Classical Dimer Model

ynopsis Demonstration of Simulation via HTML5

经典二聚体模型的介绍与简单展示

Reporter：李佳泽

复旦大学物理系 19300200008

Contents

- Background
- Algorithm
- Demonstration via HTML5 pages

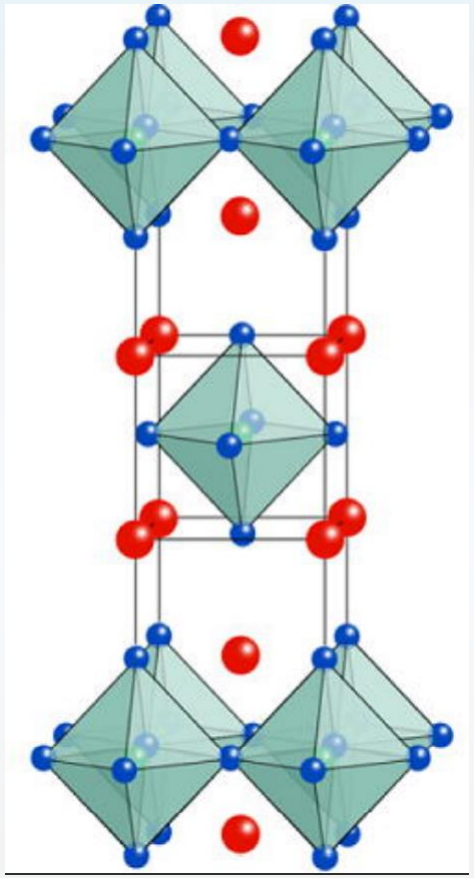
Background

1

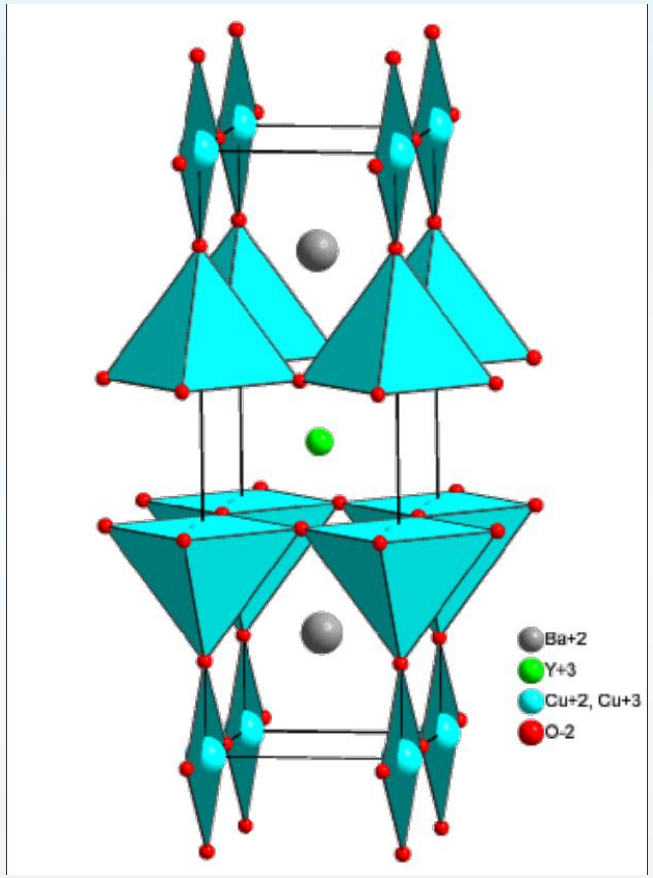
- History
- Models

Resonating Valence Bond

P.W. Anderson

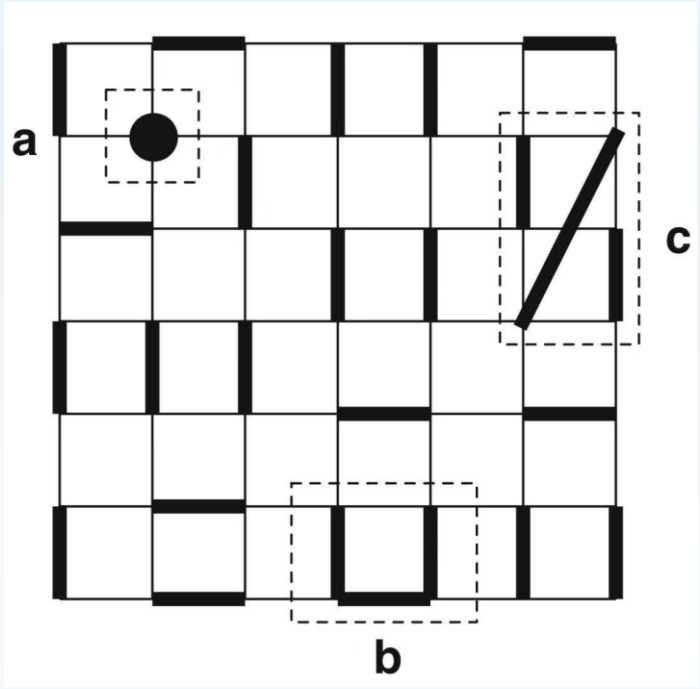


$La_{1.85}Sr_{0.15}CuO_4$

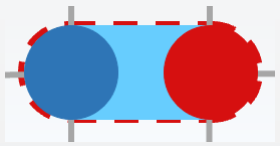


$YBa_2Cu_3O_7$

Resonating Valence Bond



P. W. Anderson, Science237, 1196(1985a)
P. W. Anderson et al, J. Phys. :Condens. Matter16(2004)

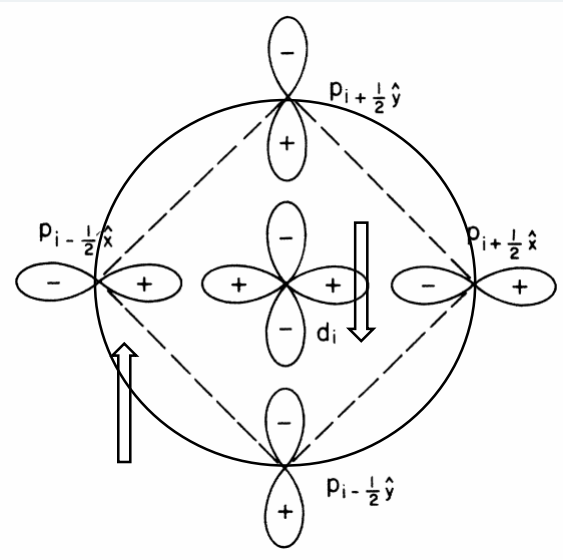


$$= \frac{|\uparrow\downarrow> - |\downarrow\uparrow>}{\sqrt{2}}$$

The phase is a s=1/2 Mott insulator, highly fluctuating quantum spin liquid.

