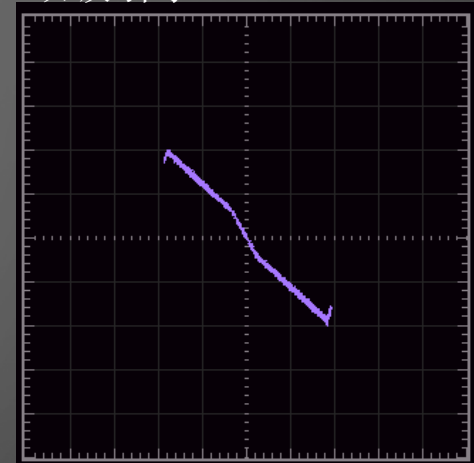
- ◆ 内置信号扫描



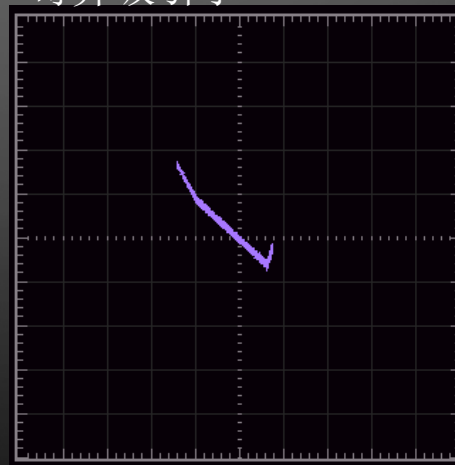
初始状态



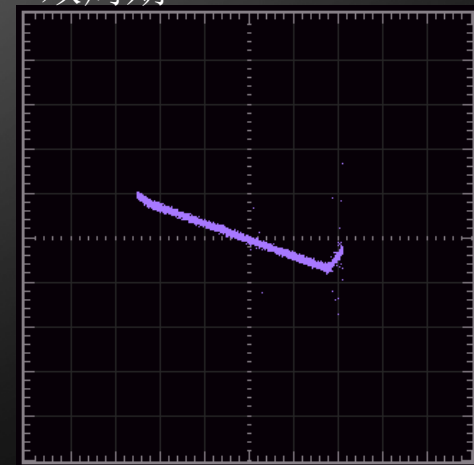
双吸引子



奇异吸引子



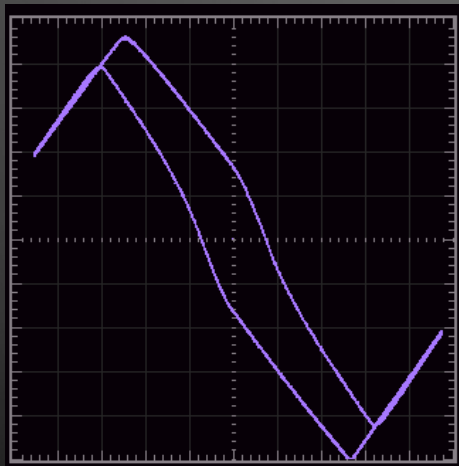
