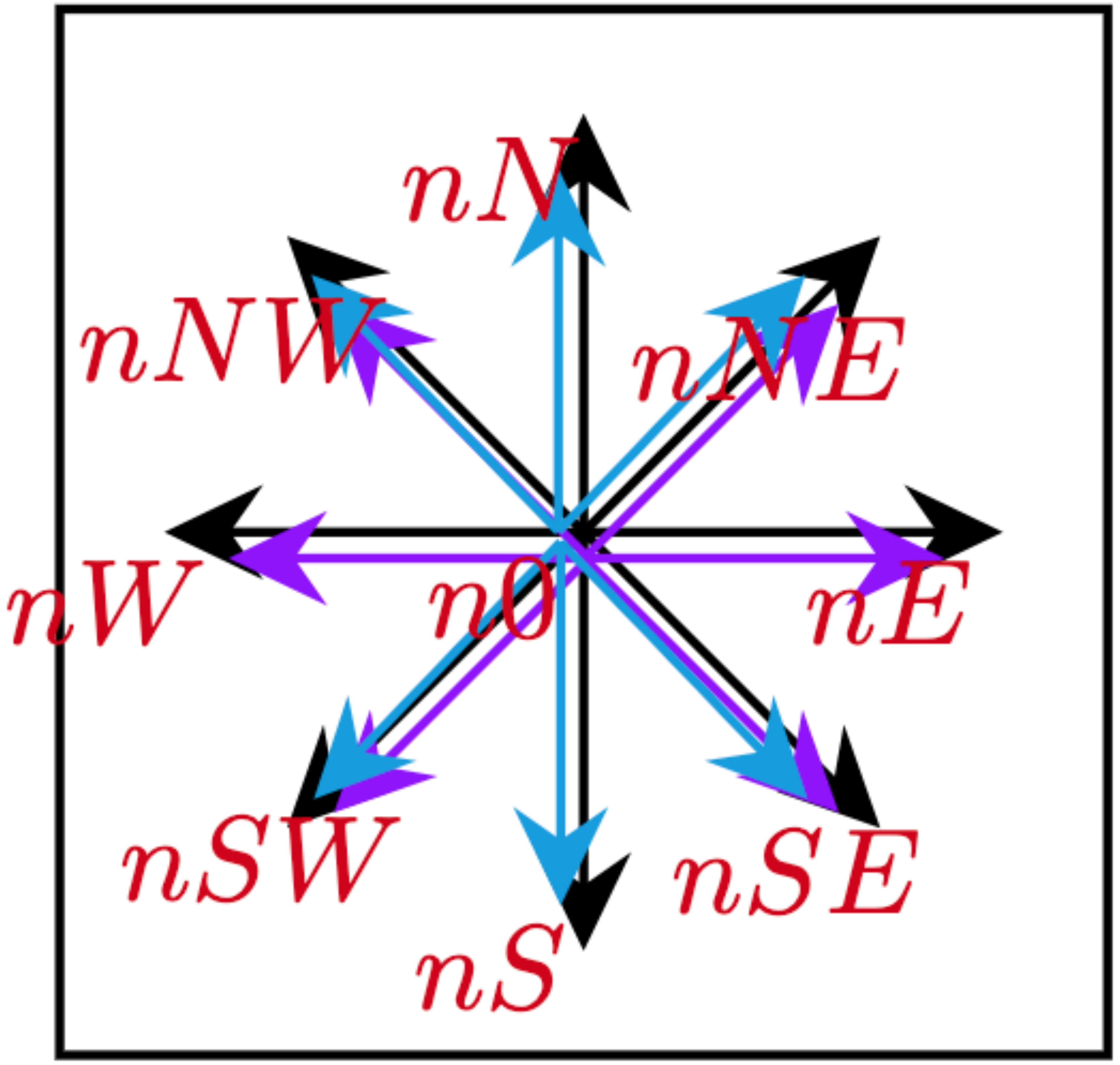
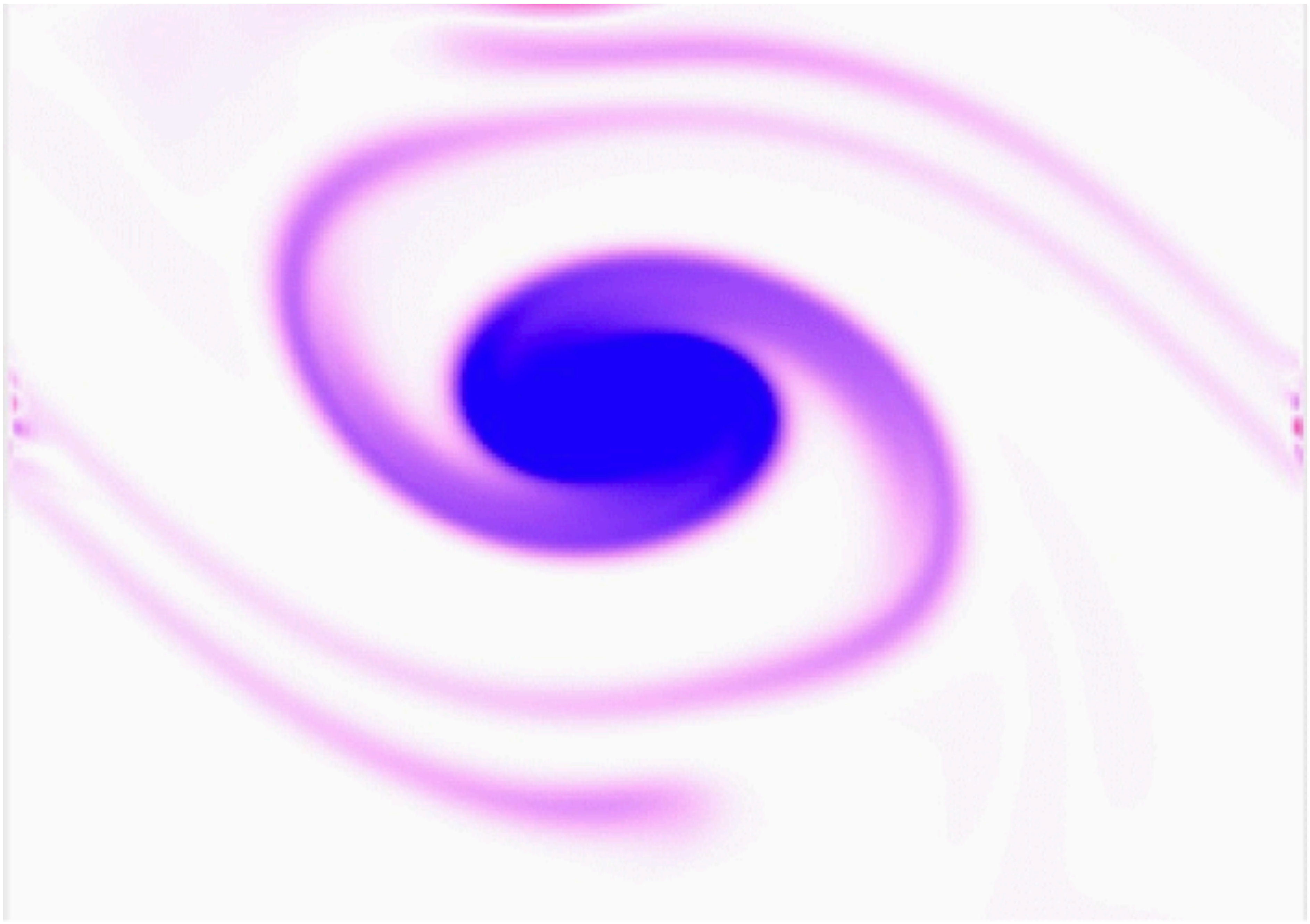


$$f_i(\vec{r}+\vec{e}_i\delta t, t+\delta t)-f_i(\vec{r},t) = -\frac{1}{\tau}\left(f_i-f_i^{eq}\right)$$