Advanced model has been introduced:

Zhang-Rice singlet



F. C. Zhang, T. M. Rice, Phys. Rev. B37,3759(1988)

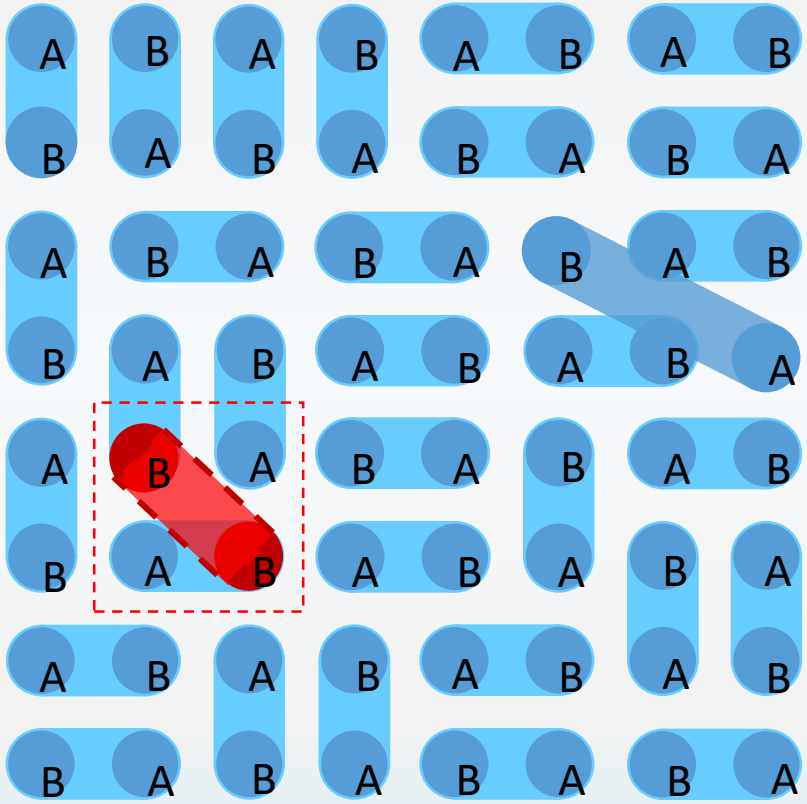
Others:
Phase string effect—Z. Y. Weng, et al, Phys. Rev. L80,5401(1998)

Bosonic liquid—S. T. Jiang, L. Zou, W. Ku. Phys. Rev. B99, 104507(2019).

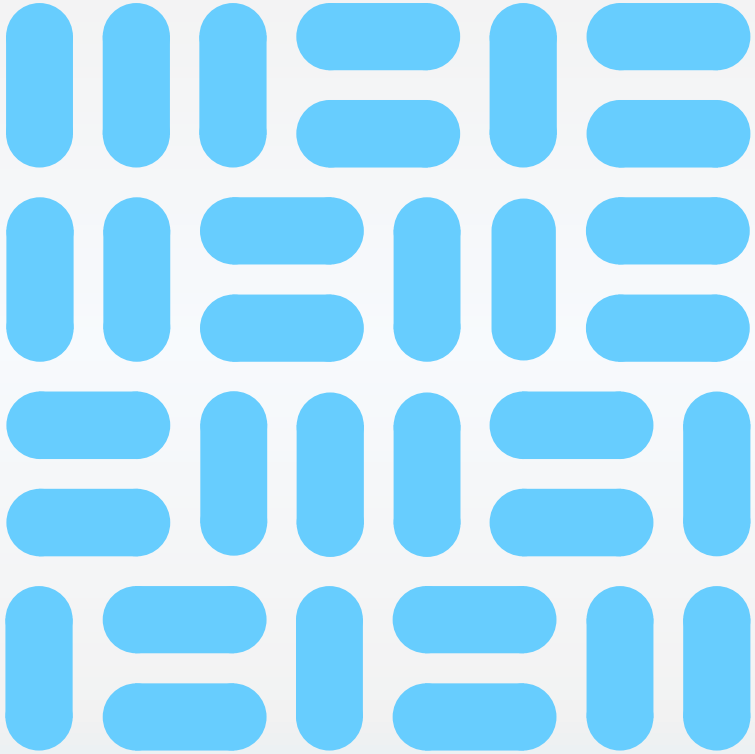
……

An effective low energy model introduced by D.S. Rokhsar and S.A. Kivelson.

Dimer Model



Geometric constraints



Compact dimer model

Dimer Model

Hamiltonian:

Simplification:

Implication:

$$H = \sum_{plaquettes} V(|=\rangle\langle=| + |\shortparallel\rangle\langle\shortparallel|) \quad (1)$$

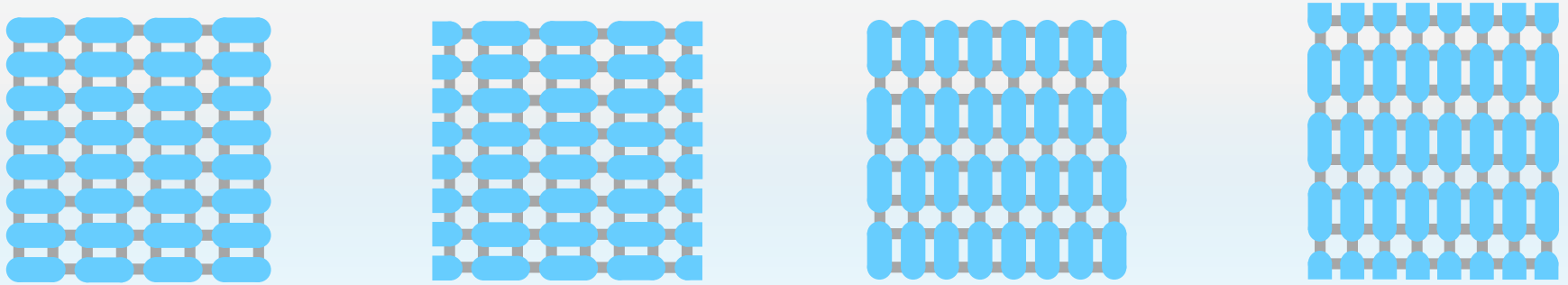
$$(2)\ |\shortparallel\rangle\langle\shortparallel|] \quad (1)$$