双周期



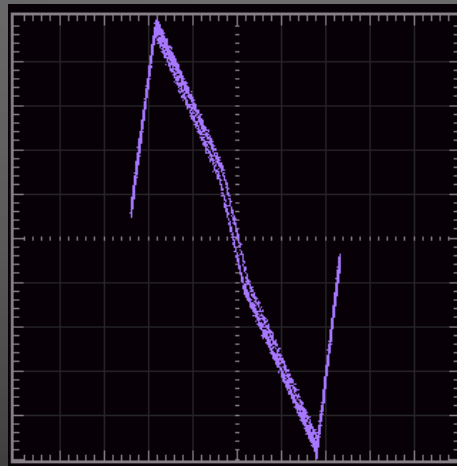
改变R的大小

同为初始状态下

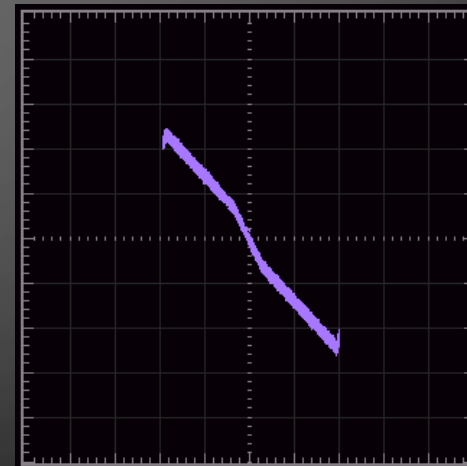
R=1000 Ω



R=200 Ω



R=100 Ω



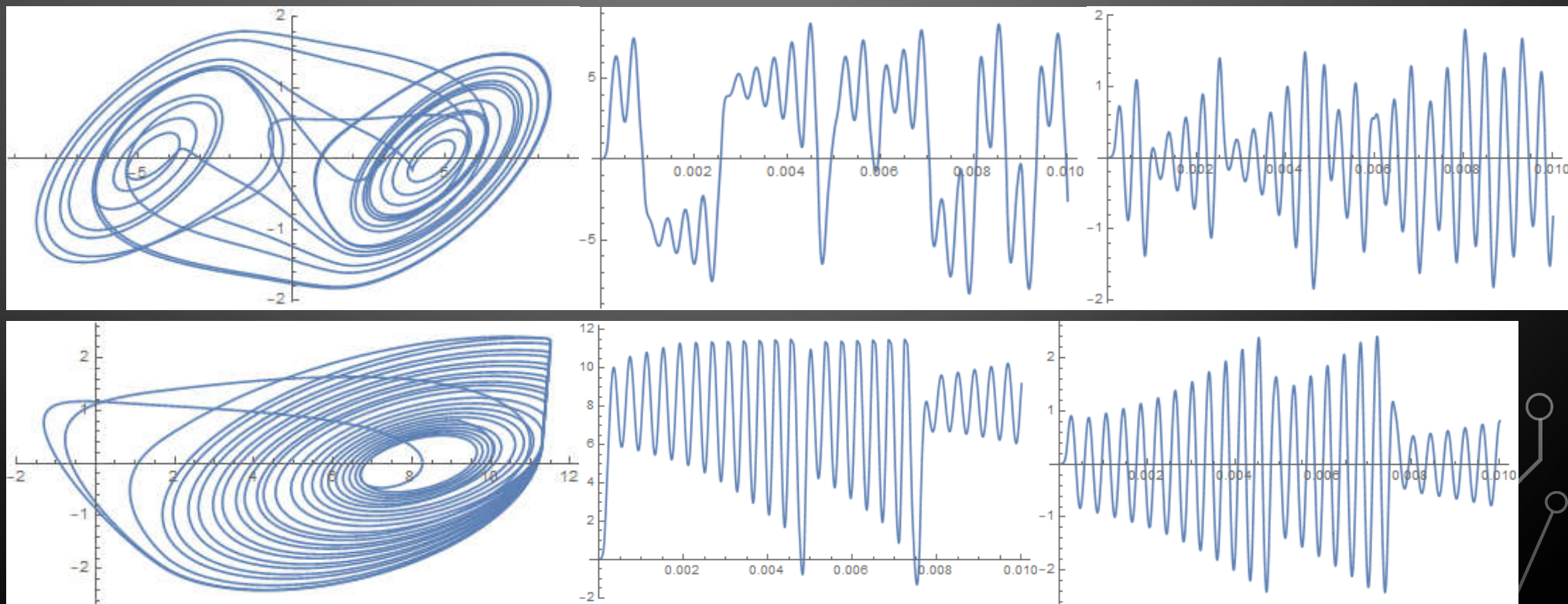
R=1000 Ω 时 无法通过G调制使出现双吸引子