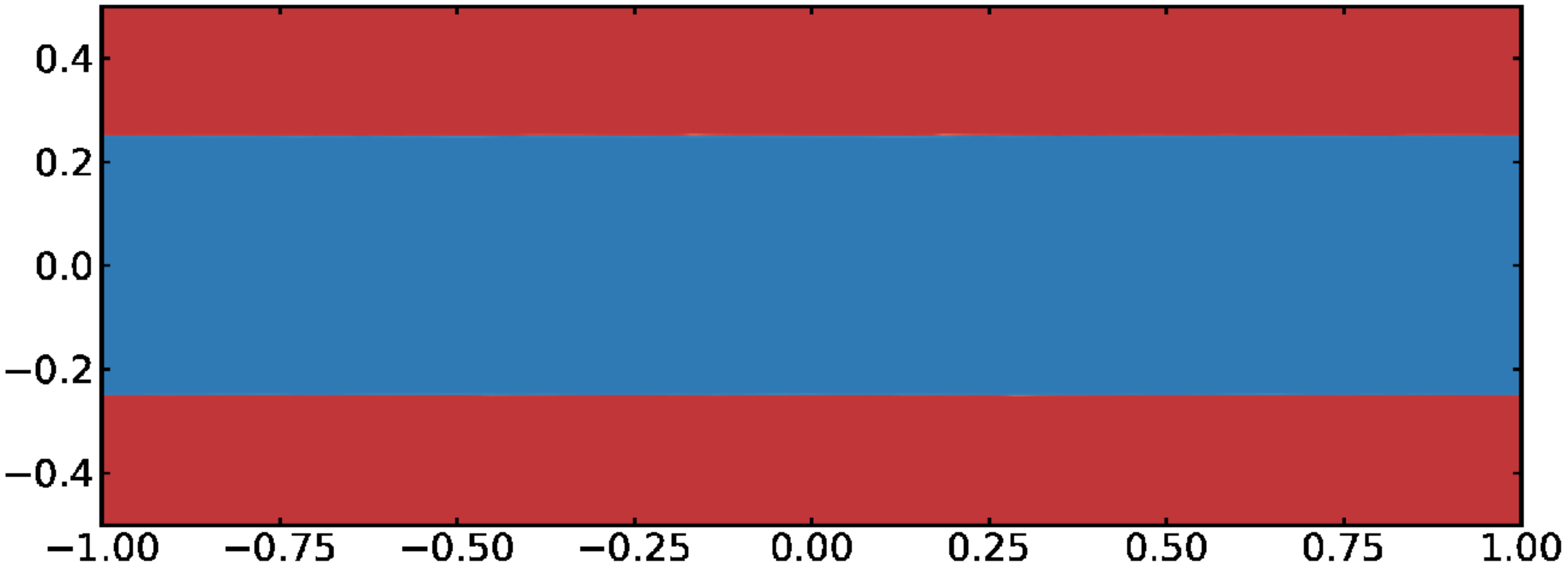
Lattice Boltzmann Method for Fluid Dynamics with Astrophysical Application

18307130056 Haiyang Wang(王海洋)

2022.4.13

Fluid Dynamics: Perfect for Illustration

Example: Kelvin-Helmholtz Instability



From Athena++ simulation

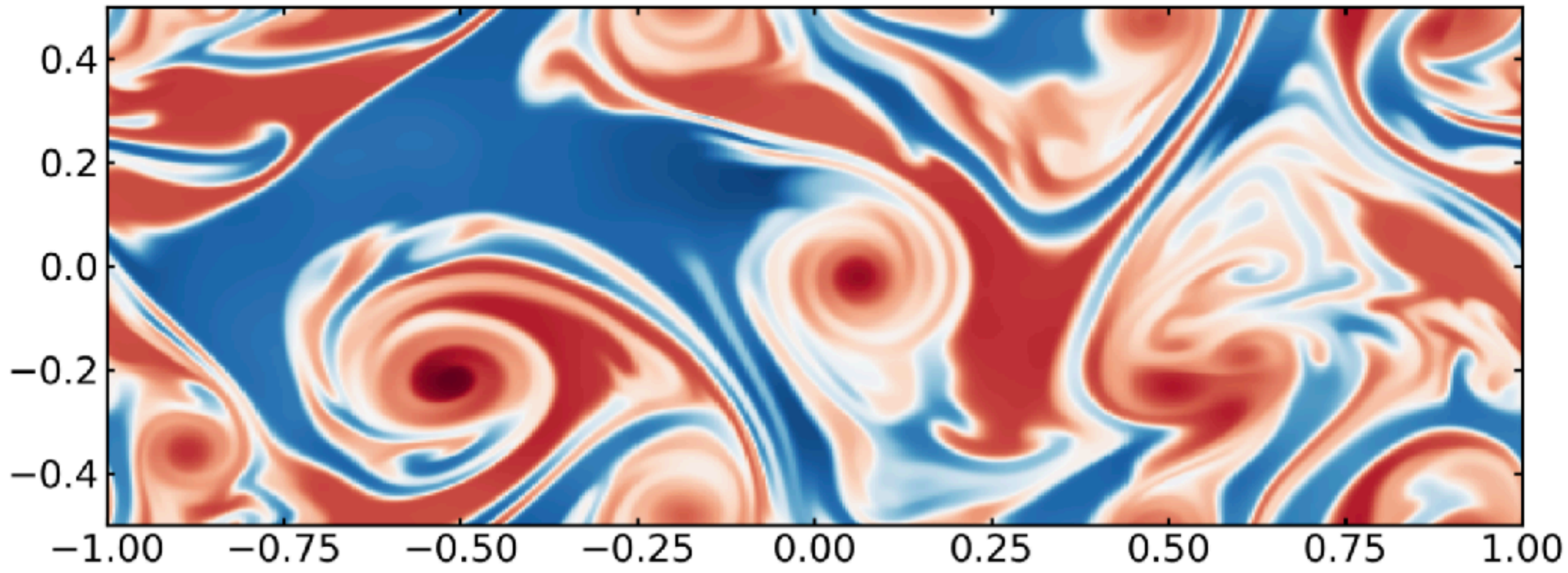
Finite-Volume Grid Code(C++), 8 CPU hours

Conventional Fluid Dynamics?