Partition Function:

$$Z = \sum_{state} exp[-\frac{k}{T}(N(=) + N(\shortparallel))] \quad (3)$$

2-dim order parameter:

$$\boldsymbol{\mu} = (\epsilon_1 N(-), \epsilon_2 N(|)) \quad (4)$$

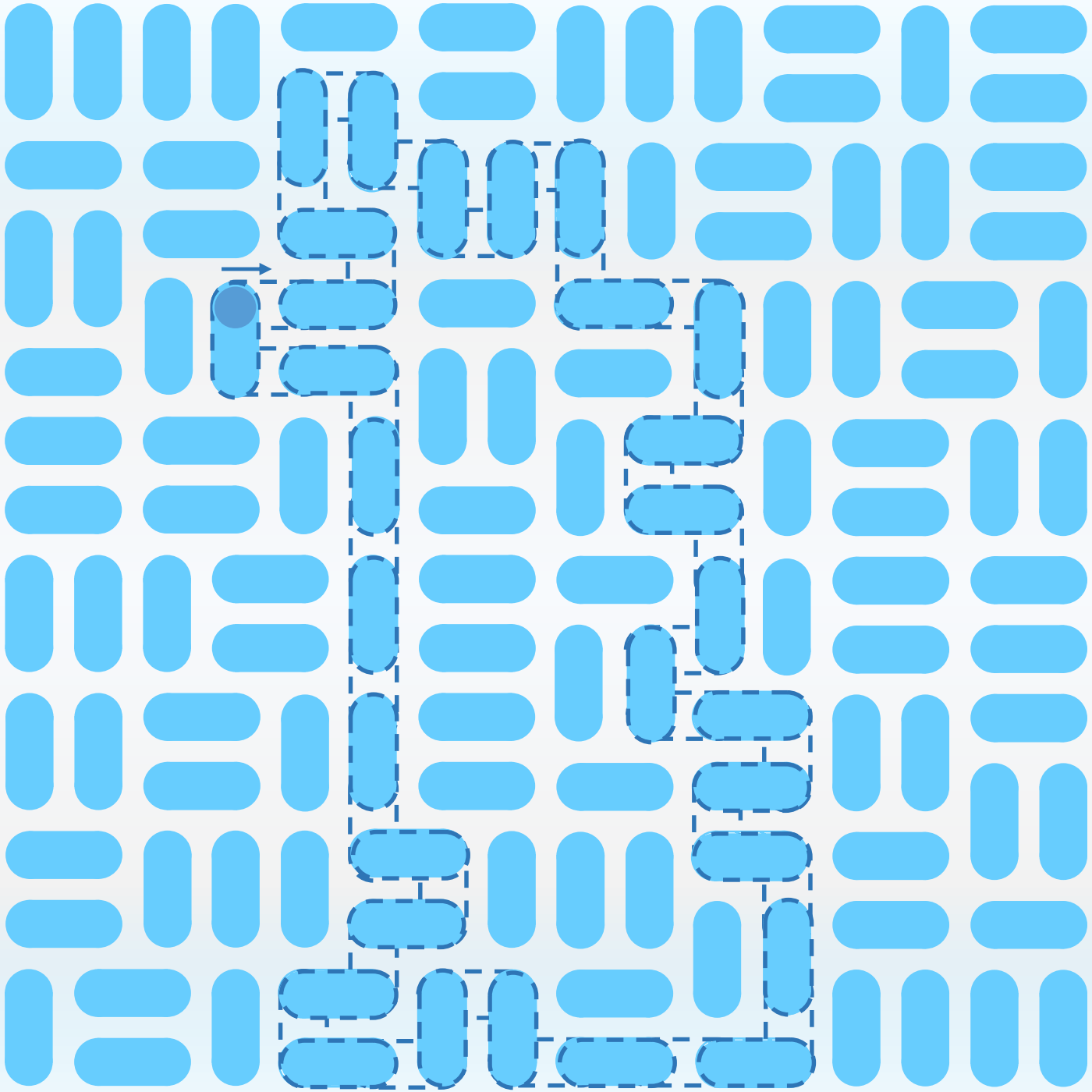


Algorithms

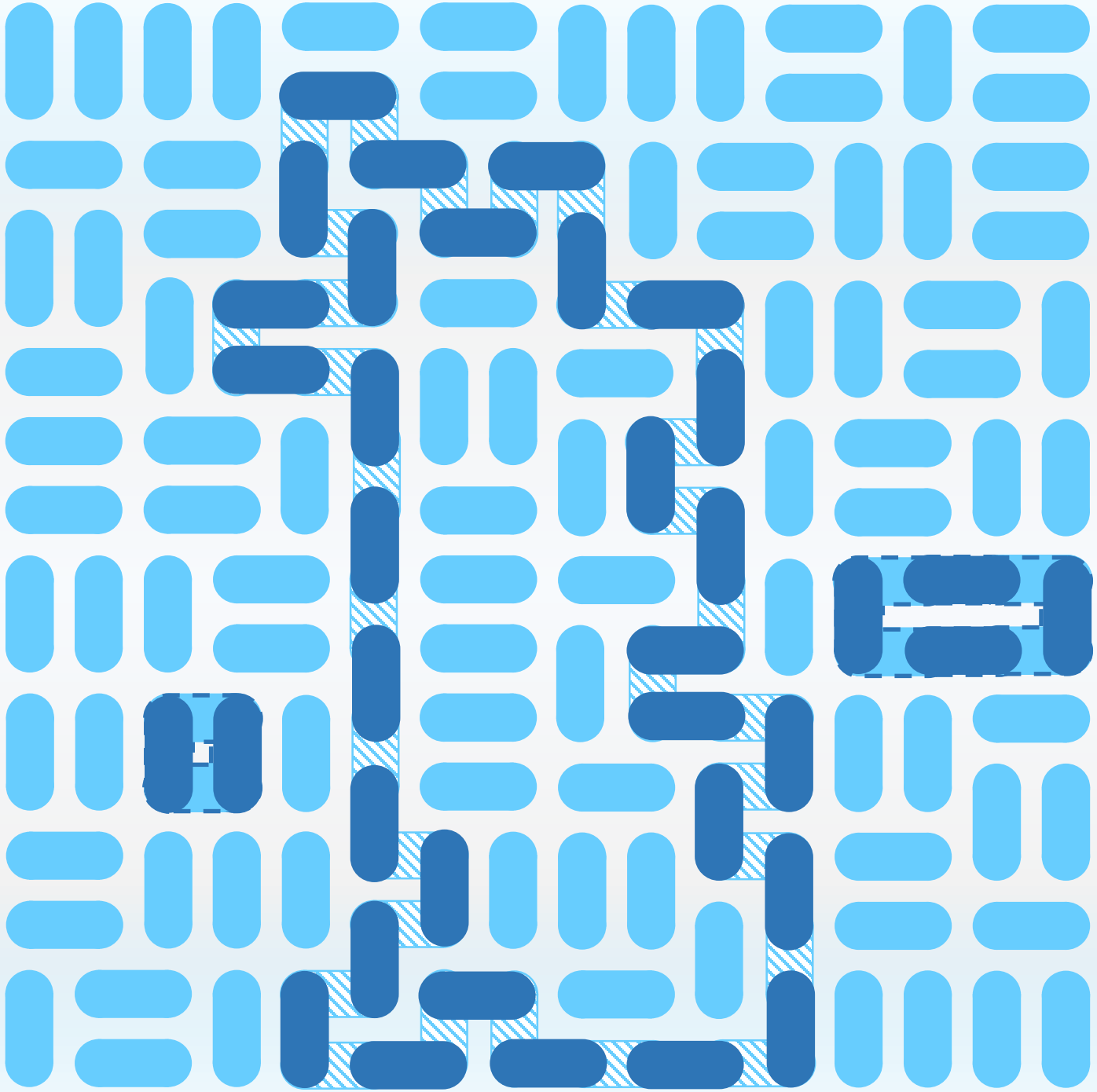
21

- **Flippable Loop Algorithm**
- Pocket Edged Algorithm
- Directed Loop Algorithm
- Detailed Balance

