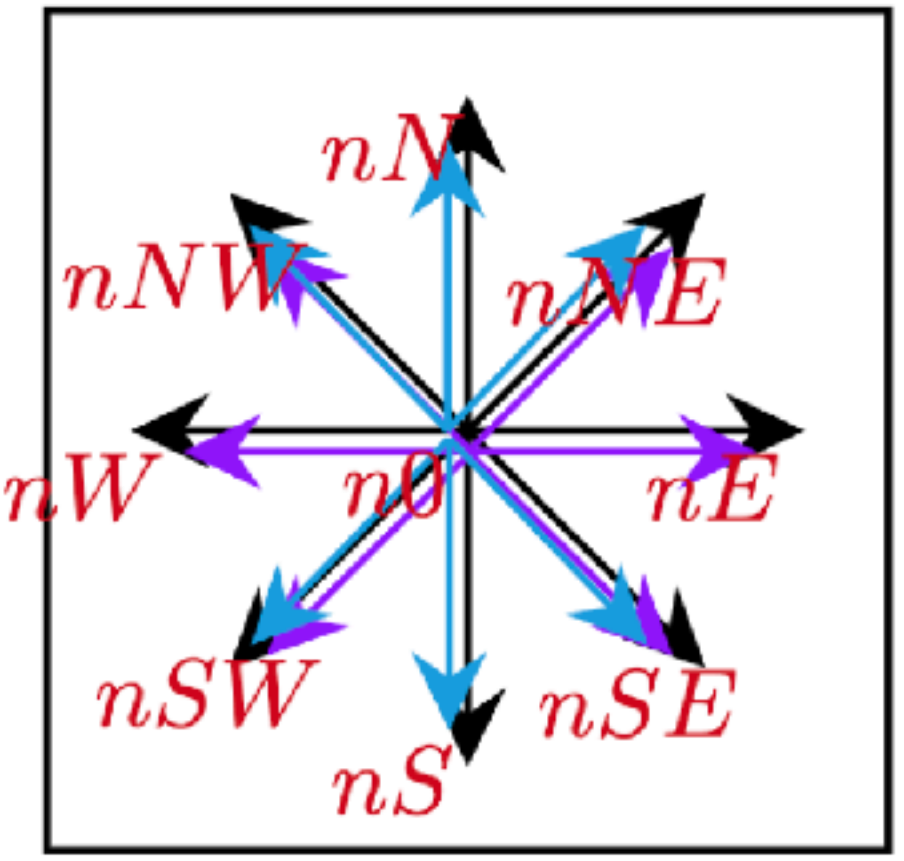
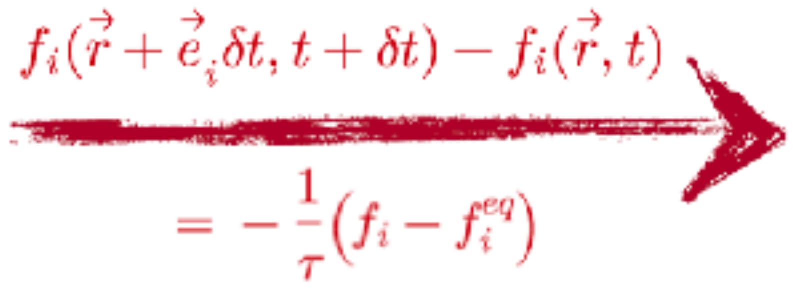
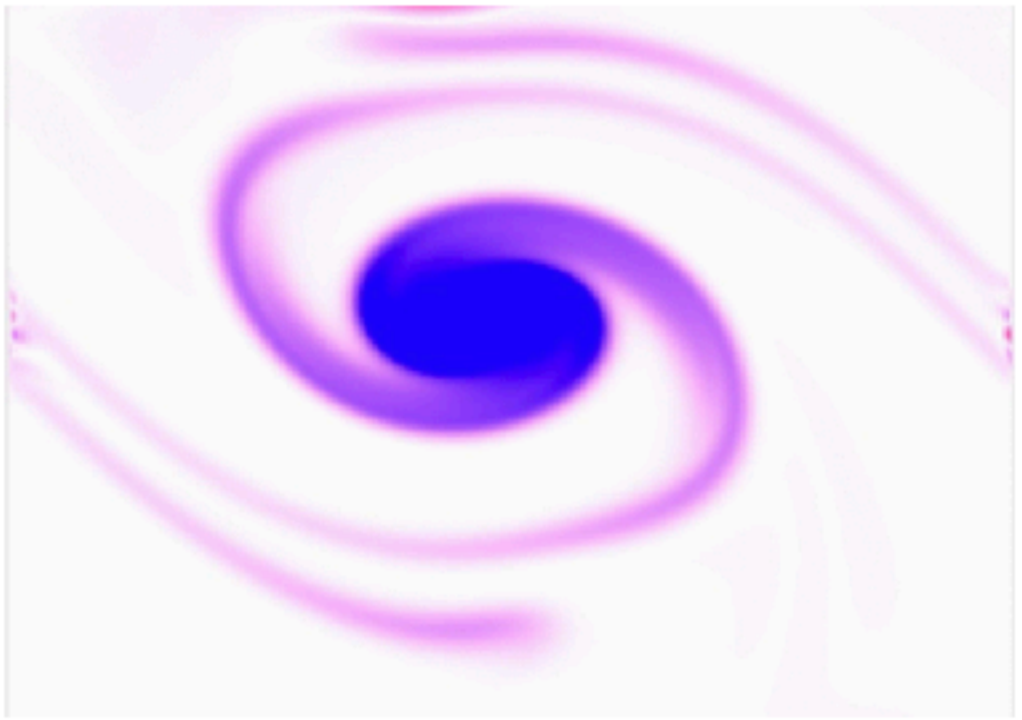
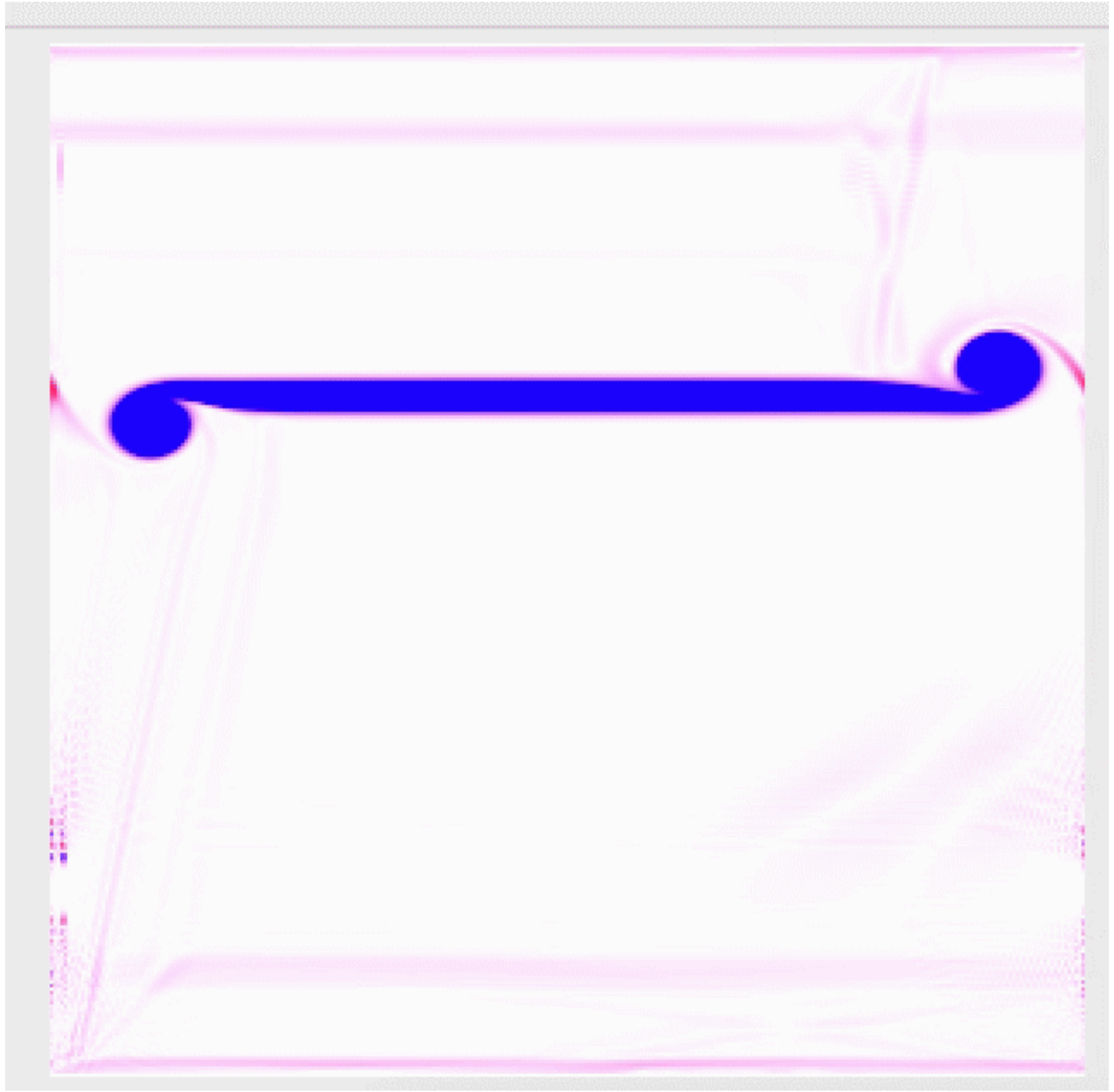
Circumbinary discs: Linear Theory For AM Transfer

