Experiments of Modern Physics I Report

on Lock-in Amplifier

Zhe Li `16307110295`

Department of Data Science
Fudan University

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Outline

1. Background & Frontier Application
2. Principle of LIA
3. Analysis and Conclusion
    - Detection of Weak Signals
    - Multi-Harmonic Measurement of Weak Signals
    - Micro-Impedance Measurement
    - Measurement of Diode Junction Capacitance
    - Resistance Thermal Noise Measurement

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Background

- Signal and Noise

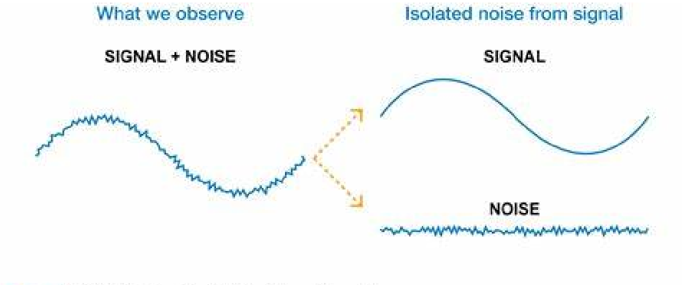
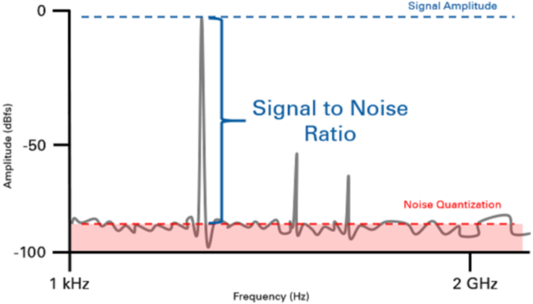


FIGURE 2. Isolated Noise from Signal

Background

- *SNR* and *SNIR*



$$SNR = \frac{S}{N}; \qquad SNIR = \frac{SNR_o}{SNR_i}$$

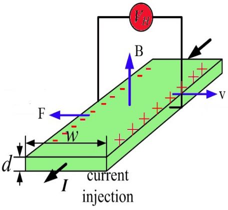
Background

Weak Signal Detection

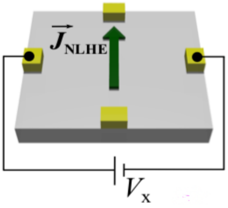
Method

- Filtering
- Null Method
- Modulation-Demodulation Method
- **Lock-in Amplifier**(*LIA*)

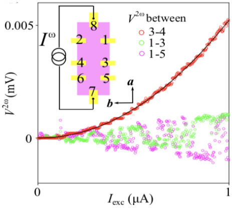
Frontier Application



(a) Hall effect



(b) Nonlinear Hall effect



(c) $V^{2\omega} \sim I_{\mathrm{exc}}$

$$V = \frac{KI}{d} B$$

$$V \propto I^2 = [I_0 \cos(\omega t)]^2 = \frac{1}{2} I_0^2 (\cos(2\omega t) + 1)$$

Ma Q , Xu S Y , Shen H , et al. Observation of the nonlinear Hall effect under time reversal symmetric

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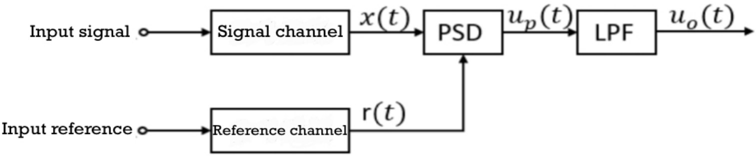
Principle of LIA