Directed loop algorithm

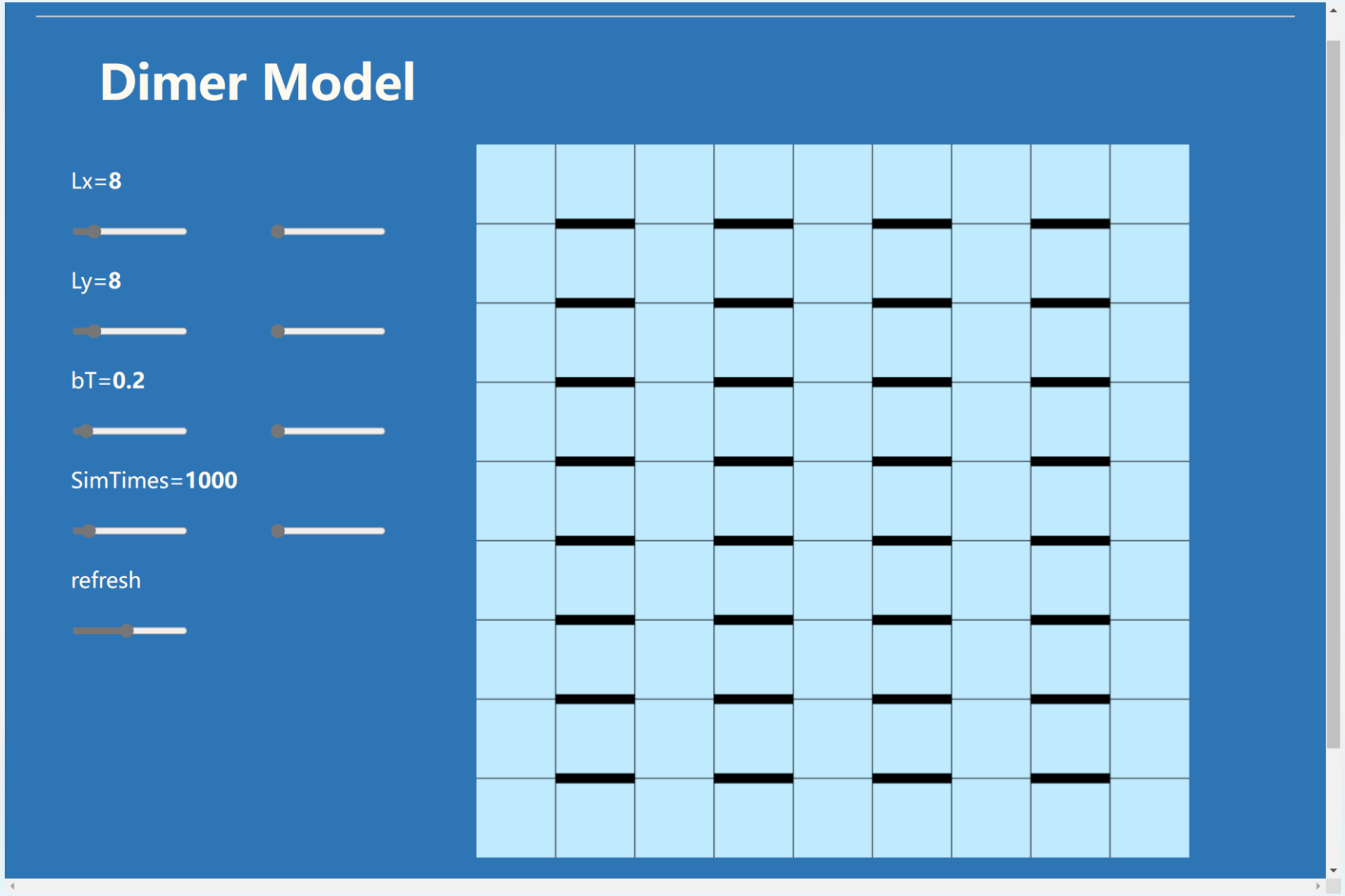


Directed loop algorithm

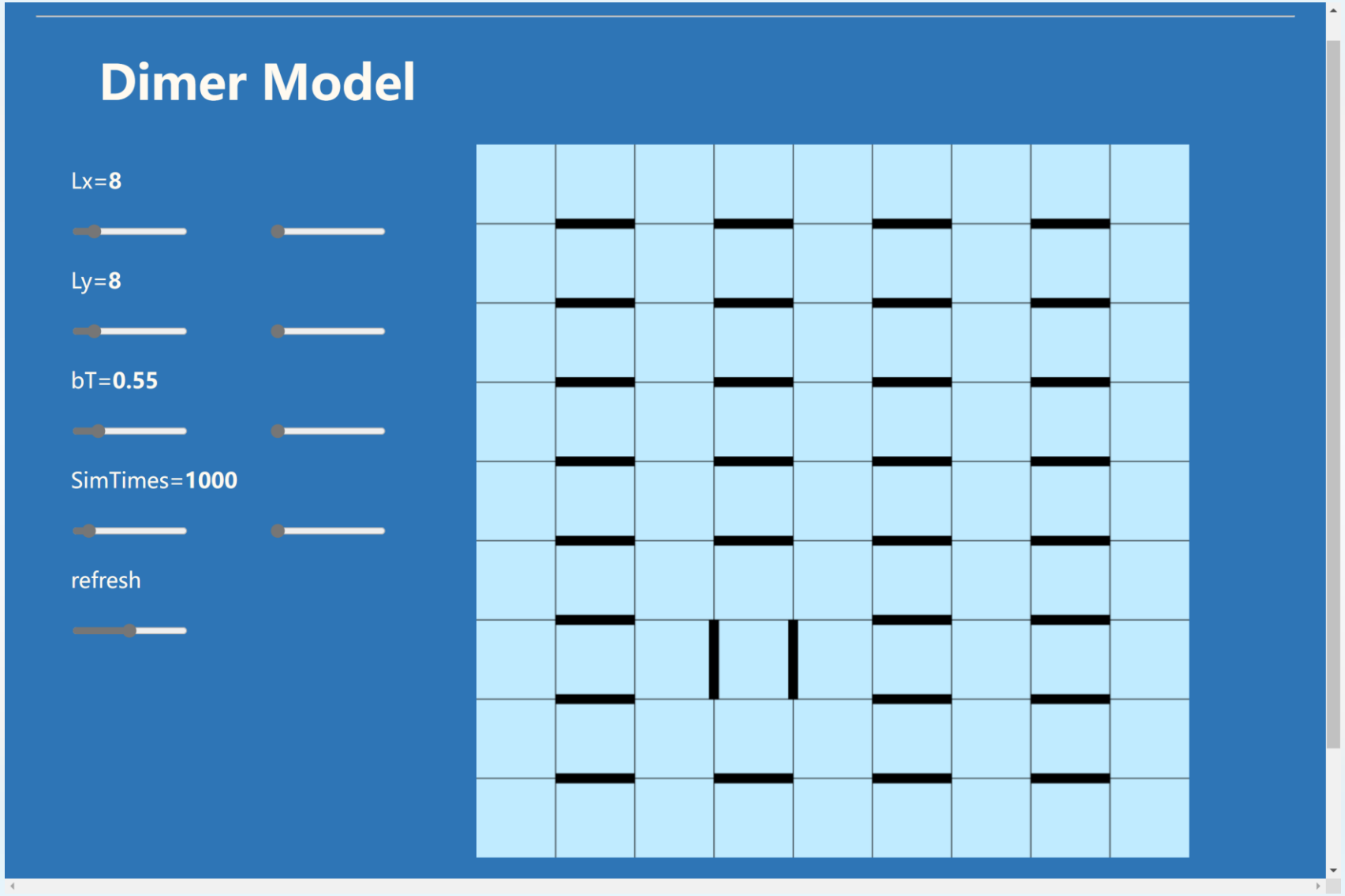