2022.5.26 Haiyang Wang





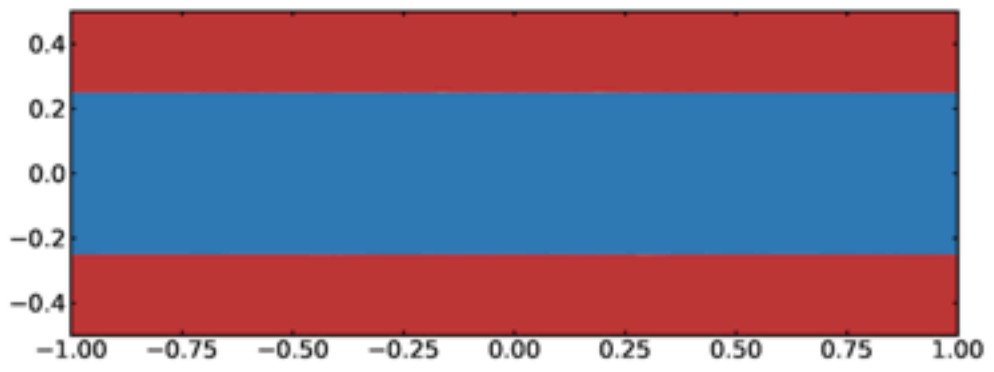




Lattice Boltzmann Method for Fluid Dynamics with Astrophysical Application

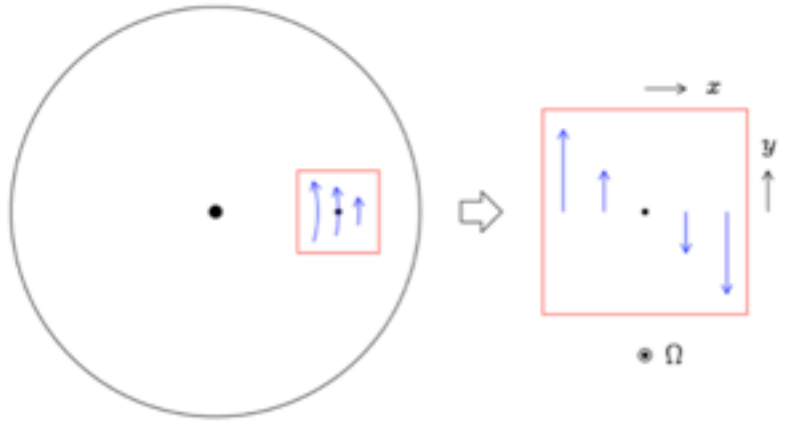
Caveats & TODO

1. Kelvin-Helmholtz Instability



(From Athena++ simulation)

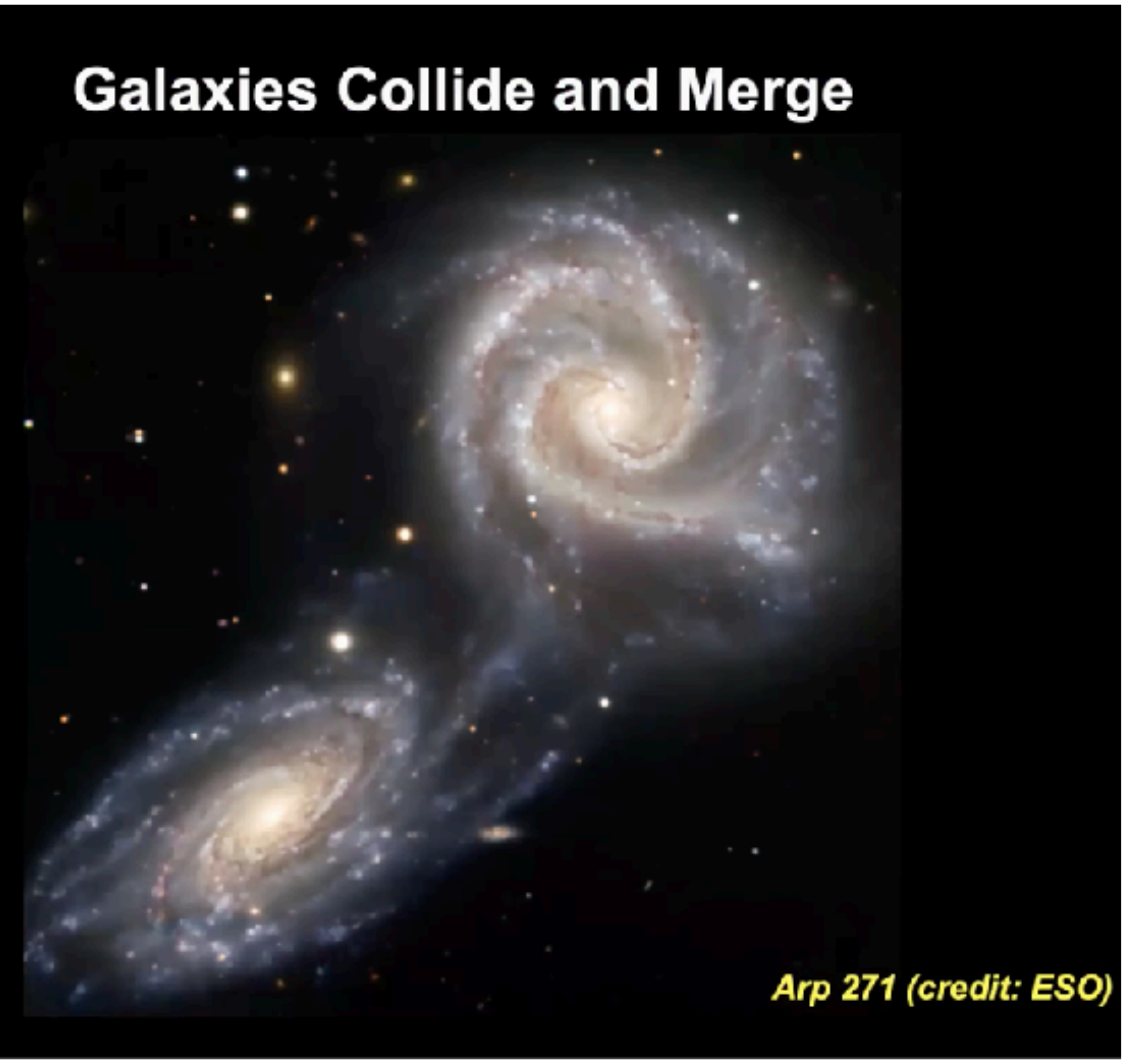
2. Shearing box implementation
   (Local study for Circumstellar disk)