数值模拟

- 典型混沌信号

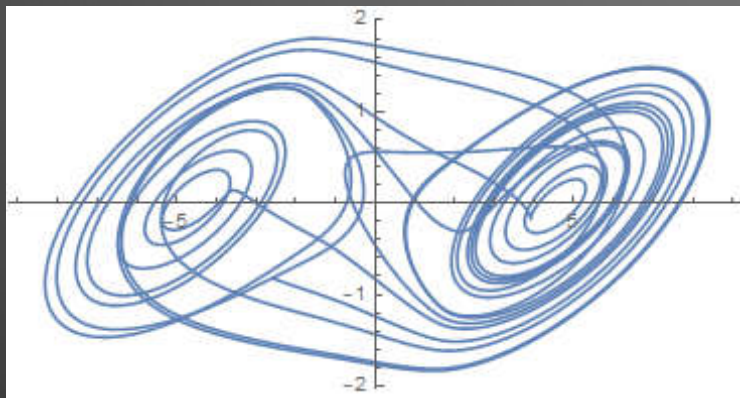
U1

U2

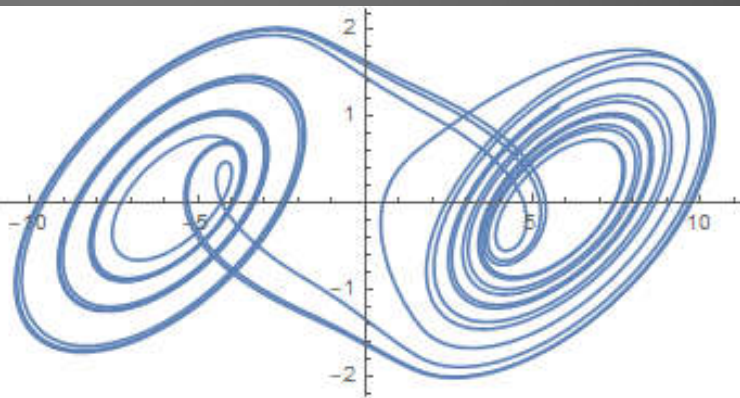


G调制

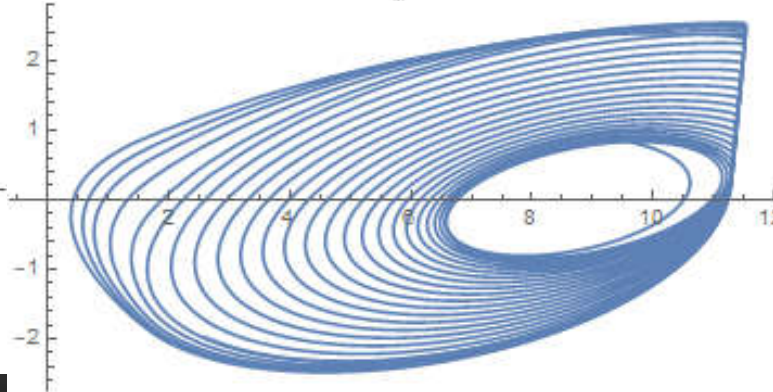
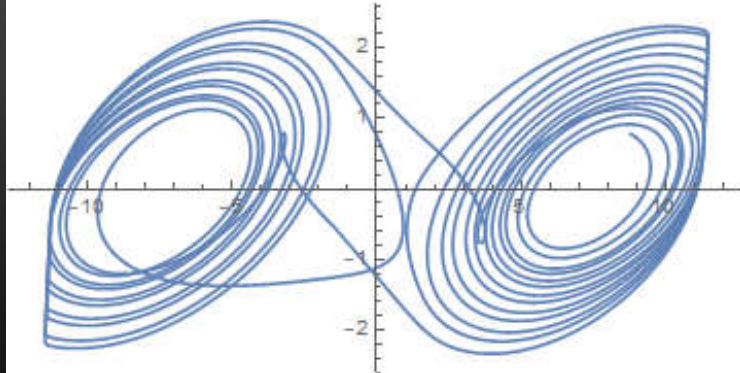
1/1904