Demonstration via HTML5 pages

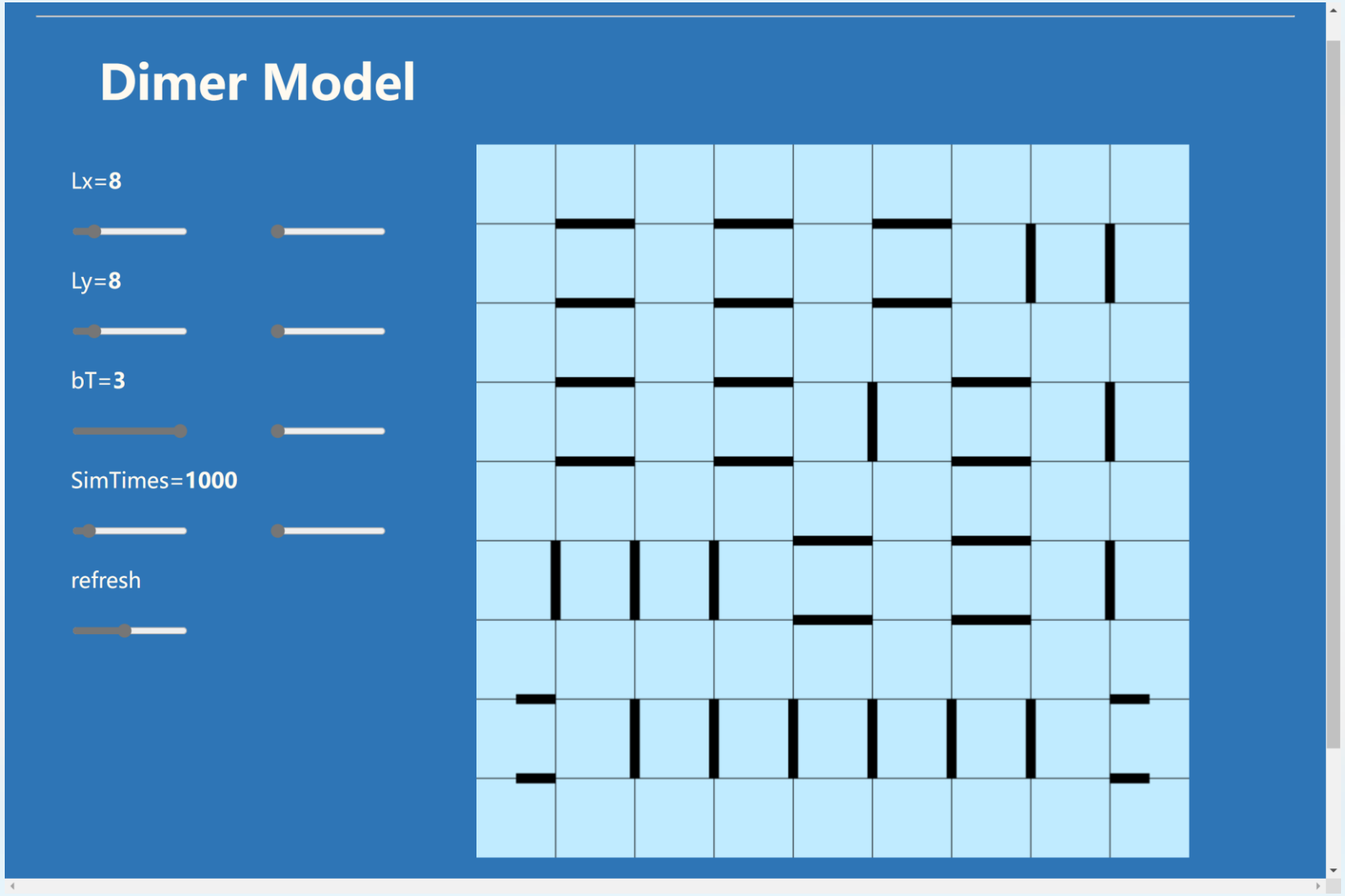
Some results (capture from HTML5 pages)