LIA is composed of

(1) Signal Channel
(2) Reference Channel
(3) **PSD**
(4) LPF



Principle of LIA

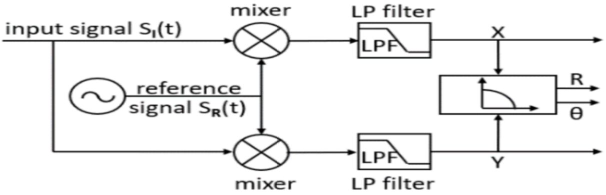
Theory

$$S_I(t) = A_I \sin(\omega t + \varphi) + B(t)$$

$$S_{R_0}(t) = A_R \sin(\omega t + \delta)$$

$$S_{R_1}(t) = A_R \cos(\omega t + \delta)$$

$$\Rightarrow \begin{cases} X = \frac{1}{2} A_I A_R \cos(\varphi - \delta) \\ Y = \frac{1}{2} A_I A_R \sin(\varphi - \delta) \\ R = \frac{\sqrt{2(X^2+Y^2)}}{A_R} \\ \theta = \varphi - \delta = \tan^{-1} \frac{Y}{X} \end{cases}$$



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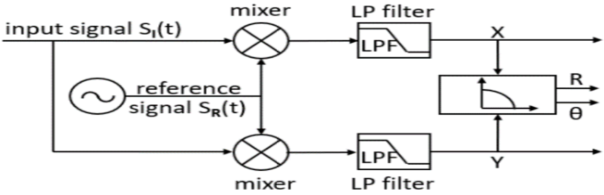

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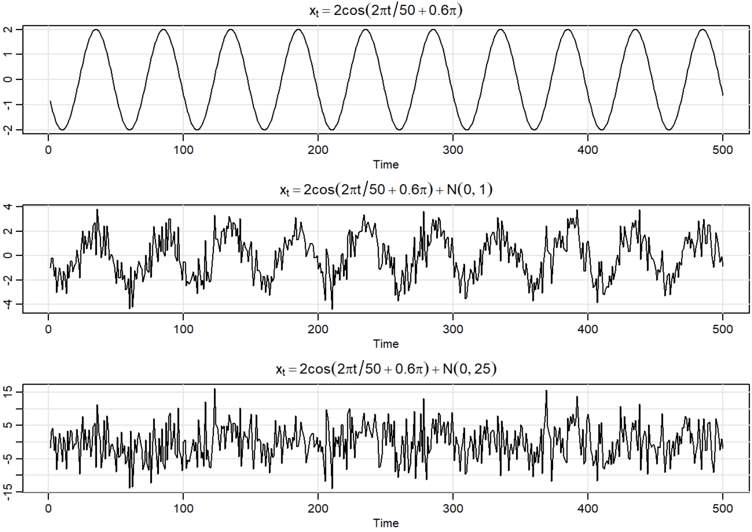
Detection of Weak Signals

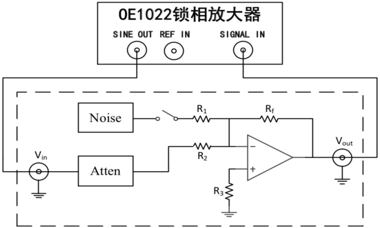


$x_t = 2\cos(2\pi t/50 + 0.6\pi)$

$x_t = 2\cos(2\pi t/50 + 0.6\pi) + N(0, 1)$

$x_t = 2\cos(2\pi t/50 + 0.6\pi) + N(0, 25)$

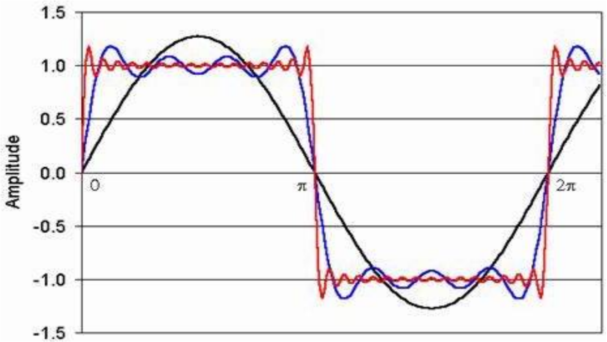
Detection of Weak Signals



Weak Signal Detection

V_{in}	V_{noise}	**SNR**/dB	R	**Stability**
1000	100	20	1010.2	0
100	100	0	100.77	0
10	100	-20	10.083 ± 0.002	0.02%
1	100	-40	1.005 ±0.005	0.5%
0.1	100	-60	0.103± 0.002	1.94%