Continuity equation: $D\rho$

3 Nonlinear Equations

Energy equation: Isothermal/Adiabatic/General

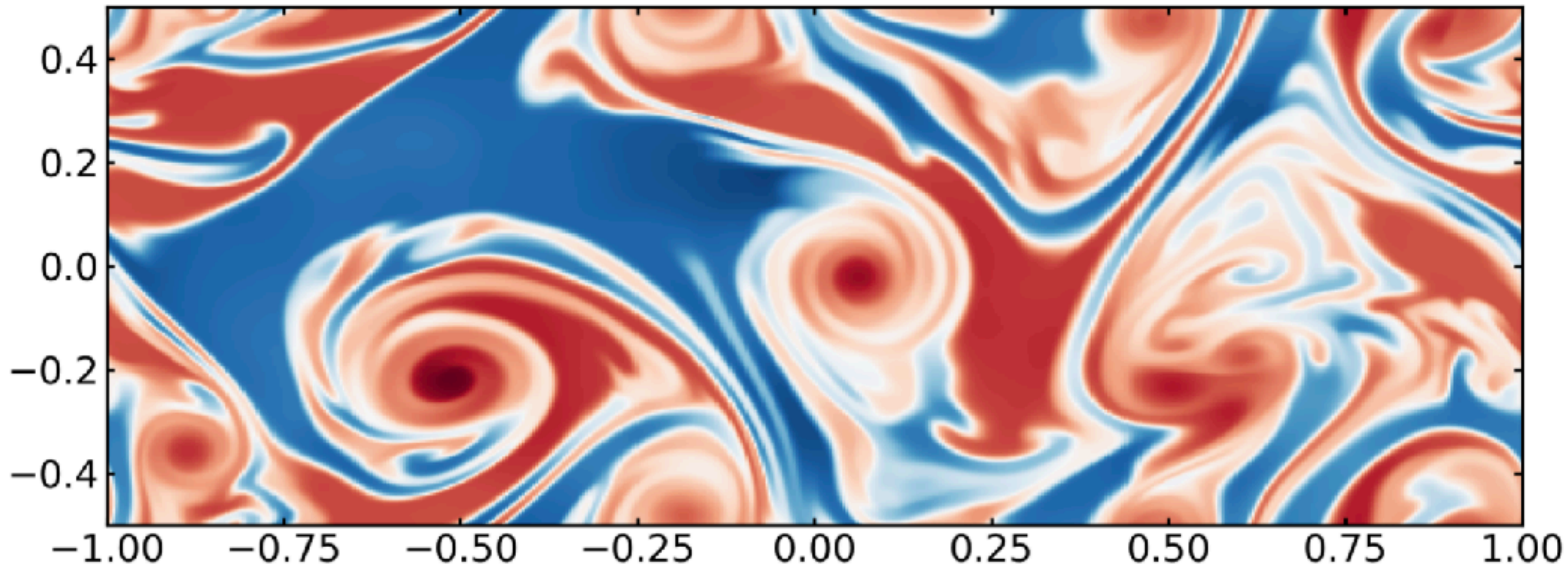


Conventional Fluid Dynamics?

Continuity equation: $\frac{D\rho}{Dt} = -\rho \nabla \cdot \boldsymbol{u}$

N-S equation: $\rho \frac{D\boldsymbol{u}}{Dt} = -\rho \nabla \Phi - \nabla P + \nabla \cdot \overleftrightarrow{T}$

Energy equation: Isothermal/Adiabatic/General



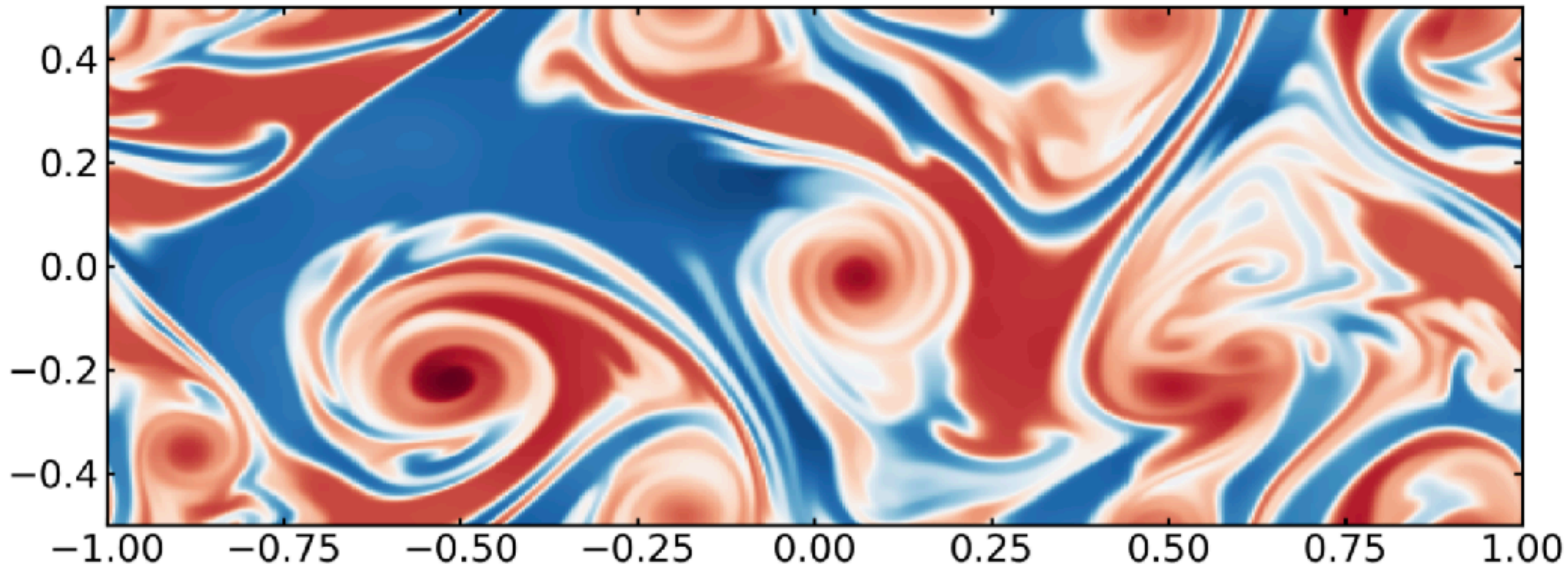
Conventional Fluid Dynamics?

Continu… $D\rho$ …

N…

Discretisation

Ener… …ral



Conventional Fluid Dynamics?

$$\frac{\partial \boldsymbol{U}}{\partial t} + \frac{\partial \boldsymbol{F}}{\partial x} = 0 \text{ (conservation form PDE, 7 variables, 7 waves)}$$

$$\boldsymbol{U} = \begin{bmatrix} \rho \\ M_x \\ M_y \\ M_z \\ E \\ B_y \\ B_z \end{bmatrix}, \quad \boldsymbol{F} = \begin{bmatrix} \rho v_x \\ \rho v_x^2 + P + B^2/2 - B_x^2 \\ \rho v_x v_y - B_x B_y \\ \rho v_x v_z - B_x B_z \\ (E + P^*) v_x - (\boldsymbol{B} \cdot \boldsymbol{v}) B_x \\ B_y v_x - B_x v_y \\ B_z v_x - B_x v_z \end{bmatrix}$$

$$U_i^{n+1/2} = U_i^n - \frac{\Delta t}{2\Delta x}\left(F_{i+1/2}^n - F_{i-1/2}^n\right)$$

$$U_i^{n+1} = U_i^n - \frac{\Delta t}{\Delta x}\left(F_{i+1/2}^{n+1/2} - F_{i-1/2}^{n+1/2}\right)$$

Conventional Fluid Dynamics?