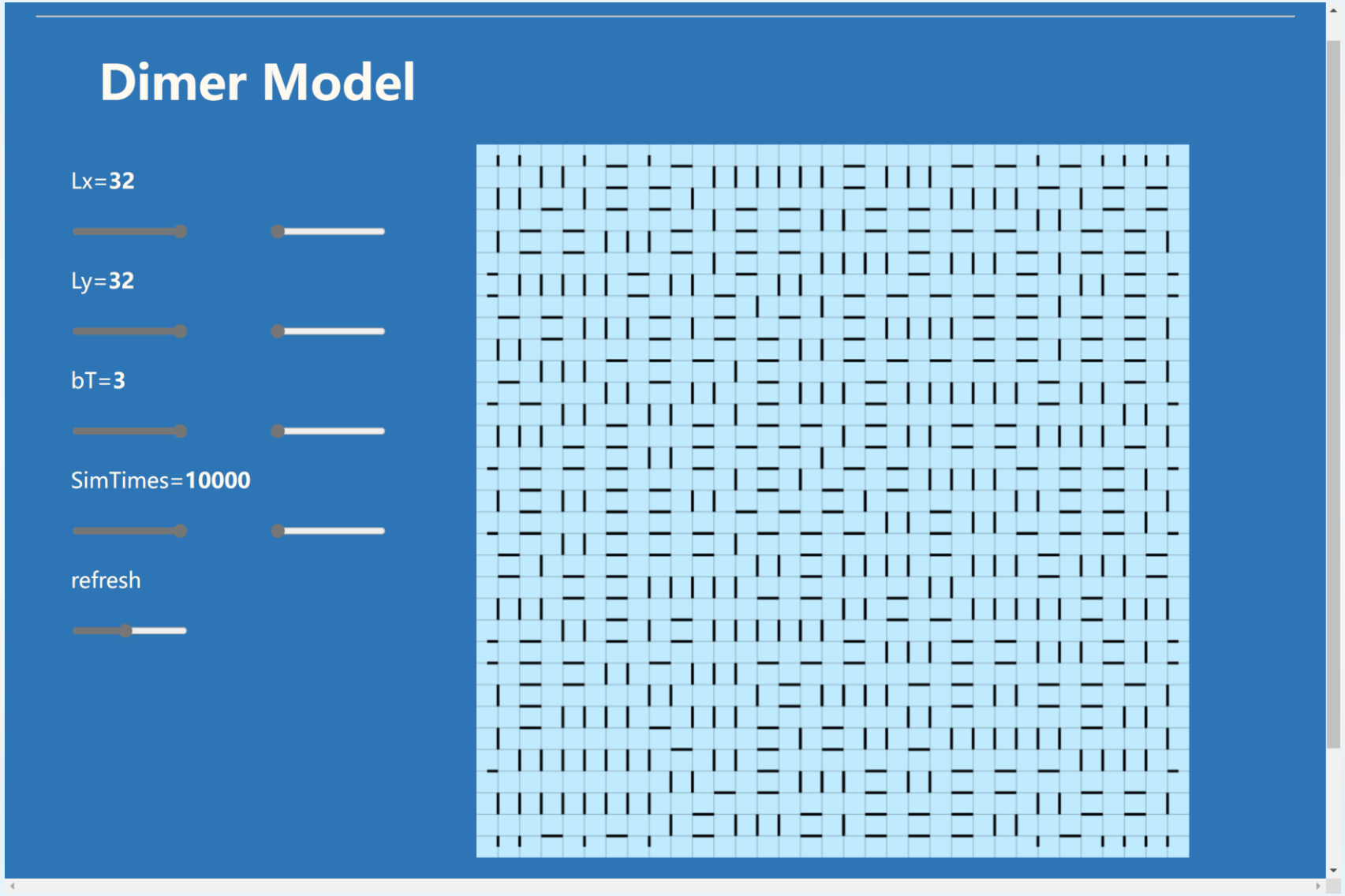
Some results (capture from HTML5 pages)