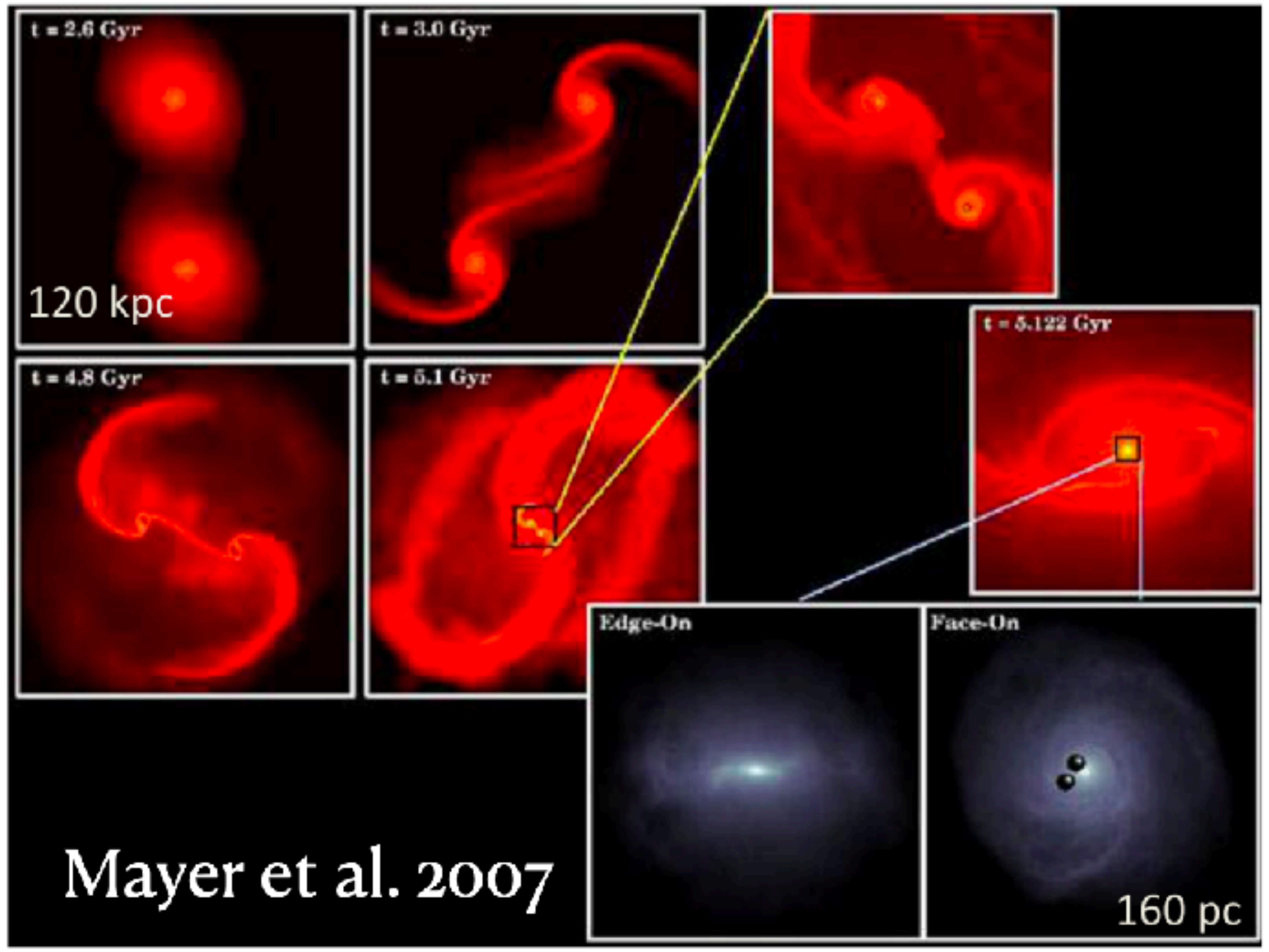


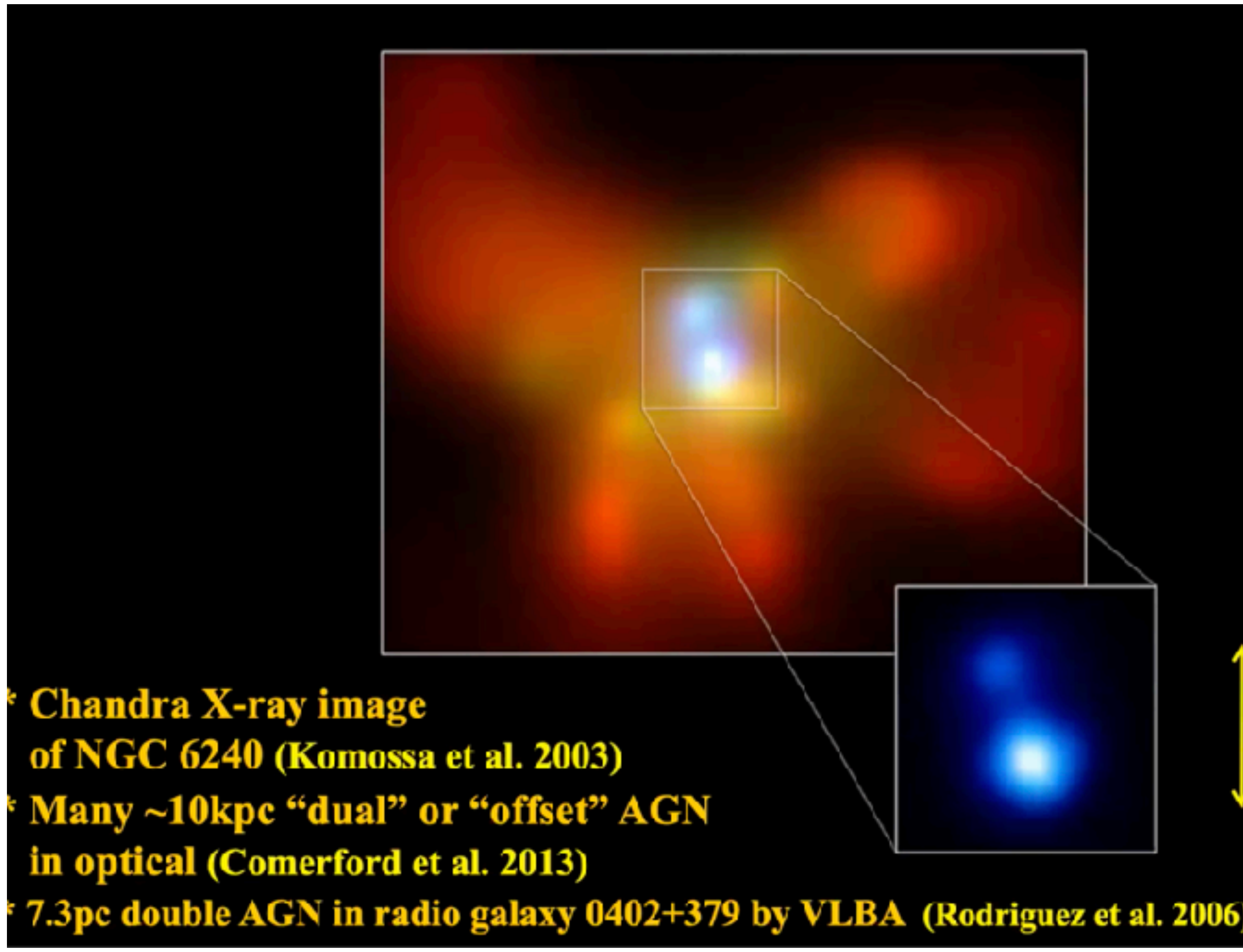
Why we are interested in CBDs?

Supermassive Binaries Black Hole(SMBBHs)

"Final parsec" Problem/how to bring SMBBHs together?







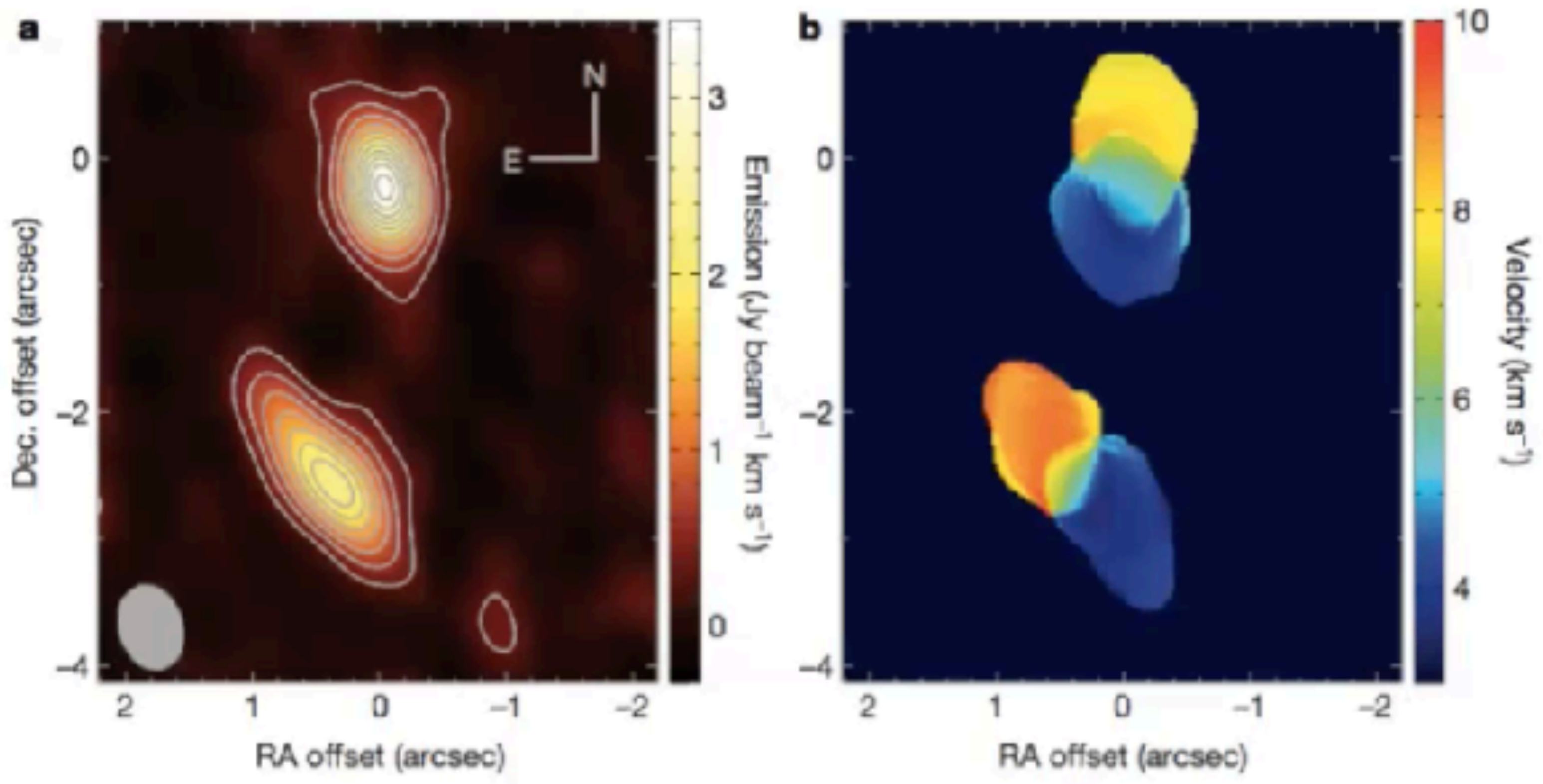
Why we are interested in CBDs?