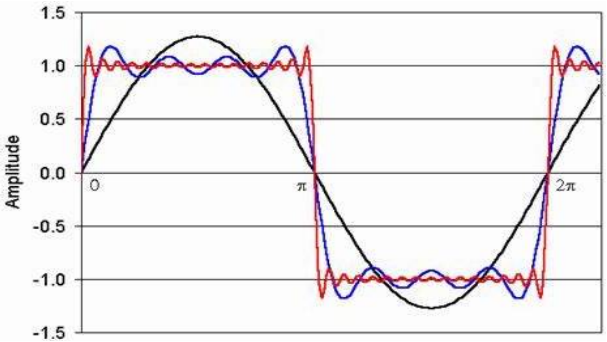
Multi-Harmonic Measurement of Weak Signals



Fourier expansion of square wave

$$f(t) = \frac{2E}{\pi} \sum_{k=0}^{\infty} \frac{sin[(2k+1)\omega t]}{2k+1} \quad \Rightarrow f_n(t) = \frac{2E}{n\pi} \sin(n\omega t)$$

Multi-Harmonic Measurement of Weak Signals



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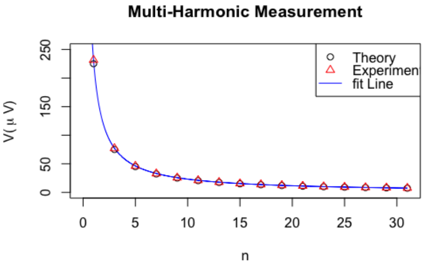
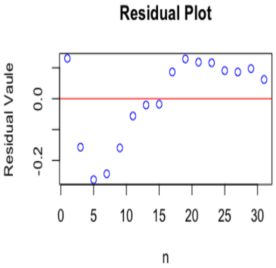
Multi-Harmonic Measurement of Weak Signals

Fitting Function

$$V = \frac{V_p}{\sqrt{2}} = \frac{2E}{\sqrt{2}n\pi} = \frac{\sqrt{2}E}{\pi} \cdot \frac{1}{n} \Rightarrow \qquad \mathbf{V} = \frac{\mathbf{a}}{\mathbf{n}} + \mathbf{b}$$



(d) Fitting Plot



(e) Residual Plot

Multi-Harmonic Measurement of Weak Signals

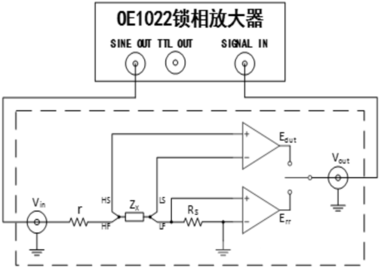
Fitting Result

	Estimaye	**Std.Error**	$p-value$
a	231.3	0.2	$< 2\times 10^{-16}$
b	0.036	0.042	0.408

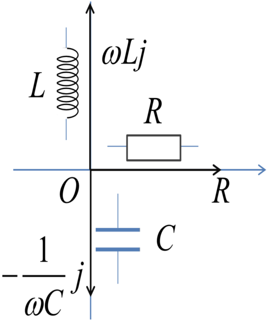
$$a = \frac{\sqrt{2}E}{\pi} \quad \Rightarrow \quad E = (513.8 \pm 0.5)\mu V; \qquad \eta = 2.8\%$$