1/2000



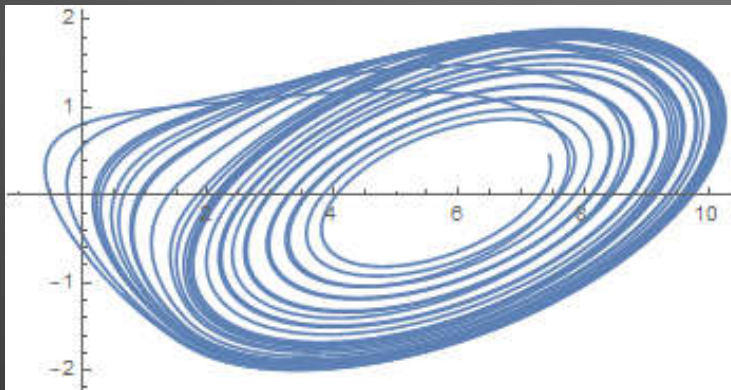
1/2150



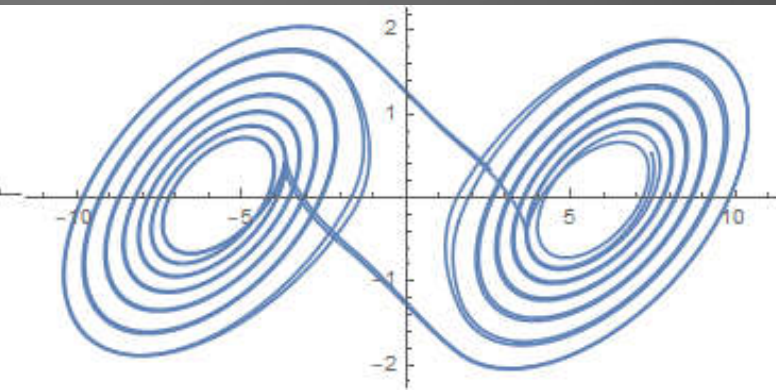
1/2100

C调制 (C2)