$\frac{\partial U}{\partial} + \frac{\partial F}{} = 0$ (conservation form PDE. 7 variables. 7 wa

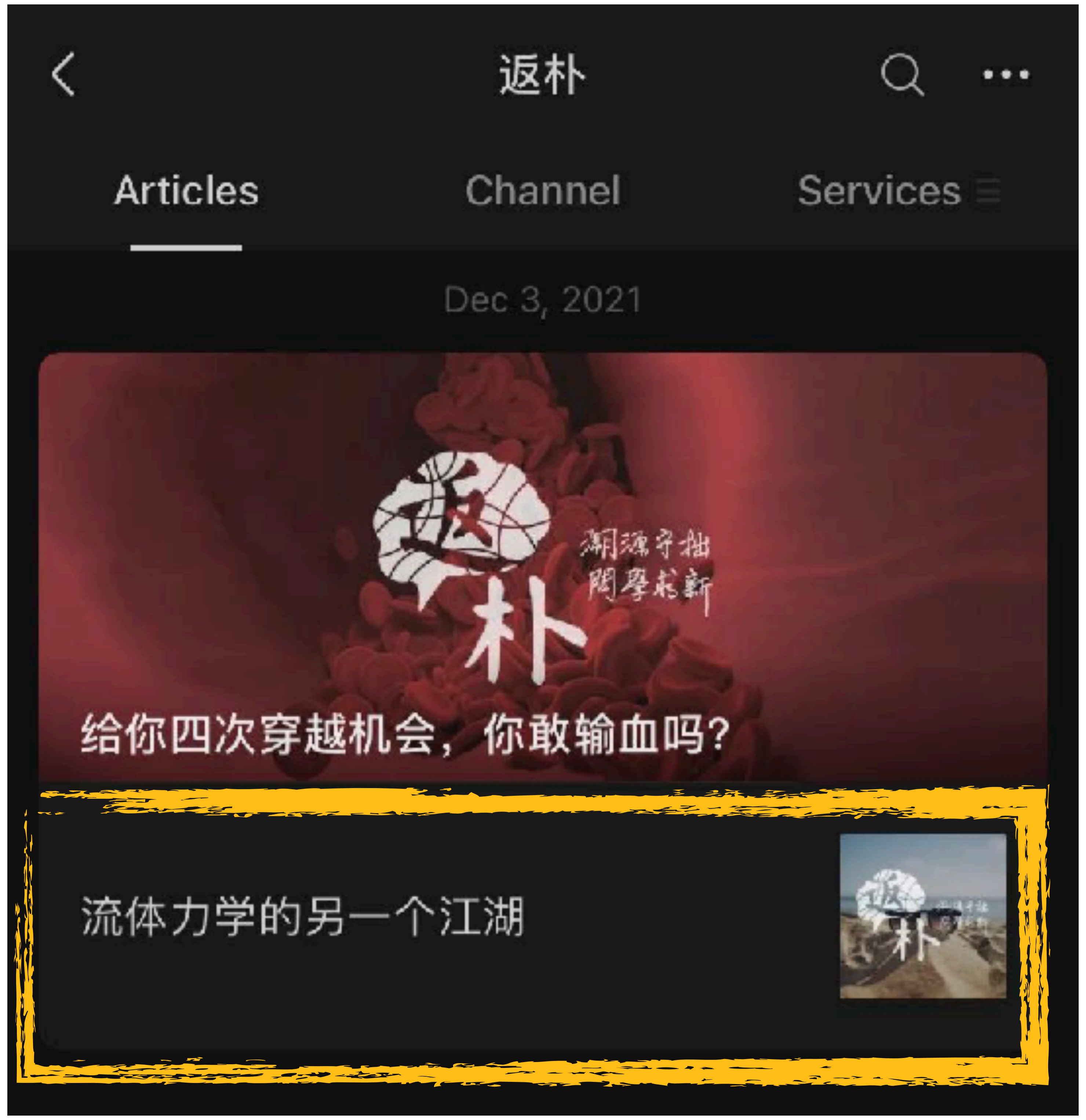
Way too complicated

Too slow for javascript

$\left[B_z \right]$ $\left[B_z v_x - B_x v_z \right]$

LBM: Our Savior!!!

Lattice Boltzmann Method(晶格玻尔兹曼法)



Mesoscopic

Discretised sub-grid distribution

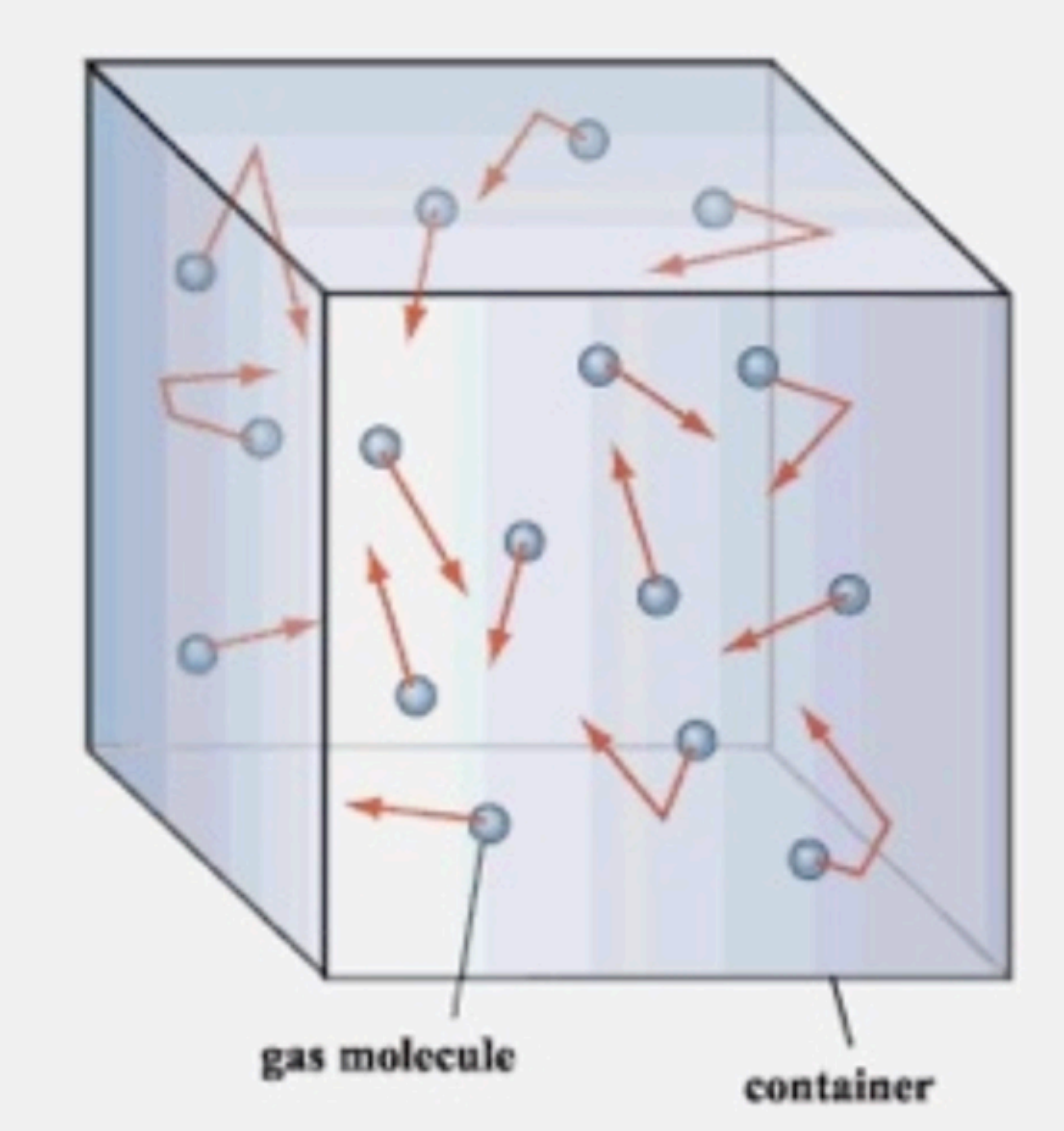
Macroscopic

(NS equation)