A 95% confidence interval of b: $(-0.4632, 1.1832)$

Micro-Impedance Measurement



(f) Schematic Diagram



(g) Impedance phase

Z_X: The impedance of measured component
R_S: The standard resistance

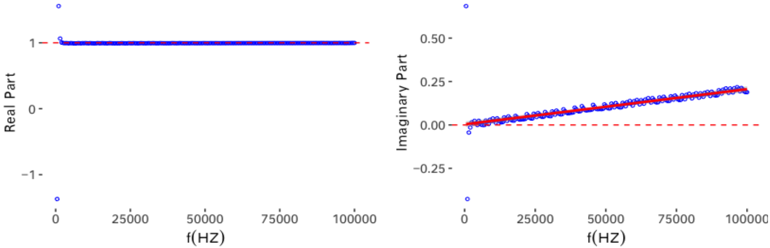
Micro-Impedance Measurement

Theory

$$Z_x = \frac{R_s\left(X_{dut} + Y_{dut} i\right)}{X_s + Y_s i} = \frac{R_s\left(X_{dut} X_s + Y_{dut} Y_s\right)}{X_s^2 + Y_s^2} + \frac{R_s\left(Y_{dut} X_s - X_{dut} Y_s\right)}{X_s^2 + Y_s^2} i = \mathrm{Real}_x + \mathrm{Image}_x\, i$$

Ideal Resistance: $\mathrm{Real}_x = R;\ \mathrm{Image}_x = 0$
Ideal Capacitance: $\mathrm{Real}_x = 0;\ \mathrm{Image}_x = \frac{1}{2\pi f C}$
Ideal Inductance: $\mathrm{Real}_x = 0;\ \mathrm{Image}_x = 2\pi f L$

Resistance($R = 1\Omega$)

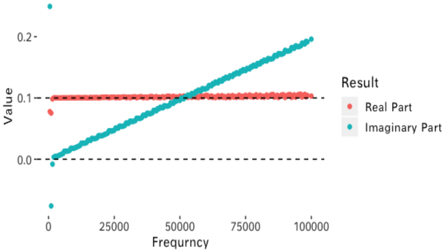


(h) Real Part (i) Imaginary Part

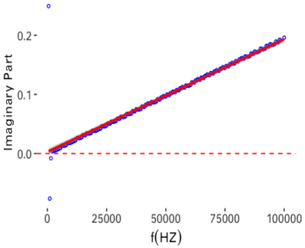
Fitting Result

	Estimate	**Std.Error**	$p-value$
a	2.08×10^{-6}	3×10^{-8}	$< 2 \times 10^{-16}$
b	-2.4×10^{-4}	1.6×10^{-3}	0.876

Resistance($R = 0.1\Omega$)



(j) Real Part

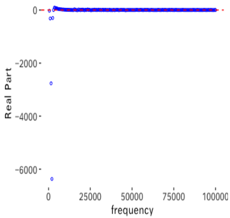


(k) Imaginary Part

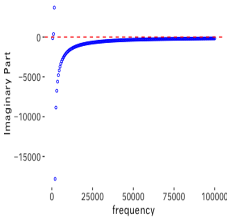
Fitting Result

	Estimate	**Std.Error**	$p-value$
a	1.952×10^{-6}	3×10^{-9}	$< 2 \times 10^{-16}$
b	-2.2×10^{-4}	1.8×10^{-4}	0.242

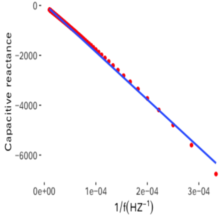
Capacitance($C = 10nf$)



(l) Real Part



(m) Imaginary Part

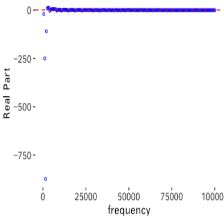


(n) Linear Regreesion

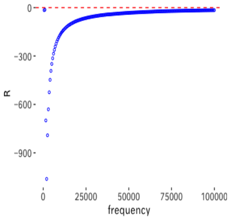
Fitting Result

	Estimate	Std.Error	$p-value$
a	-1.742×10^7	9.807×10^3	$< 2 \times 10^{-16}$
b	5.09	0.252	$< 2 \times 10^{-16}$