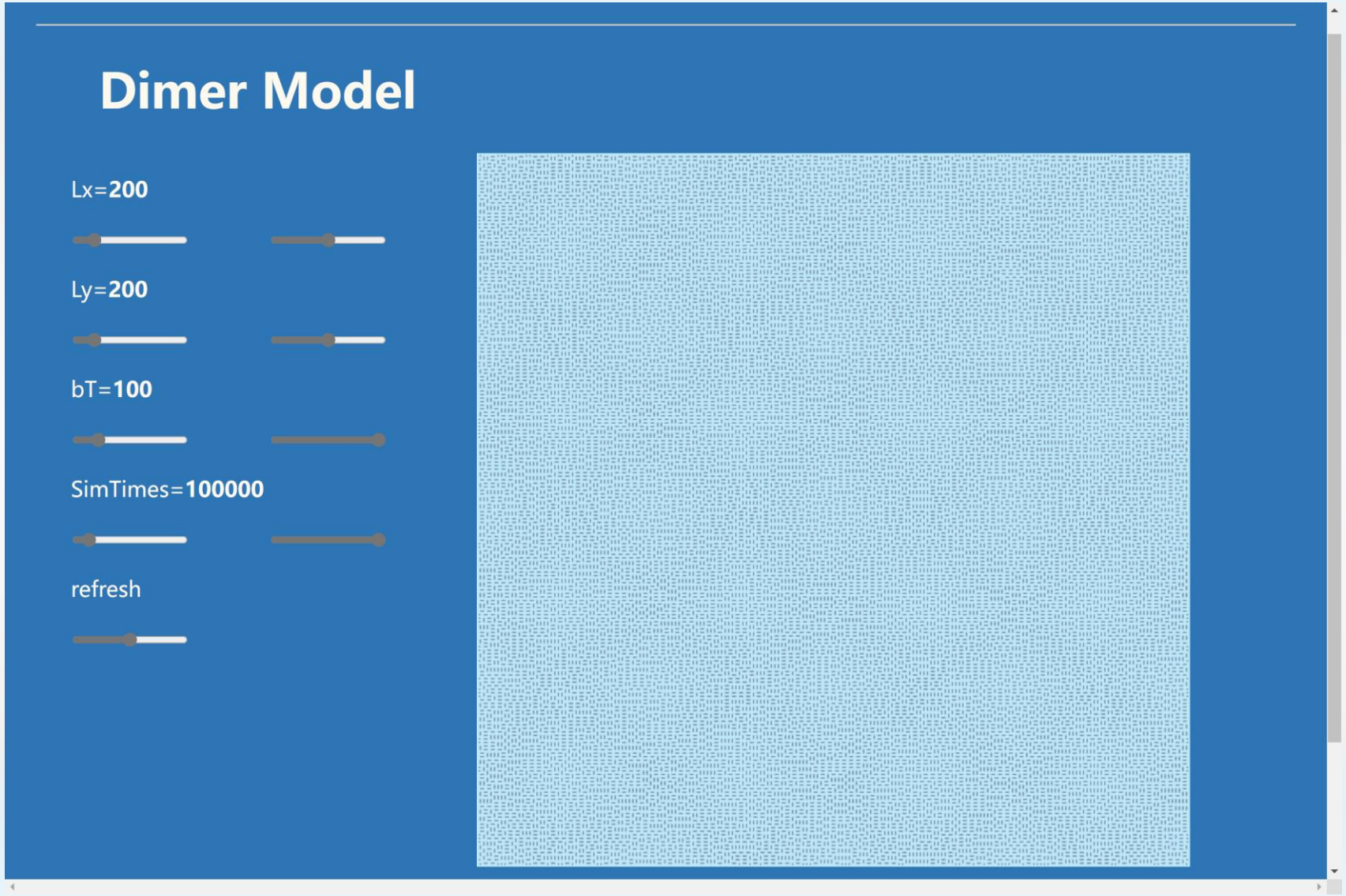
Some results (capture from HTML5 pages)



Some results (capture from HTML5 pages)

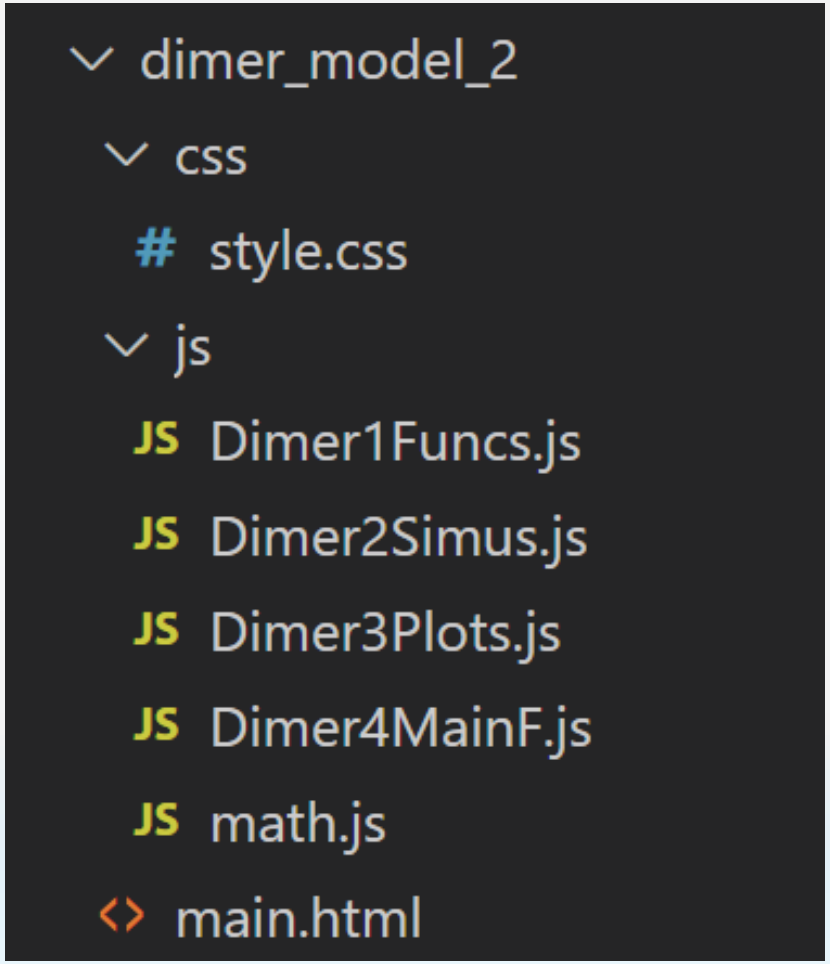


Some results (capture from HTML5 pages)



Some codes

procedural framework



Some codes

HTML5 and JavaScript framework

main.html

dimer_model_2 > main.html > html

```html
<!DOCTYPE html>
<html>
    <head>
        <meta charset="utf-8"/>
        <title>Dimer Model</title>
        <link rel="stylesheet" href="css/style.css">
    </head>
    <hr/>
    <body>
        <h1 class="title">Dimer Model</h1>

        <div class="left">···
        </div>

        <div class="middle">···
        </div>
        <script>
            function ParaDisplay(){···
        </script>

        <div class="right">
            <canvas id="statePicture" width="800" height="800"></canvas>
        </div>

        <script src="js/math.js"></script>
        <script src="js/Dimer1Funcs.js"></script>
        <script src="js/Dimer2Simus.js"></script>
        <script src="js/Dimer3Plots.js"></script>
        <script src="js/Dimer4MainF.js"></script>
    </body>
</html>
```