Young Stellar Binaries

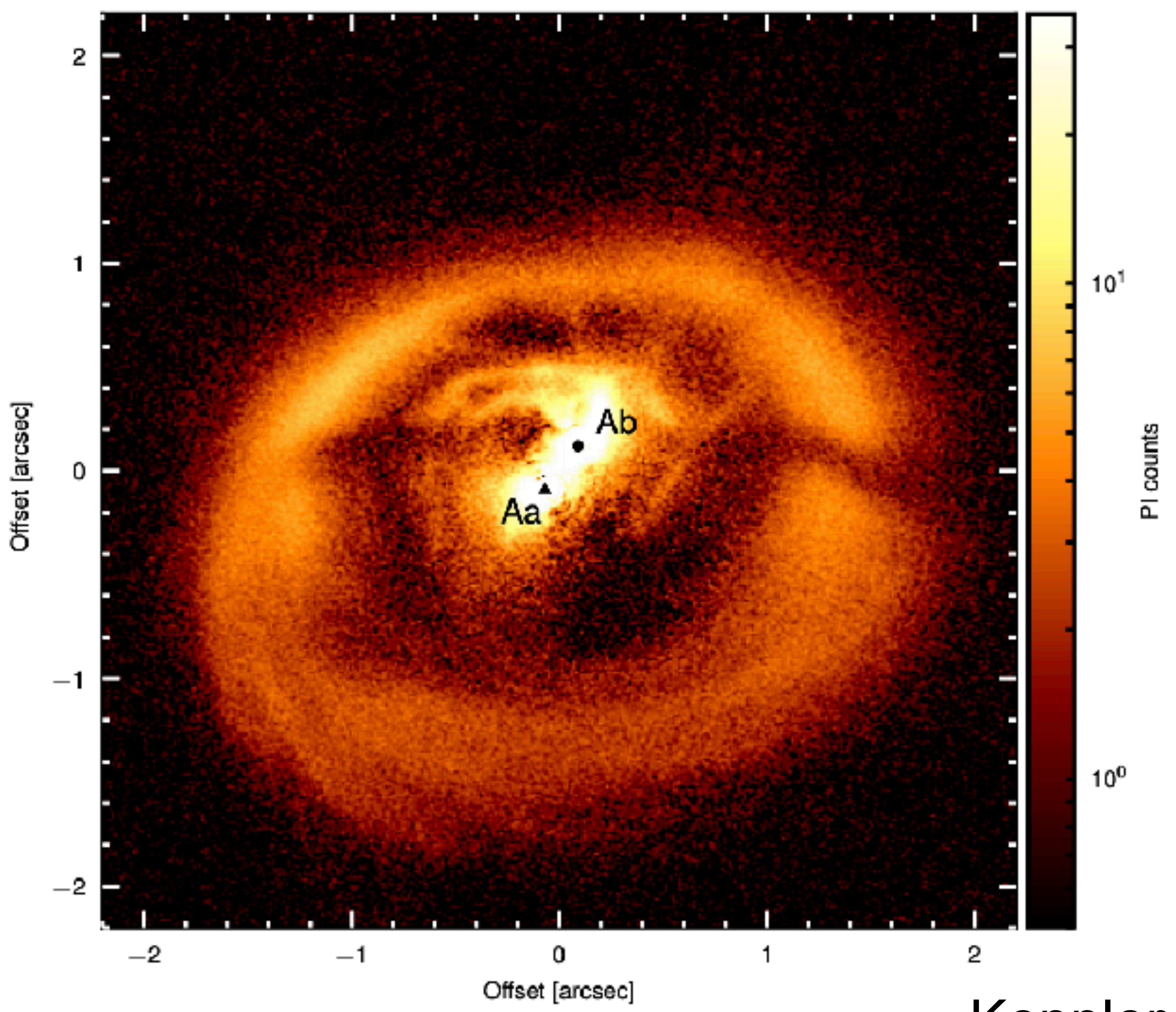
Rich in dynamical processes; enormous parameter space(e, q, h, I…)

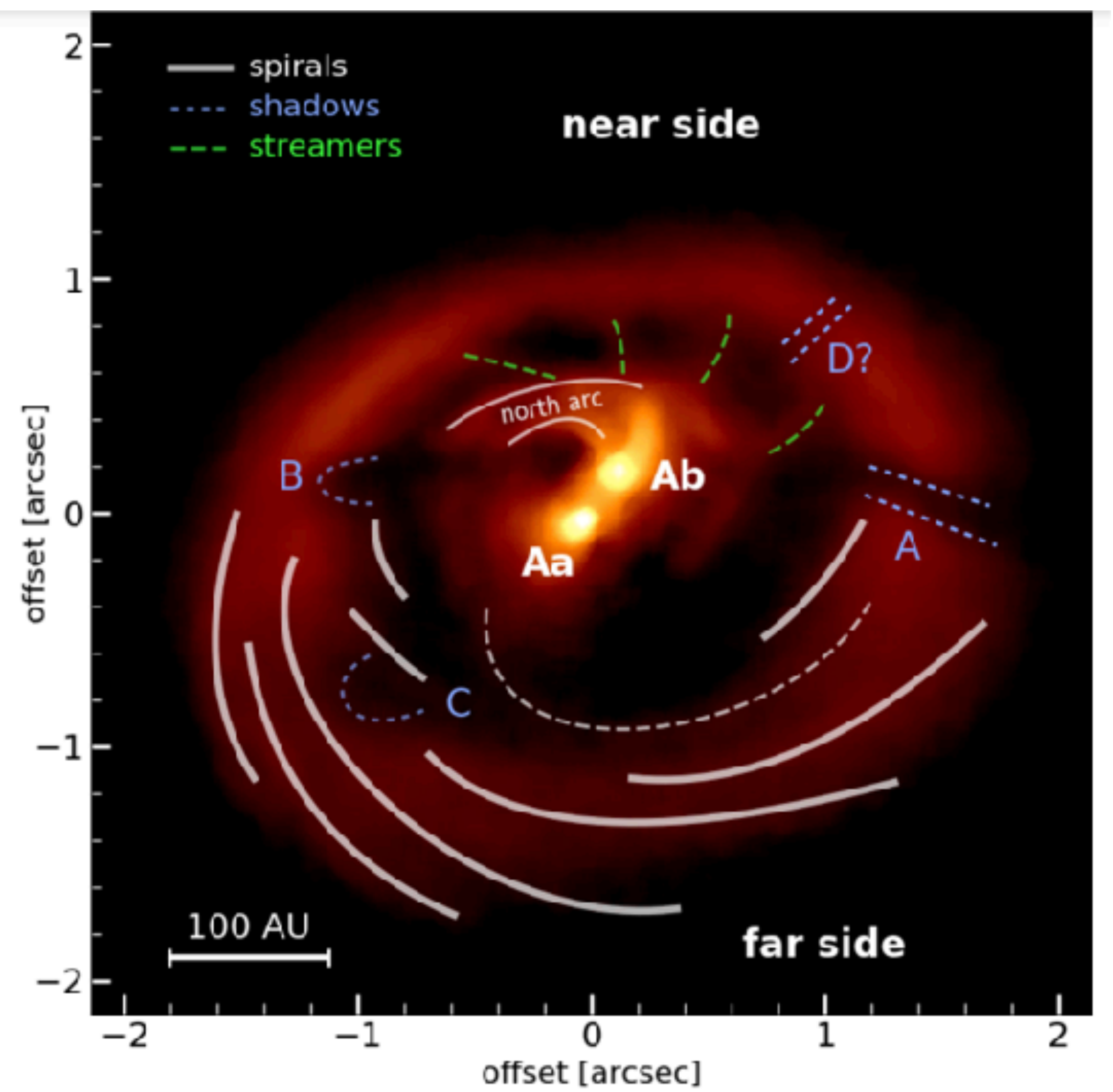
circumstellar discs

HK Tauri (a_b = 350AU)



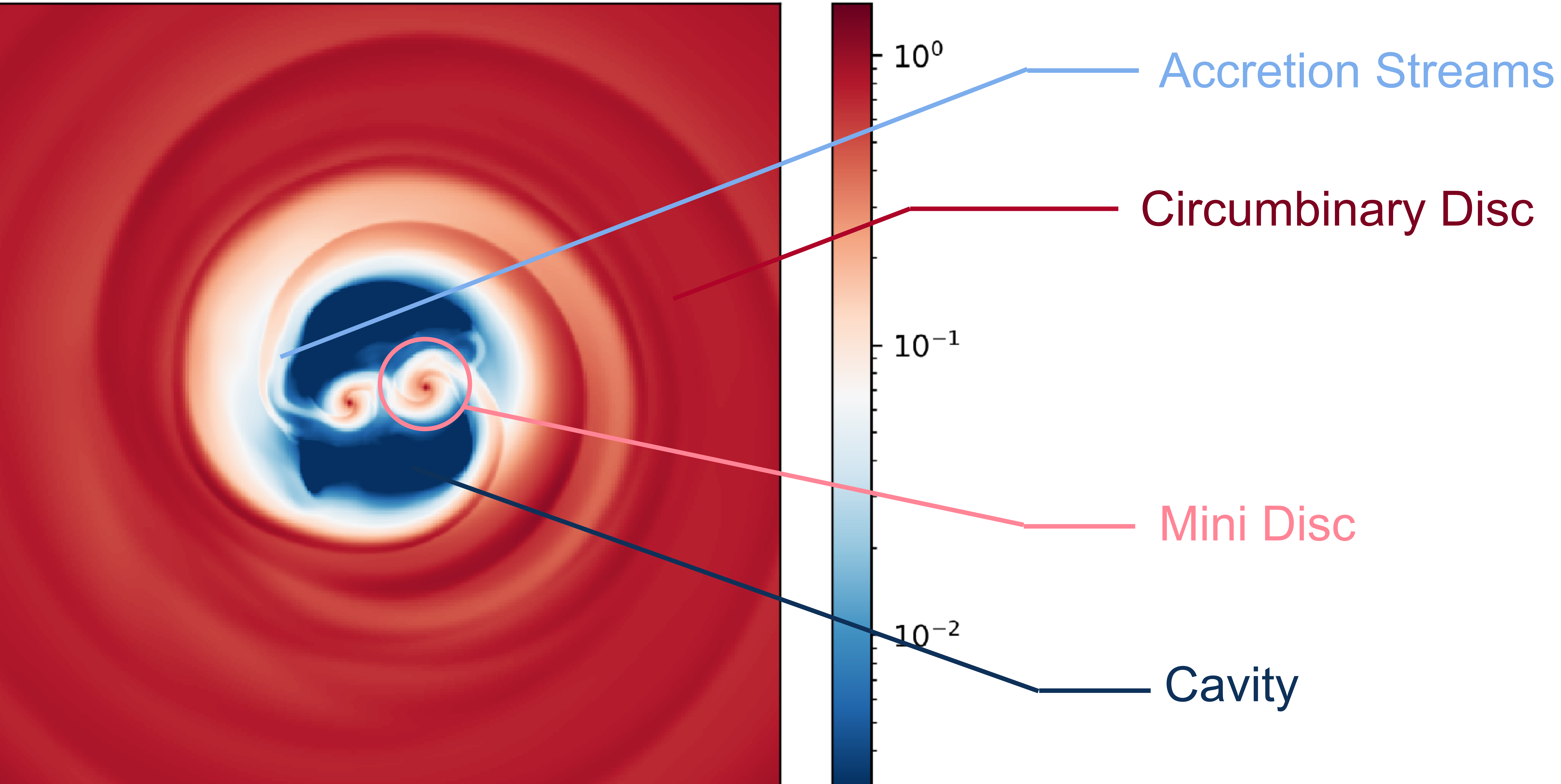
(Brinch et al. 2016)