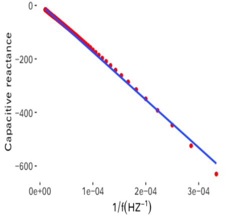
Capacitance($C = 100nf$)



(o) Real Part



(p) Imaginary Part



(q) Linear Regreesion

Fitting Result

	Estimate	Std.Error	$p-value$
a	-1.738×10^6	8.45×10^2	$< 2 \times 10^{-16}$
b	4.4	0.2	$< 2 \times 10^{-16}$

Summary

Summary

	$R(\Omega)$	$C(nf)$	$L(\mu H)$	η
$R = 1\Omega$	0.98 ± 0.17	0	0.331	2%
$R = 0.1\Omega$	0.102 ± 0.002	0	0.310	2%
$C = 10nf$	0	9.14	0	8.6%
$C = 100nf$	0	91.57	0	8.4%

Conclusion

(1) Quantitative agreement has been obtained between theory and experiment

(2) The Resistance is Impure

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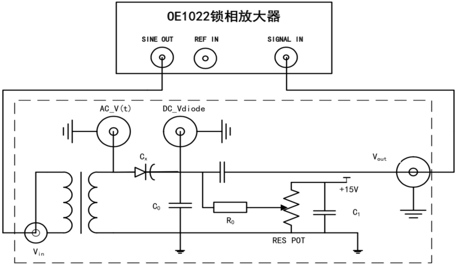
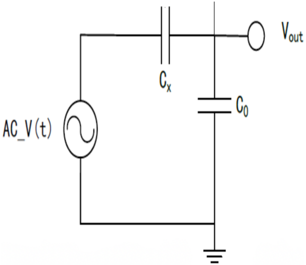
Measurement of Diode Junction Capacitance



(r) Schematic Diagram

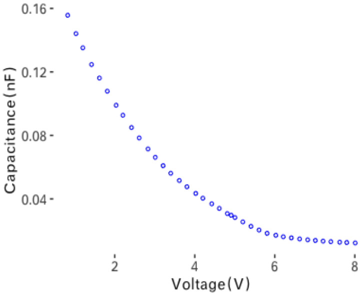


(s) Schematic Diagram

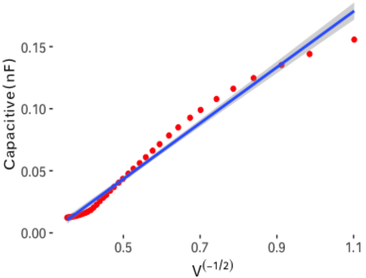
Theory Formula: $C_X = \sqrt{A\frac{\varepsilon S q N_0}{2V}} \propto \sqrt{\frac{1}{V}}$

Experiment Formula: $C_X = \frac{V_{out}}{V_{sine} - V_{out}} \cdot C_0$

Measurement of Diode Junction Capacitance



(t) **"C-V"** Plot



(u) Linear Regression

Measurement of Diode Junction Capacitance

Fitting Result

	Estimate	Std.Error	$p-value$
a	0.225	0.006	$< 2\times10^{-16}$
b	-0.069	0.003	$< 2\times10^{-16}$

Summary

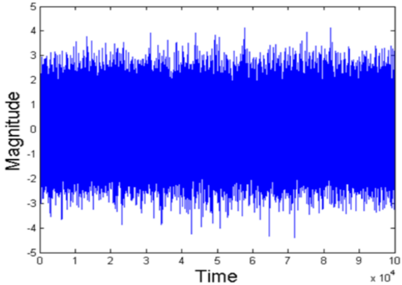
$$C_X = \frac{(2.25\pm0.006)\times10^{-10}}{\sqrt{V}} - (6.9\pm0.3)\times10^{-11}$$