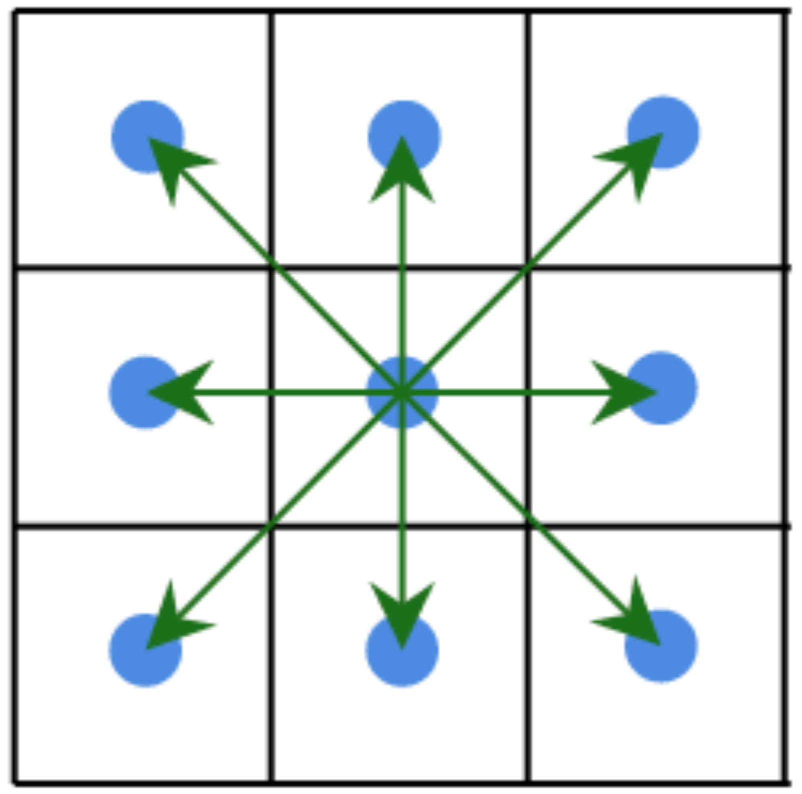
Microscopic

(Kinetic equation)

Lattice Boltzmann Method: Algorithm



Random direction



$\vec{e}_0 = (0, 0)$

$\vec{e}_E = (1, 0)$

$\vec{e}_E = (0, 1)$

$\vec{e}_W = (-1, 0)$

$\vec{e}_S = (0, -1)$

$\vec{e}_{NE} = (1, 1)$

$\vec{e}_{NW} = (-1, 1)$

$\vec{e}_{SW} = (-1, -1)$

$\vec{e}_{SE} = (1, -1)$

Certain directions: D2Q9

Sub-grid particle ensemble can only go to discretized direction in one timestep

Lattice Boltzmann Method: Algorithm

Initialisation $\rho = \sum_i f_i, \ \vec{u} = \sum_i \frac{f_i \vec{e}_i}{\rho}$

↓

"Stream"

$$f_i\left(\vec{r} + \vec{e}_i \delta t, t + \delta t\right) = f_i(\vec{r}, t)$$

↓

"Thermal equilibrium"

$$F(\vec{v}) = \frac{m}{2\pi kT} \exp\left[-\frac{m|\vec{e}_i c - \vec{u}|^2}{2kT}\right]$$

"Collide"

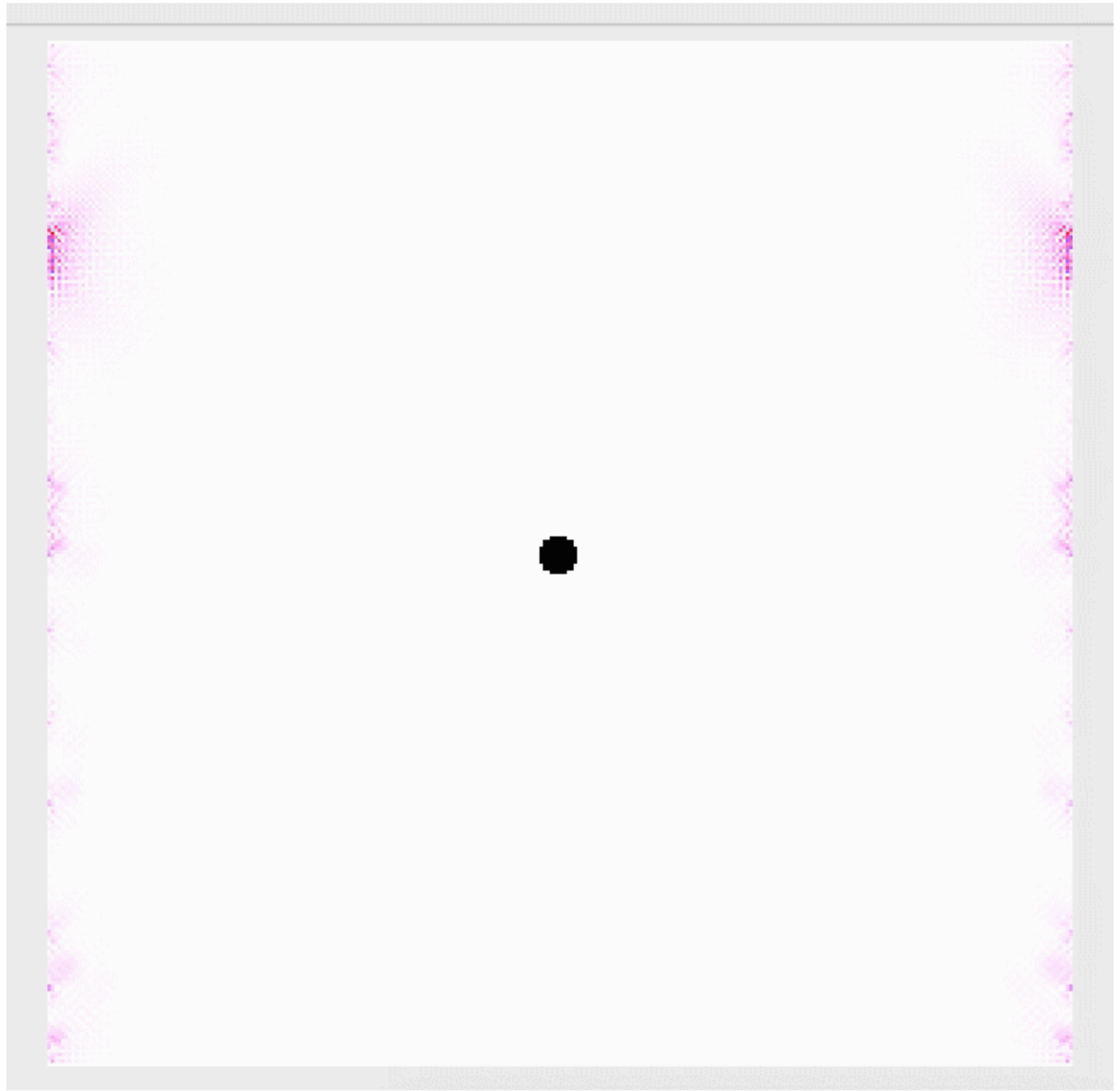
$$f_i(\vec{r} + \vec{e}_i \delta t, t + \delta t) - f_i(\vec{r}, t) = -\frac{1}{\tau}\left(f_i - f_i^{eq}\right)$$

← Macroscopic Quantities

"What you see"