Keppler et al. 2020, SPHERE Observation of GG Tau

CBD-Binary interaction



I. Hydrodynamics Simulations


Before 2016 or so
Inspiral
Recently
Outspiral
More Recently🤣
(e,h,M1/M2)
Inspiral
End?
/Endless debates?

Resonances: Coplanar Binaries

Decomposing the gravitational potential

$$\Phi(r,\theta,t) = \sum_{ml} \phi_{ml}(r)\exp[i(m\theta - l\Omega_B t)],$$

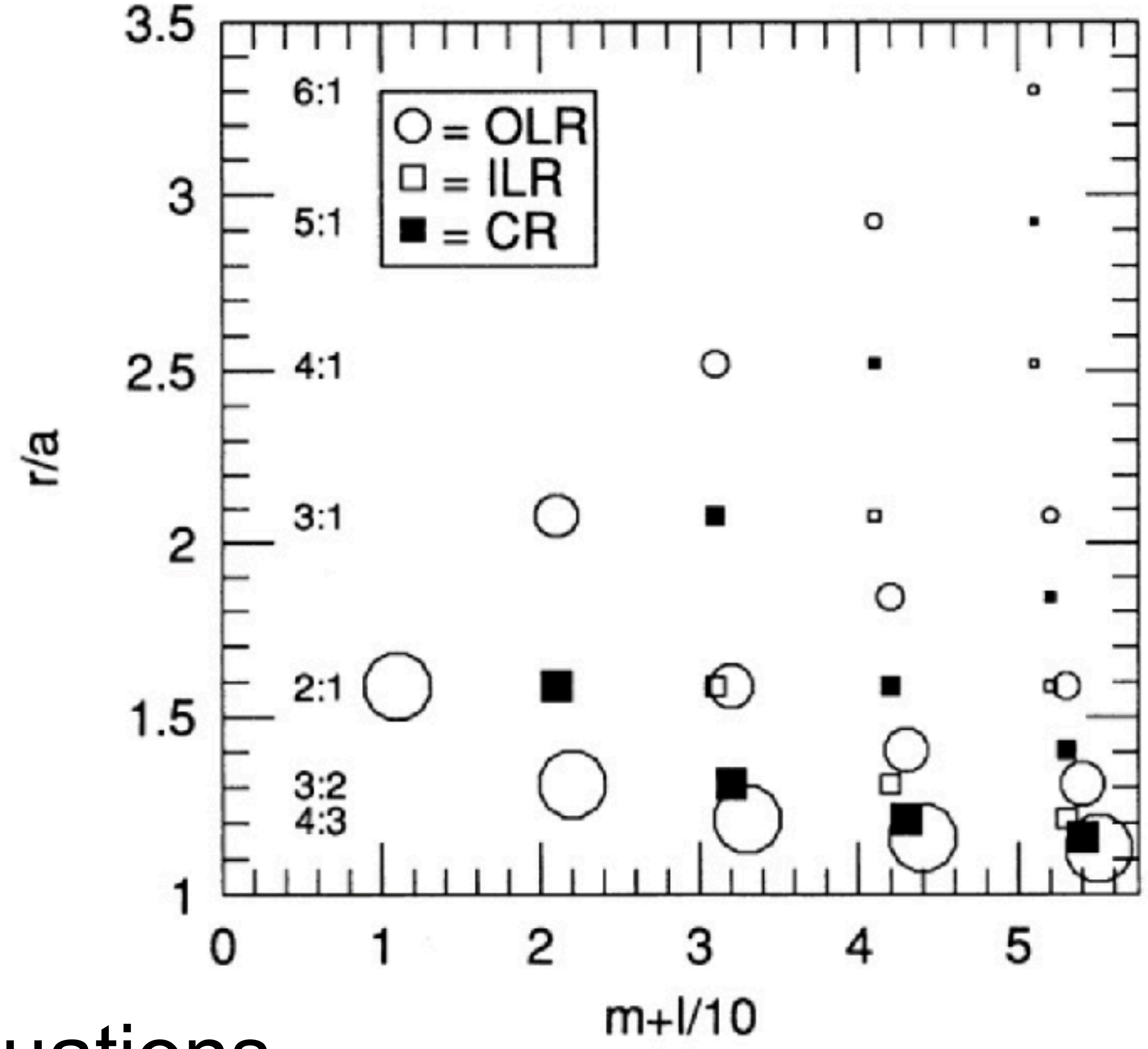
where $\Phi = -\sum_{j=1}^{j=2} \mu_j GM / |r - r_j|,$

The coefficients have the form:

$$\phi_{ml} = \frac{1}{2\pi^2}\int_0^{2\pi} d(\Omega_B t)\int_0^{2\pi} d\theta\Phi\cos(m\theta - l\Omega_B t).$$

This is what we put in the excitation&eigenmode equations

Strength/Location of Resonances



Artymowicz & Lubow, 1994

'Gap opening criteria' $3\pi\alpha_v(H/r)^2 = T/(\sigma\Omega^2 r^4),$

Linear Analysis: Multiple Spiral Arms in CBD

Continuity equation: $$\frac{D\rho}{Dt} = -\rho\nabla\cdot\boldsymbol{u}$$

N-S equation: $$\rho\frac{D\boldsymbol{u}}{Dt} = -\rho\nabla\Phi - \nabla P + \nabla\cdot\overset{\leftrightarrow}{T}$$

Energy equation: Isothermal/Adiabatic/General

Linear Analysis: Multiple Spiral Arms in CBD

$$
\begin{cases}
\dfrac{\partial u_1}{\partial t} + \Omega \dfrac{\partial u_1}{\partial \varphi} - 2\Omega v_1 = -\dfrac{\partial \Phi}{\partial r} - \dfrac{1}{\Sigma_0}\dfrac{\partial p_1}{\partial r} + \dfrac{\Sigma_1}{\Sigma_0^2}\dfrac{\partial p_0}{\partial r} \\
\dfrac{\partial v_1}{\partial t} + \Omega \dfrac{\partial v_1}{\partial \varphi} + \dfrac{\kappa^2}{2\Omega} u_1 = -\dfrac{1}{r}\dfrac{\partial}{\partial \varphi}\left(\Phi + \dfrac{p_1}{\Sigma_0}\right) \\
\dfrac{\partial \Sigma_1}{\partial t} + \Omega \dfrac{\partial \Sigma_1}{\partial \varphi} + \dfrac{1}{r}\dfrac{\partial}{\partial r}(r\Sigma_0 u_1) + \dfrac{1}{r}\dfrac{\partial}{\partial \varphi}(\Sigma_0 v_1) = 0. \\
\dfrac{\partial S_1}{\partial t} + \Omega \dfrac{\partial S_1}{\partial \varphi} + u_1 \dfrac{\partial S_0}{\partial r} = 0
\end{cases}
$$