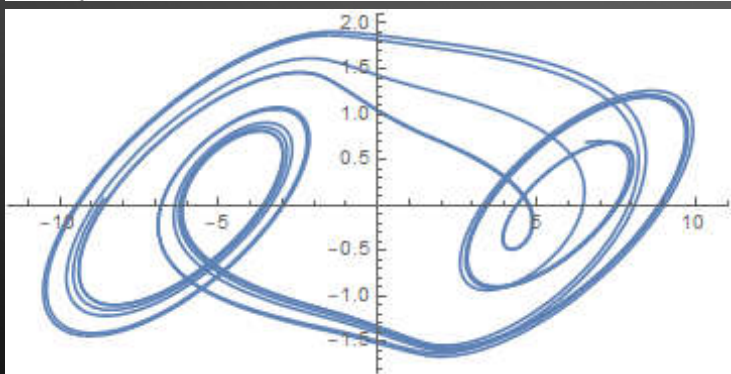
68.2nF



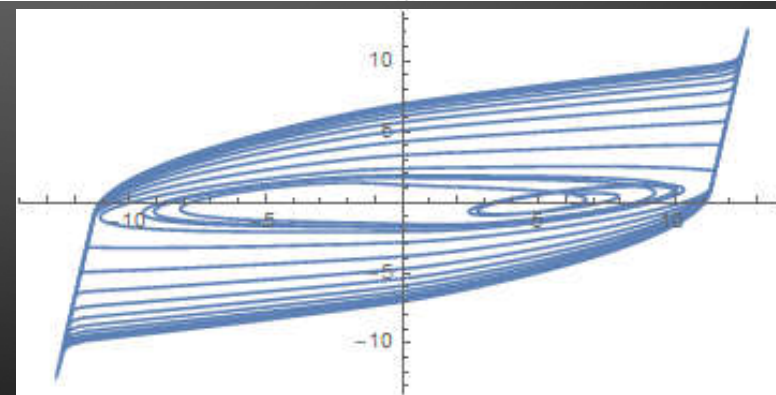
75nF



300nF

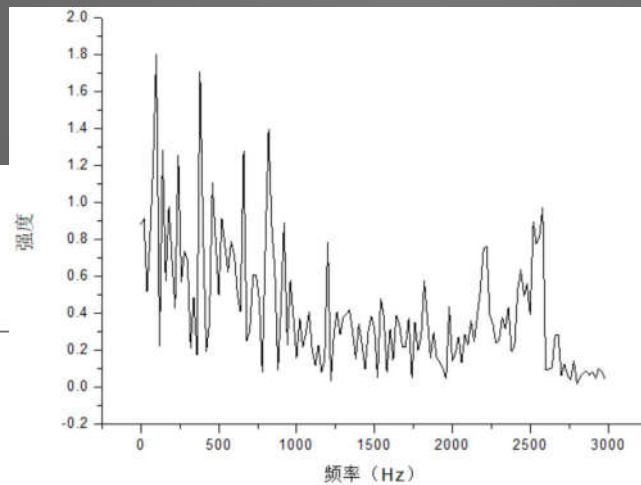
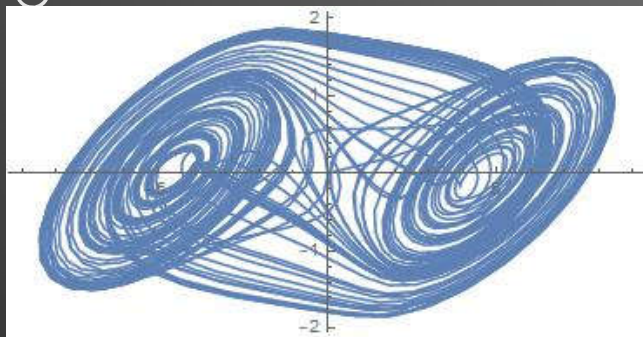


400nF



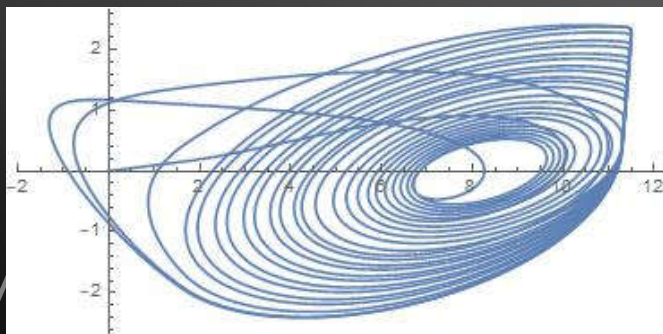
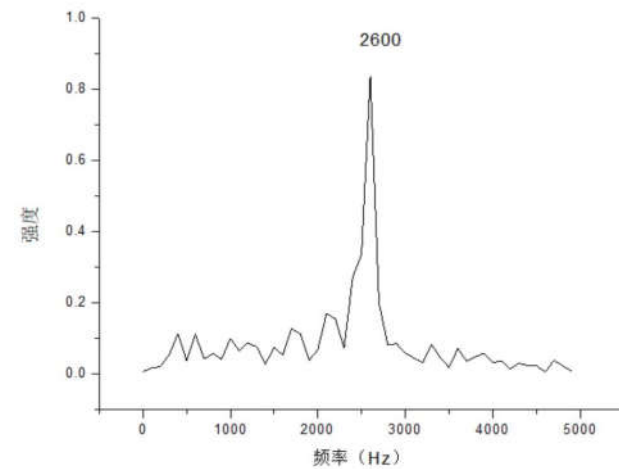
时域信号的频谱分析

$G=1/1904$



U1

U2



$G=1/2126$