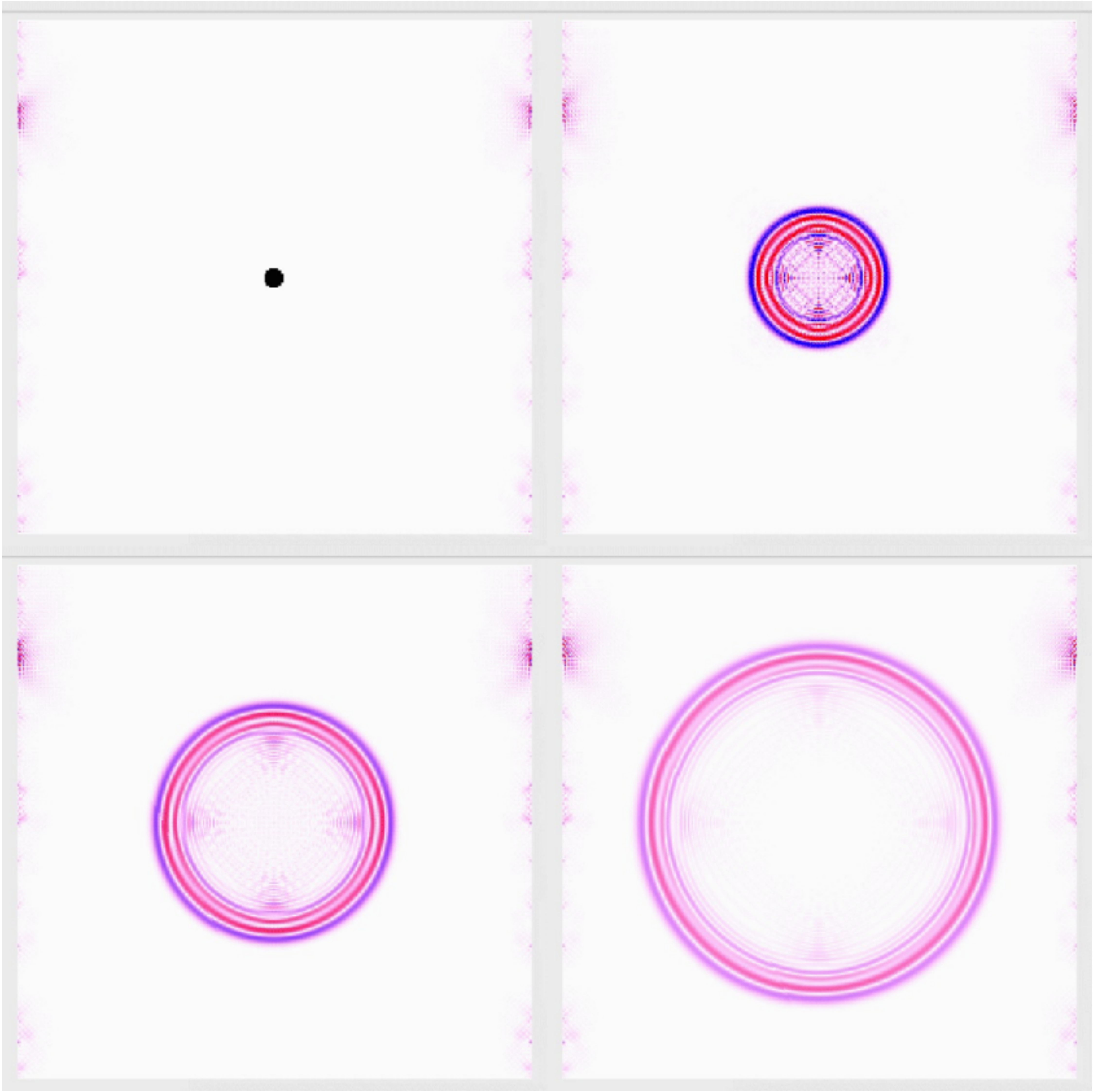
Algorithm Implementation

LBM in action - Results: I. Supernova explosion

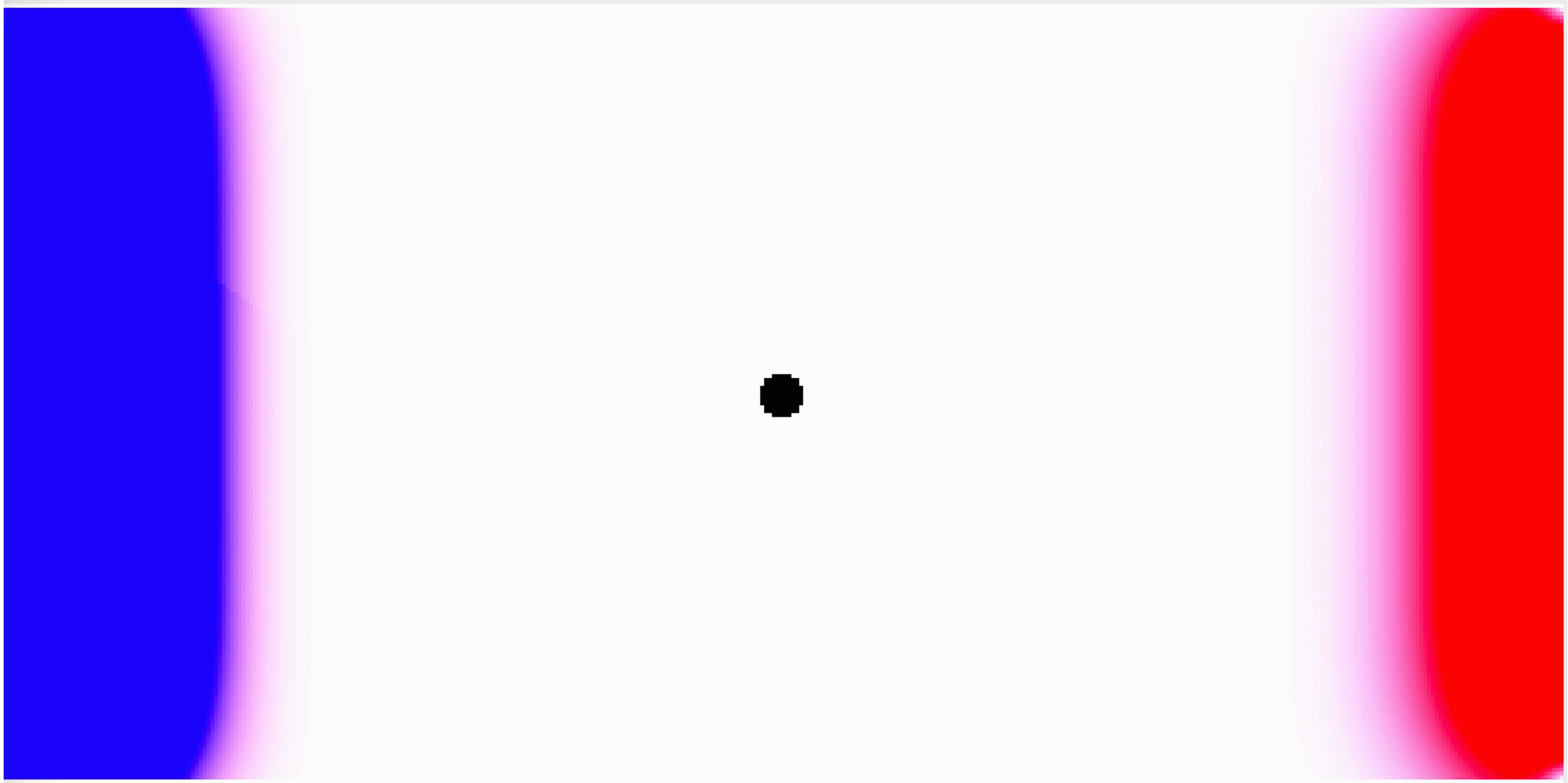


2-dimensional spherical shock

LBM in action - Results: I. Supernova explosion

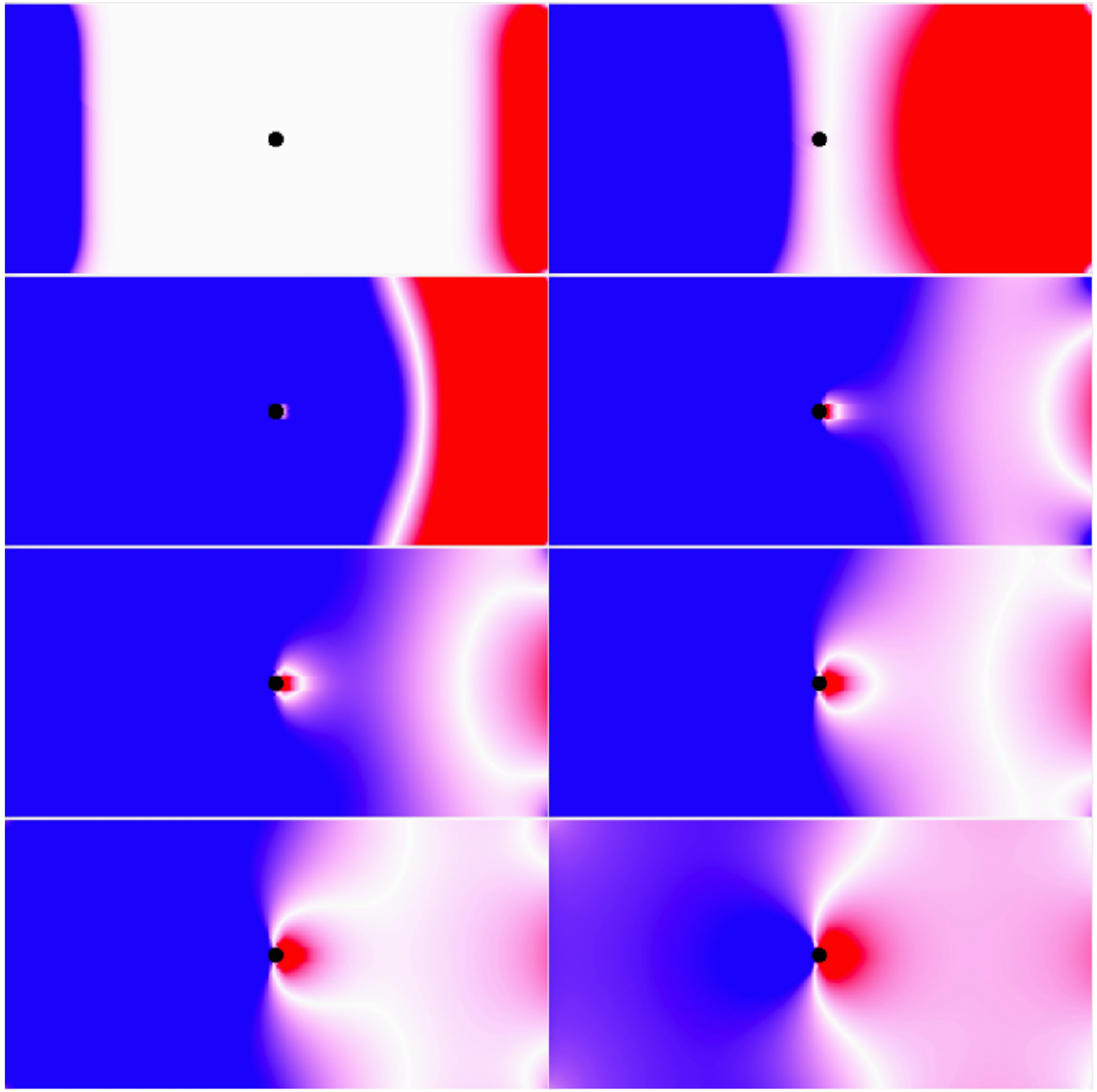


LBM in action - Results: II. Bondi-Hoyle-Lyttleton accretion

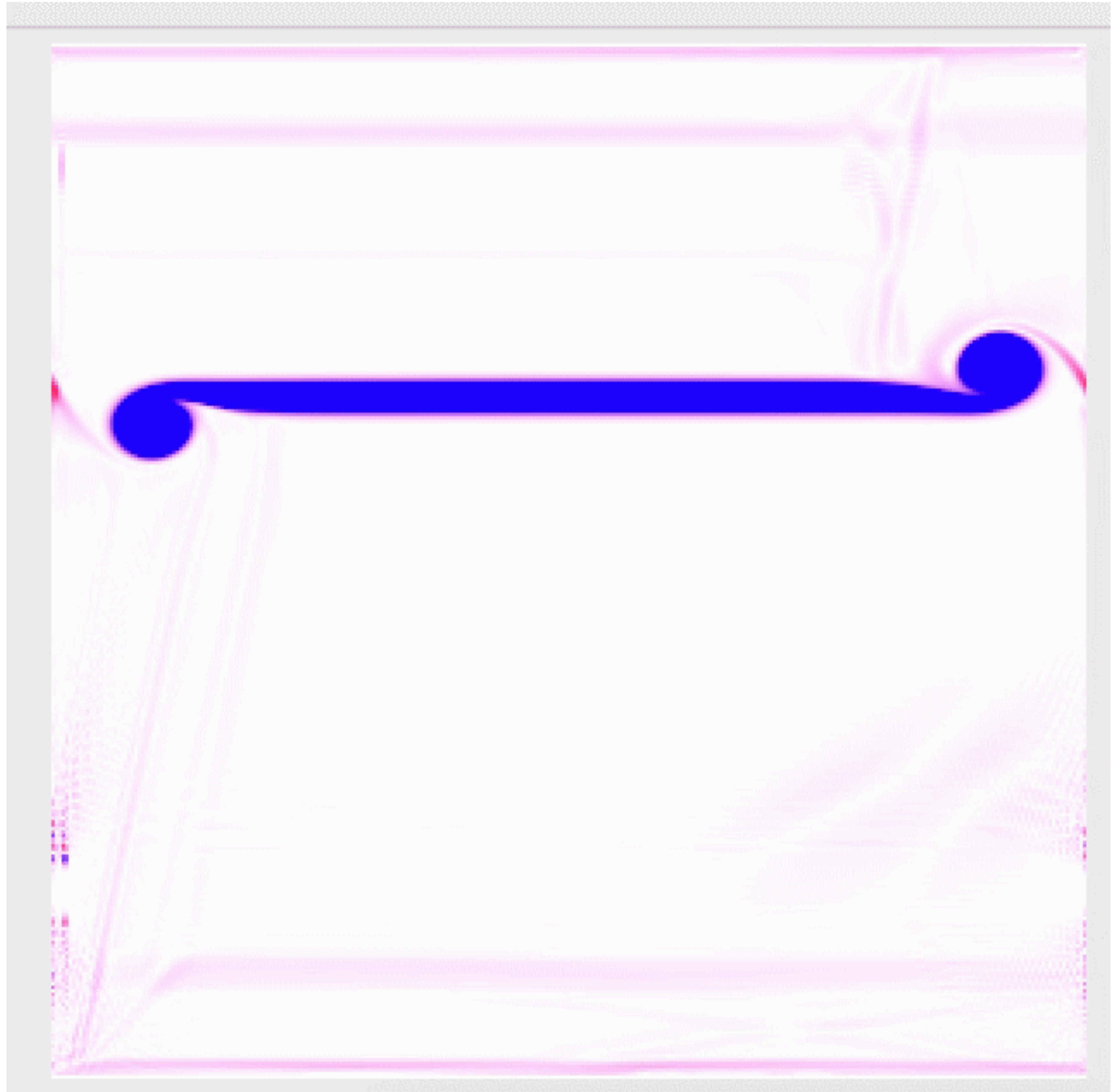


Planet encounters solar wind

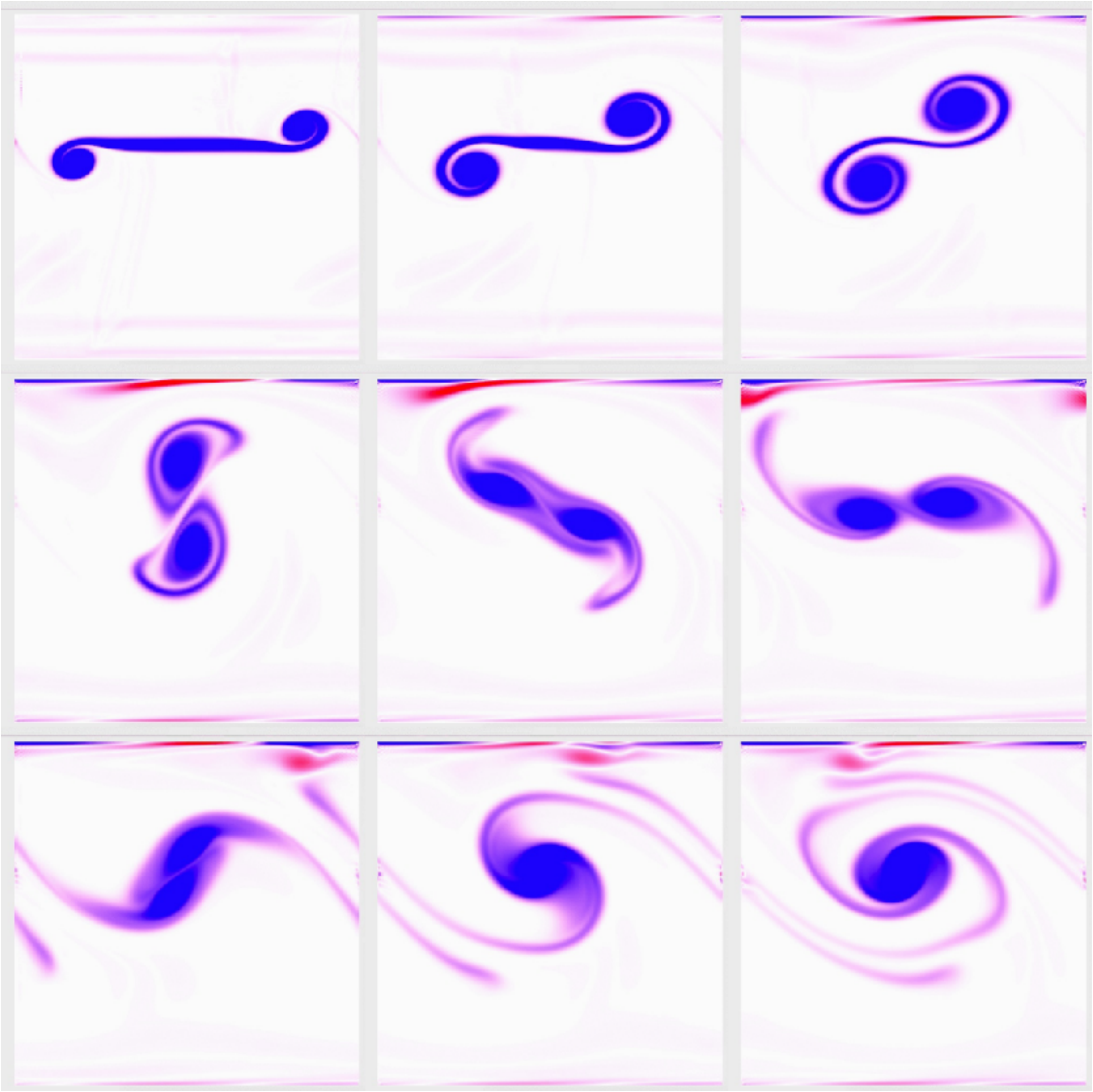