Standard procedure of linear analysis

↓ $\Psi = p_1 / \Sigma_0$

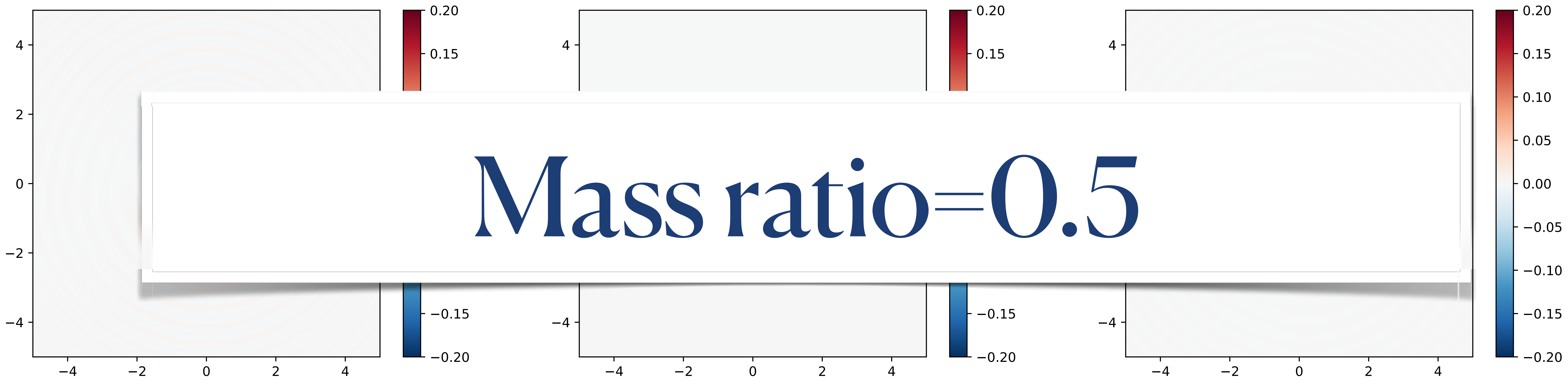
$$
r^2(\Phi + \Psi)'' + r(\mathcal{B} + \mathcal{S})(\Phi + \Psi)' - r\mathcal{S}\Psi' + \mathcal{C}\Psi + \mathcal{D}\Phi = 0
$$

Eigen-Mode equation

Strictly solve for all modes, ∀m=0,±1,±2,…

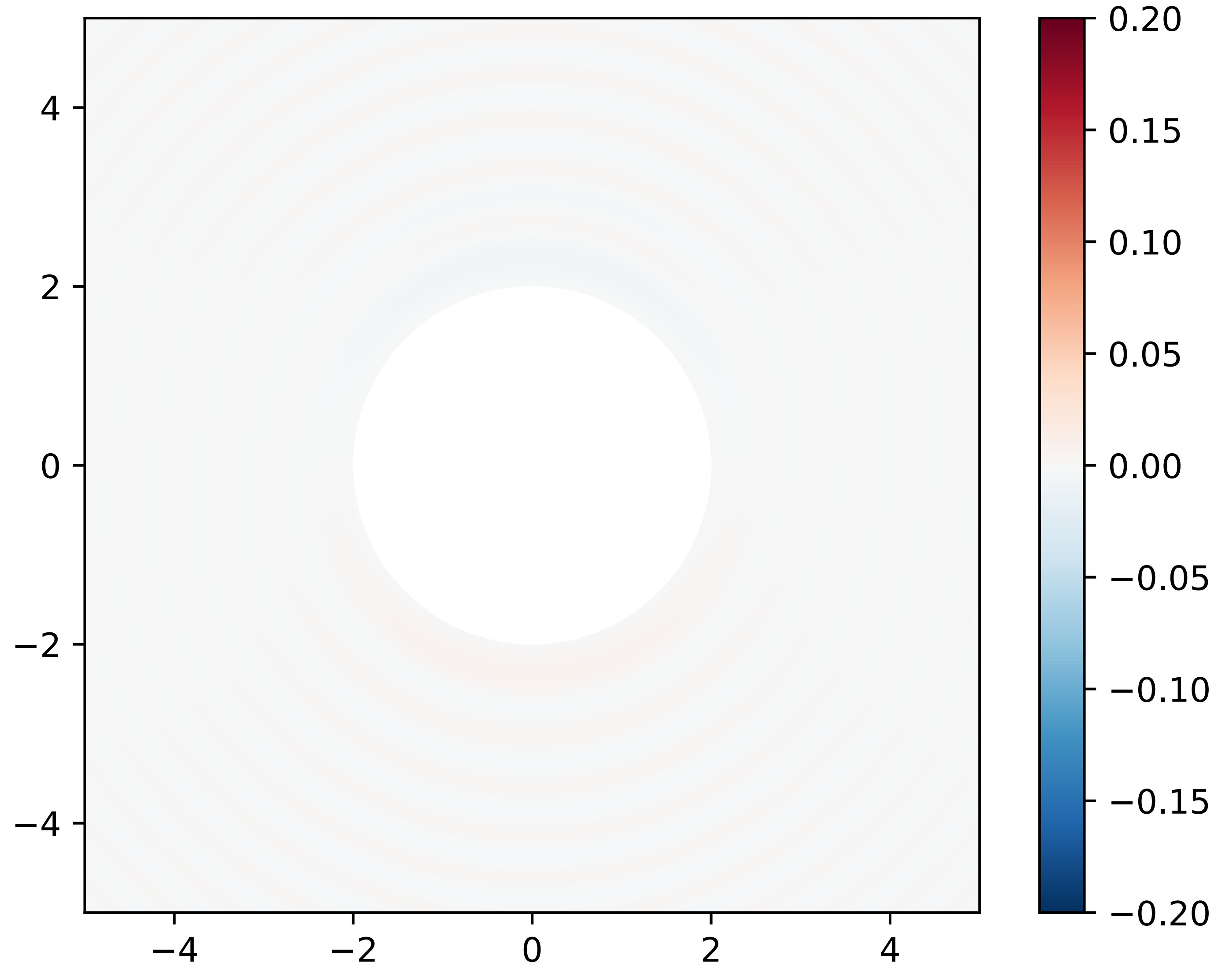
Linear Calculation Results: Adding up the Modes

