Courseware report – *Critical Phenomena and its simulation on finite size systems*

Zheng Zhou

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    - *Long-range correlation*
    - *Scale invariance*
  - Demonstrate the finite size behavior

Structure of the courseware

1. Introduction to critical phenomena and Ising model
2. Demonstration of correlation and scaling invariance
3. Introduction to finite size scaling

Simulation widget 1 – Dynamics and scaling invariance in critical systems

- Design : Full scale simulation – zoom-in of 1/4 region – zoom-in of 1/16 region
- Demonstration of clusters with the feature of the ordered phase emerges which do not have determined position and boundary, but dynamically evolve over time and are nested.
- Demonstration of scaling invariance – if we zoom in a specific region, we would find similar structural characteristics.

Simulation widget 2 – Correlation function and correlation length

- Design : updating configuration – simulated measurement of correlation function and correlation length(and its plot in log scale and log-log scale)
- Demonstrate the different behavior of correlation through tuning the temperature
  - PM phase : exponential decay
  - Critical : power-law decay
  - FM phase : not decay to zero.
- Demonstrate the divergence of correlation length at critical point.

Simulation widget 3 – Finite size behavior of the critical system

- Design : updating configuration – simulated measurement of physical quantities and their scaling behavior (data collapse)
- Demonstrate that singularities do not exist in finite size systems
- Demonstrate that the curves of different sizes are of the same shape and collapse with each other after 'streching' transformation.

Appendix : Monte Carlo Method

$$\langle \mathcal{O} \rangle = \frac{\sum_{\{\sigma\}} \mathcal{O}_\sigma \exp(-E_\sigma/kT)}{\sum_{\{\sigma\}} \exp(-E_\sigma/kT)} \quad (1)$$

- **Importance sampling** : Sample in all the possible states $\{\sigma\}$ with a weight of each state $W(\sigma) = \exp(-E_\sigma/kT)$, and the expectation of observables should equal the simple algebric average of all the samples.
- **Markov process** : the configuration of the consecutive sample solely depend on the former sample

$$\sigma \to \sigma' \text{ at a probability } P(\sigma \to \sigma') \quad (2)$$

$$W(\sigma)P(\sigma \to \sigma') = W(\sigma')P(\sigma' \to \sigma) \quad (3)$$

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