Dimer4MainF.js

dimer_model_2 > js > Dimer4MainF.js > DimerMainFunc

```javascript
function DimerMainFunc(){
    let wPoint=[-1,-2];
    let vPoint=[-1,0,0,0,0,0,0];
    let Lx, Ly, TPoint, SIM_TIME;
    let temp1, temp2;

    temp1=parseInt(document.getElementById("IN_x1").value);
    temp2=parseInt(document.getElementById("IN_x2").value);
    Lx=temp1>temp2?temp1:temp2;
    temp1=parseInt(document.getElementById("IN_y1").value);
    temp2=parseInt(document.getElementById("IN_y2").value);
    Ly=temp1>temp2?temp1:temp2;
    temp1=parseFloat(document.getElementById("IN_T1").value)/100;
    temp2=parseFloat(document.getElementById("IN_T2").value)/100;
    TPoint=temp1>temp2?temp1:temp2;
    temp1=parseInt(document.getElementById("IN_S1").value);
    temp2=parseInt(document.getElementById("IN_S2").value);
    SIM_TIME=temp1>temp2?temp1:temp2;

    let caseOfEProb=new Array(78125);
    let caseOfWProb=new Array(2);
    let state=new Array(Lx*Ly);
    EP_EProbSave(caseOfEProb, TPoint, vPoint);
    EP_WProbSave(caseOfWProb, wPoint);

    for(let a=0; a<Lx; a++){
        for(let b=0; b<Ly; b++){
            state[a*Ly+b]=(a%2)*6;
        }
    }
    for(let iSim=0; iSim<SIM_TIME; iSim++){
        EP_Body(state, caseOfEProb, caseOfWProb, Lx, Ly);
    }
    let canvasState=document.getElementById("statePicture");
    let contextState=canvasState.getContext("2d");
    contextState.clearRect(0,0,800,800);
    LatticePlot(Lx, Ly, contextState);
    StatePlot(state, Lx, Ly, contextState);
}
```

Some codes

The functions realizing simulation

Dimer1Funcs.js

dimer_model_2 > js > Dimer1Funcs.js > BSC_CriterionUpdate

```
1   //return an int form 0 to (range-1) at random.
2   function BSC_RandRangeI(range){ ···
6   }
7
8   //print the matrix with numbers and spaces
9   function BSC_MatrixPrint(array, ORDERx, ORDERy){ ···
28  }
29
30  //assign matrix_1 to matrix_2
31  function BSC_MatrixCopy(array1, array2, ORDERx, ORDERy){ ···
35  }
36
37  //the next line number of "line a" directed by the direction
38  function BSC_NextPointA(a, direction, ORDERx){ ···
63  }
64
65  //the next column number of "column b" directed by the direction
66  function BSC_NextPointB(b, direction, ORDERy){ ···
91  }
92
93  //determine whether we accept the update, return 1 or 0
94  function BSC_CriterionUpdate(caseOfDeltaE, caseOfEProb) ···
104 }
```

Dimer2Simus.js

dimer_model_2 > js > Dimer2Simus.js > EP_Body

```
1   //save all probable deltaEnergy cases
2   function EP_EProbSave(caseOfEProb, temperature, vPoint){ ···
50  }
51
52  //save the two probable cases in the softed model, considering the detailed ballance
53  //omega[0] is omega_ss, omega[1] is omega_ll
54  //caseOfWProb[0] is a sum of P_ss, caseOfWProb[1] is a sum of P_ll
55  function EP_WProbSave(caseOfWProb, omega){ ···
89  }
90
91  //return a direction number from 0 to 11, considering the detailed ballance
92  function EP_RandWalk(inwardDirc, caseOfWProb){ ···
122 }
123
124 //figure out the deltaEnergy case
125 function EP_CountA(state, coord, ORDERx, ORDERy){ ···
197 }
198
199 //figure out the deltaEnergy case
200 function EP_CountB(state, coord, ORDERx, ORDERy){ ···
349 }
350
351 //finish a whole update on state
352 function EP_Body(state, caseOfEProb, caseOfWProb, ORDERx, ORDERy){ ···
410 }
```

Some codes

Plotting codes

Dimer3Plots.js

dimer_model_2 > js > Dimer3Plots.js > StatePlot

```
1  function LatticePlot(ORDERx, ORDERy, context){ ···
20 }
21
22 function StatePlot(state, ORDERx, ORDERy, context){ ···
73 }
```

References

[1] P.W. Anderson, Resonating valence bonds: A new kind of insulator?, Materials Research Bulletin, 8 (1973) 153-160.

[2] P.W. Anderson, The resonating valence bond state in La2CuO4 and superconductivity, science, 235 (1987) 1196-1198.

[3] A. Gilabert, A. Hoffmann, M. Medici, I. Schuller, Photodoping effects in high critical temperature superconducting films and Josephson junctions, Journal of superconductivity, 13 (2000) 1-20.

[4] W. Heisenberg, Zur theorie des ferromagnetismus, Original Scientific Papers Wissenschaftliche Originalarbeiten, Springer1985, pp. 580-597.

[5] D.S. Rokhsar, S.A. Kivelson, Superconductivity and the quantum hard-core dimer gas, Physical review letters, 61 (1988) 2376.

[6] P.W. Leung, K.C. Chiu, K.J. Runge, Columnar dimer and plaquette resonating-valence-bond orders in the quantum dimer model, Physical Review B, 54 (1996) 12938.

[7] A.W. Sandvik, O.F. Syljuåsen, The Directed-Loop Algorithm, AIP Conference Proceedings, American Institute of Physics, 2003, pp. 299-308.

[8] F. Alet, Y. Ikhlef, J.L. Jacobsen, G. Misguich, V. Pasquier, Classical dimers with aligning interactions on the square lattice, Physical Review E, 74 (2006) 041124.

[9] F. Alet, J.L. Jacobsen, G. Misguich, V. Pasquier, F. Mila, M. Troyer, Interacting classical dimers on the square lattice, Physical review letters, 94 (2005) 235702.

[10]Yao H, Li J, Hou J. The Breaking of Geometric Constraint of Classical Dimers on the Square Lattice[J]. arXiv preprint arXiv:2106.09674, 2021.

Thanks!