m = 1

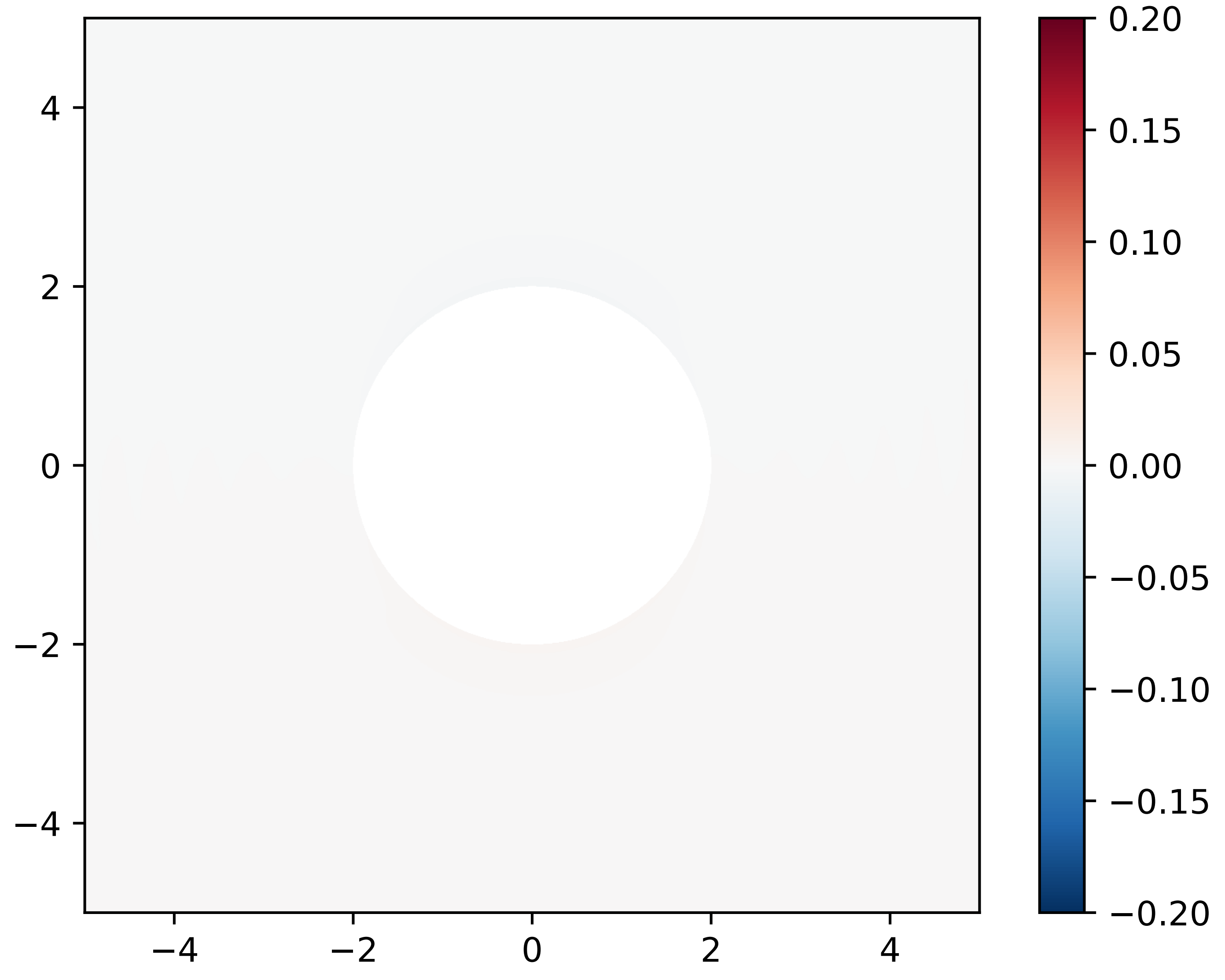


Surface Density   Azimuthal Velocity   Radial Velocity

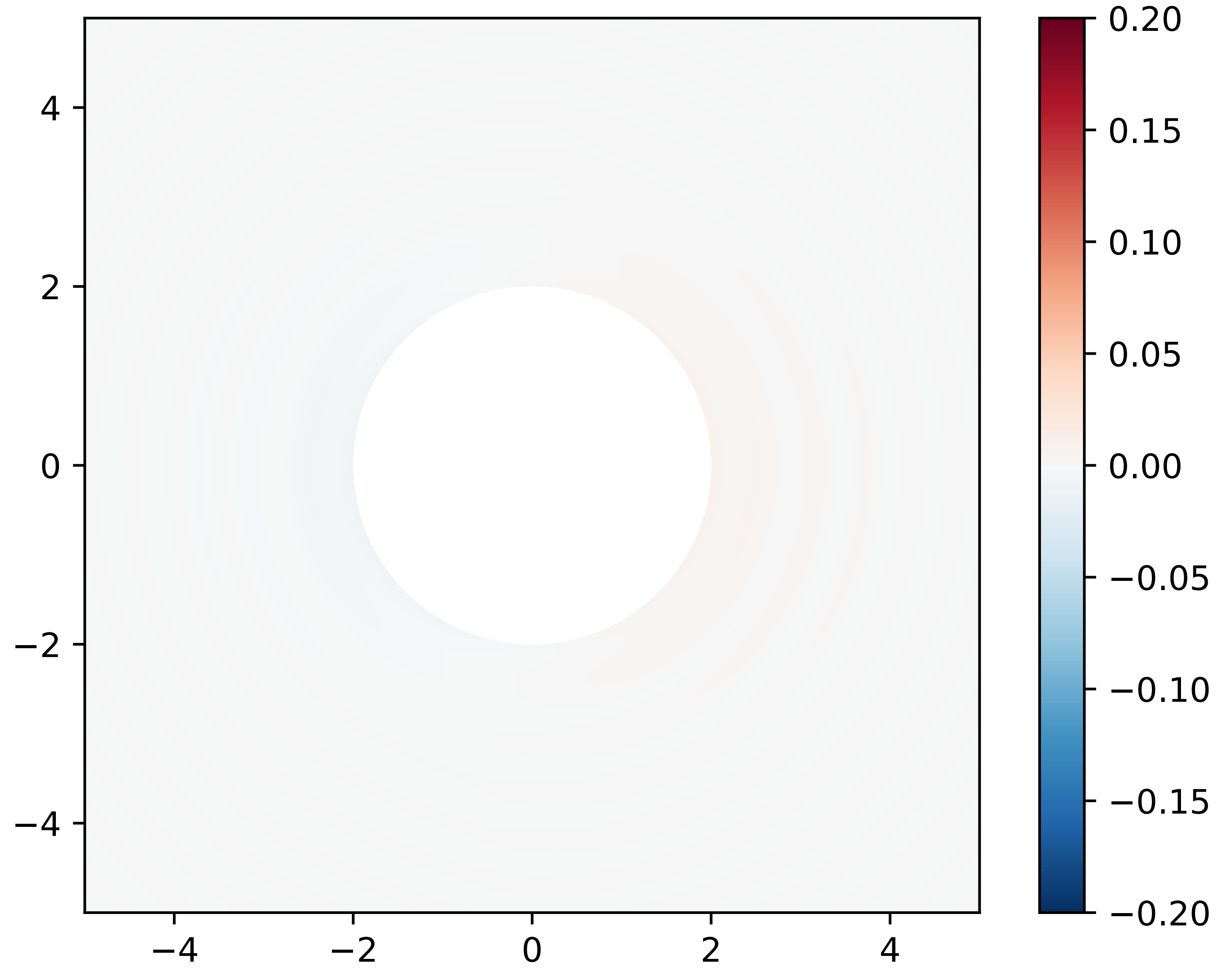
Linear Calculation Results: Adding up the Modes
$m = 1$