LBM in action - Results: II. Bondi-Hoyle-Lyttleton accretion



LBM in action - Results: III. Vorticity Presevation



LBM in action - Results: III. Vorticity Presevation



Summary

1. **Lattice Boltzmann Method:** extremely powerful for in-time simulation

2. **Algorithm implementation finished:** viscous, turbulent "streaming box"

3. **If you love equations, which are not included in the slides...**

   See my uploaded note for

   1. Details of LBM algorithm, and details of algorithm implementation
   2. Validity of LBM for fluid dynamics
   3. Future direction & Astrophysical applications

Caveats & TODO

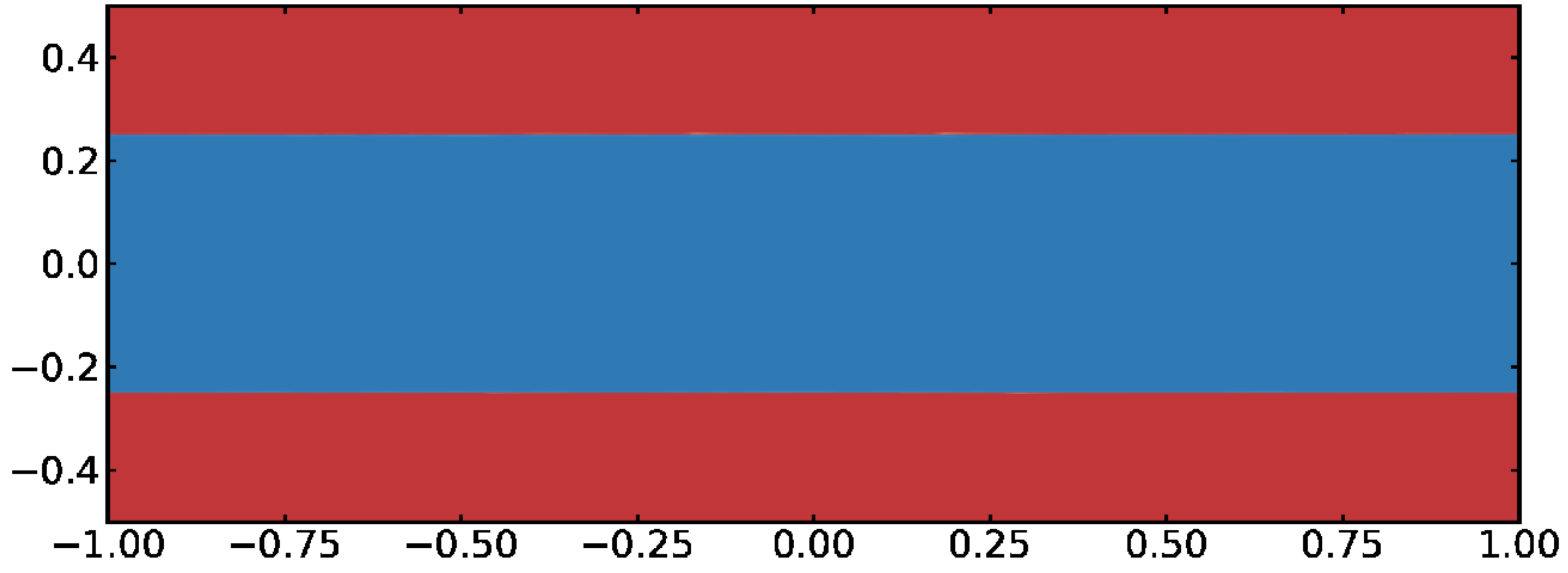
1. **No web design:** No html5 development so far

2. **Slow flow:** LBM intrinsically is not suitable for high Mach number flow

3. **Bugs...** (Some bugs are hiding in boundary conditions)

Caveats & TODO

1. Kelvin-Helmholtz Instability



(From Athena++ simulation)

2. Shearing box implementation
   (Local study for Circumstellar disk)