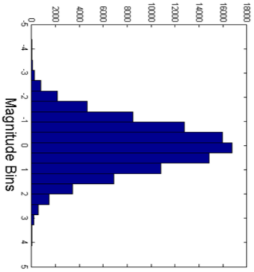
Resistance Thermal Noise Measurement

Theory

(1) $\mathrm{V} = \sqrt{(4KTRB)} \quad \Rightarrow \quad S_t(f) = 4KTR(V^2/Hz)$

(2) Thermal noise is **White Noise**

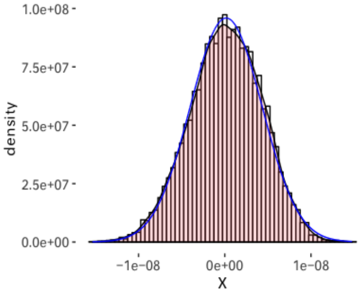




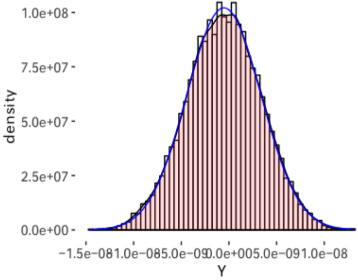
Resistance Thermal Noise Measurement

Histogram, Kernel Curve and Gaussian Curve of X & Y

$$R = 1000\Omega; \quad f = 997Hz$$



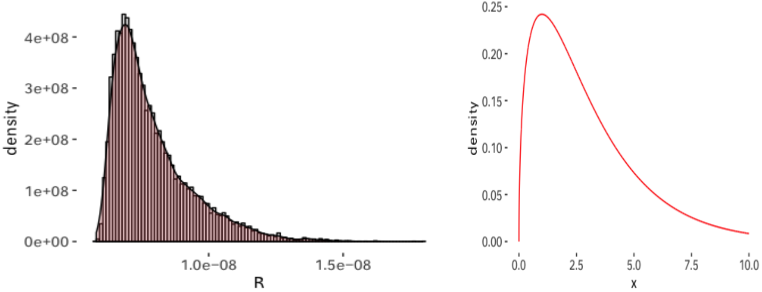
(v) Distribution of X



(w) Distribution of Y

Resistance Thermal Noise Measurement

$$R = \sqrt{X^2 + Y^2} \sim \chi^2$$



(x) Distribution of R (y) Density Curve of $\chi^2(3)$

Resistance Thermal Noise Measurement

Measuring Principle

(1) **Background Noise:** V_{BN}

(2) **Thermal Noise:** V_{TN}

(3) **Measurement:**

$$V_{SN}^2 = V_{TN}^2 + V_{BN}^2$$

Resistance Thermal Noise Measurement

Method

$$NOISE = \frac{NOISE_{sum}}{ENBW} = \frac{\text{Standard Error of data}}{ENBW}$$

Fitting Result

(nV)	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{3}{32}$	$\frac{5}{64}$
V_{SN}	10.864	7.012	6.716	6.287
Theory (V_{TN})	2.008	1.420	1.230	1.122
V'_{BN}	10.677	6.867	6.602	6.186
Theory(V_{BN})	11.875	5.938	4.453	3.711
η	10.1%	15.6%	48.3%	66.7%

Resistance Thermal Noise Measurement

Error Analysis

(1) Electromagnetic Interference

(2) Capacitive Coupling

(3) Inductive Coupling

(4) Flutter Noise

Reference

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- Wang Zixin et al. Design of teaching experiment platform for weak signal detection based on PLA [J]. Experimental technology and management, 2017.2, 34(2), 88-96.
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- Wang Zixin.OE5001 Weak Signal Detection Teaching Laboratory Box (with Digital Phase-Locked Amplifier)[J].Physical Experiments, 2017,37(11): 62.

THANKS