Surface Density

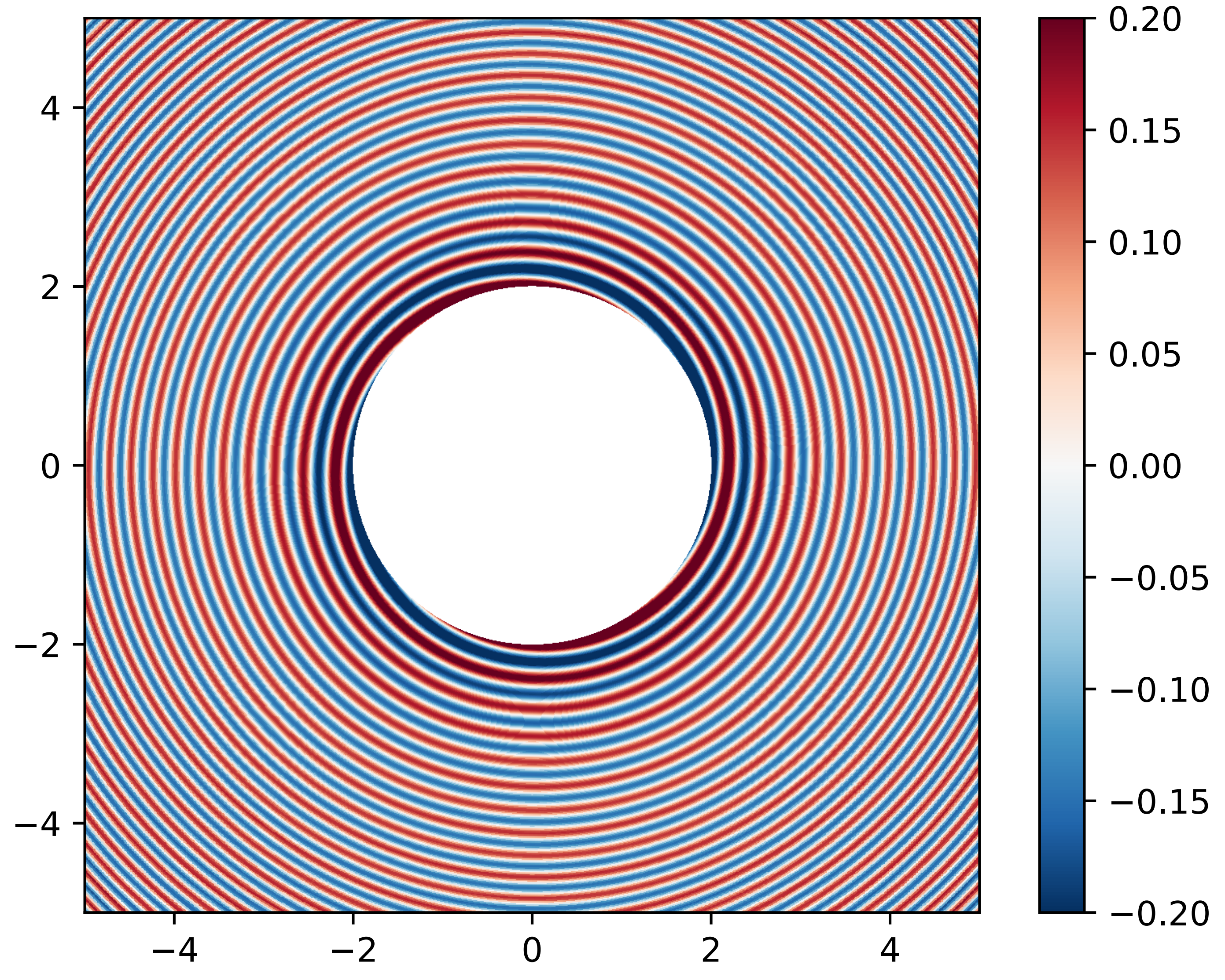


Azimuthal Velocity

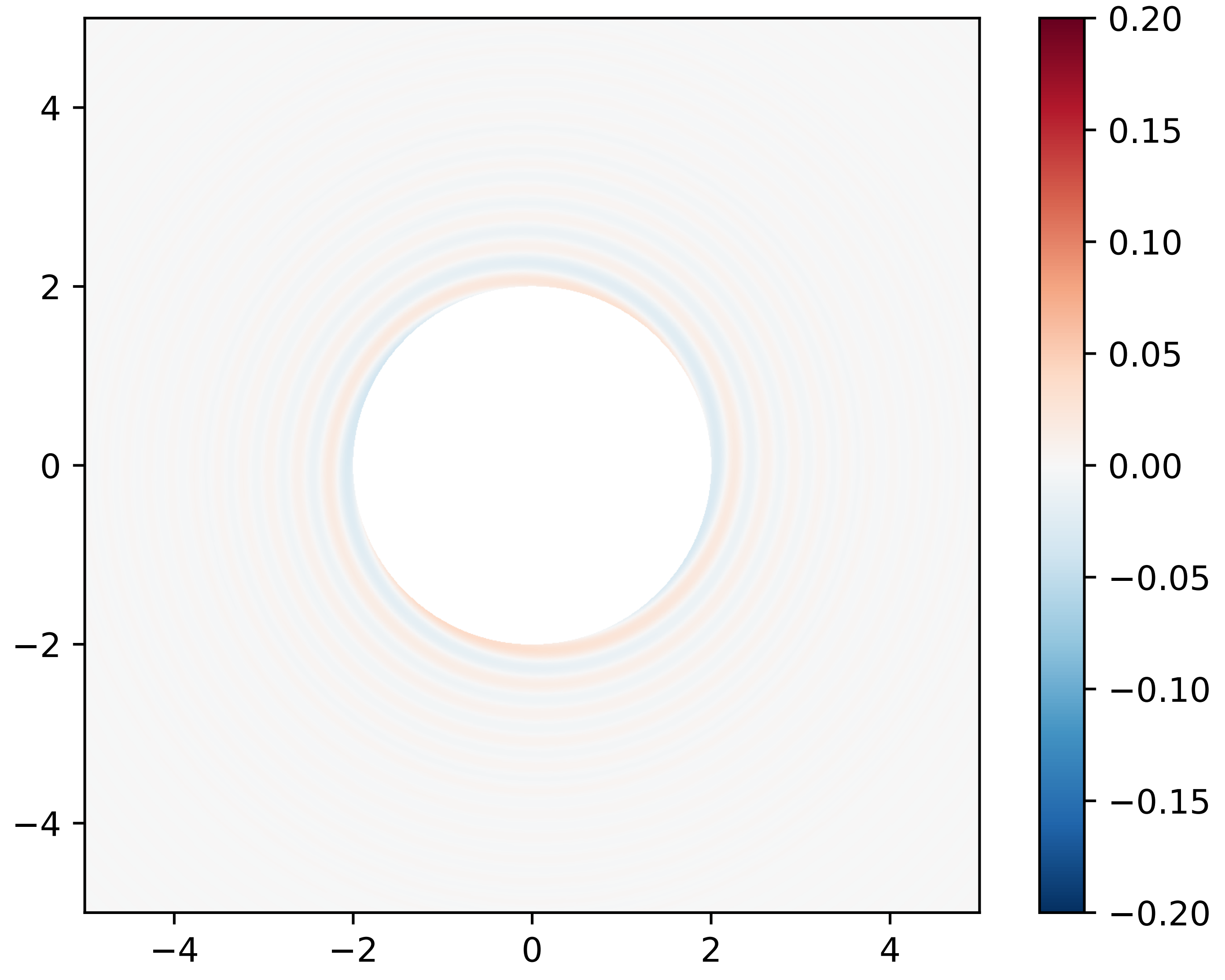


Radial Velocity

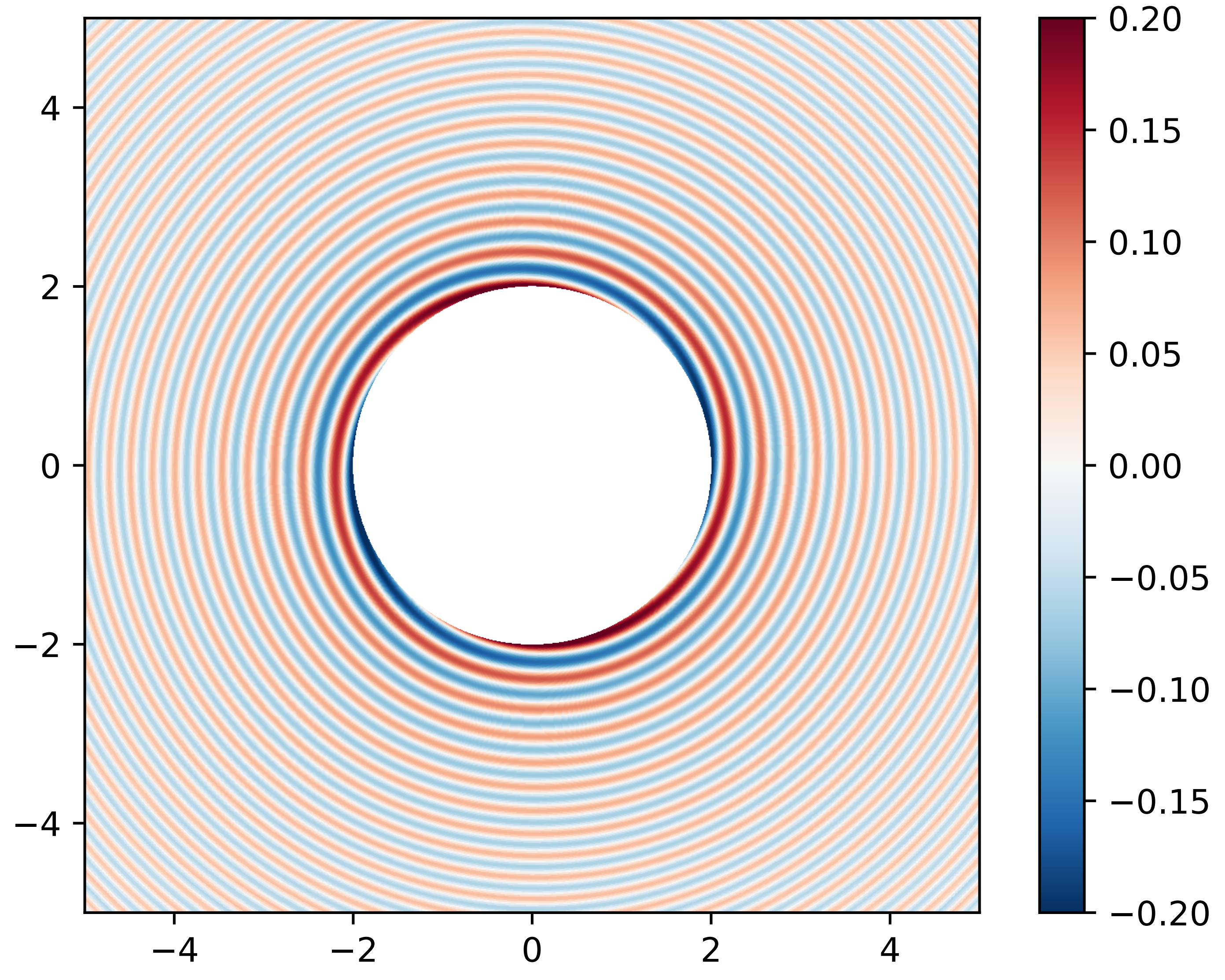
Linear Calculation Results: Adding up the Modes
m = 2



Surface Density

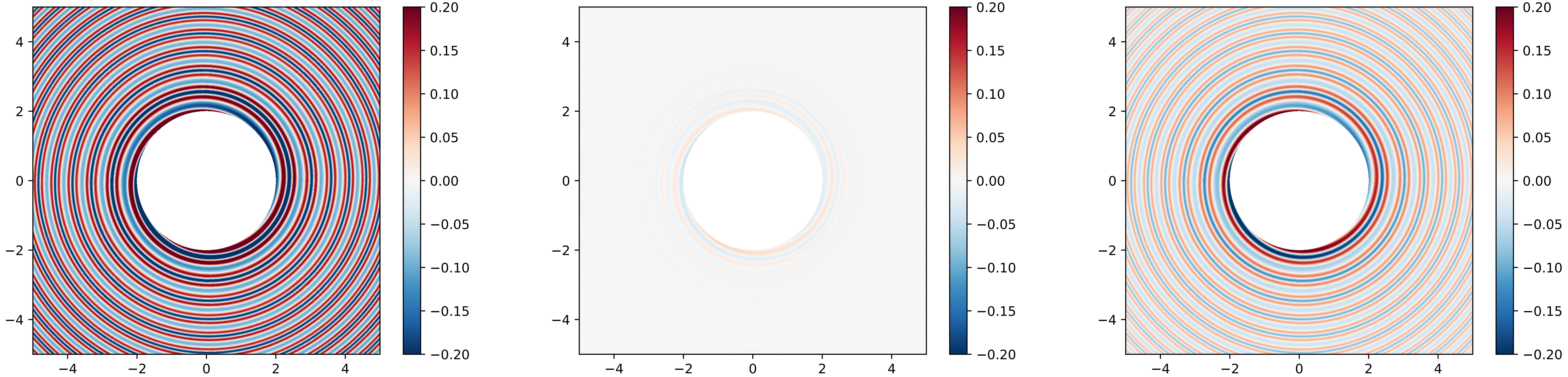


Azimuthal Velocity



Radial Velocity

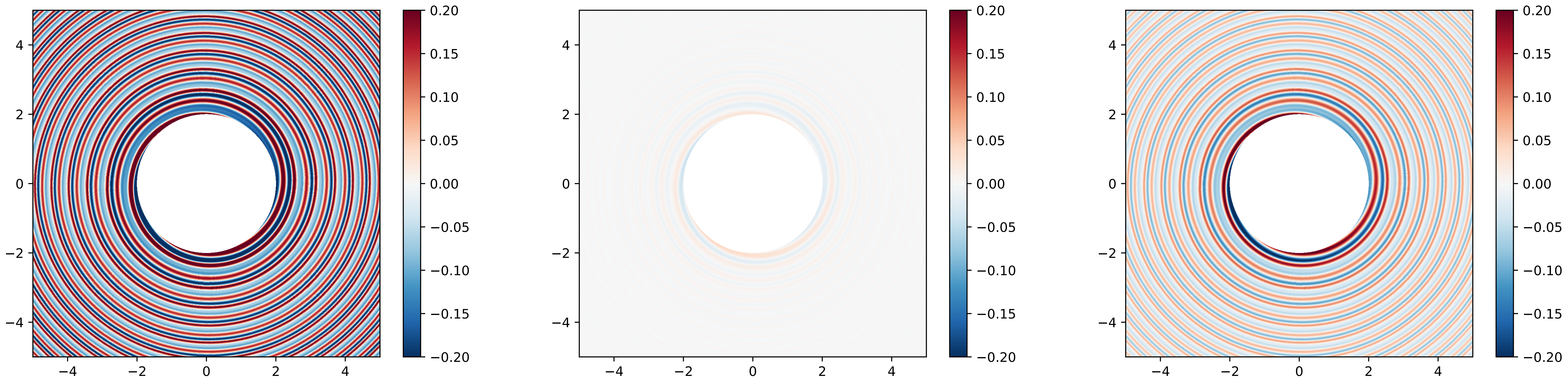
Linear Calculation Results: Adding up the Modes
m = 3



Surface Density   Azimuthal Velocity   Radial Velocity

Linear Calculation Results: Adding up the Modes

m = 4

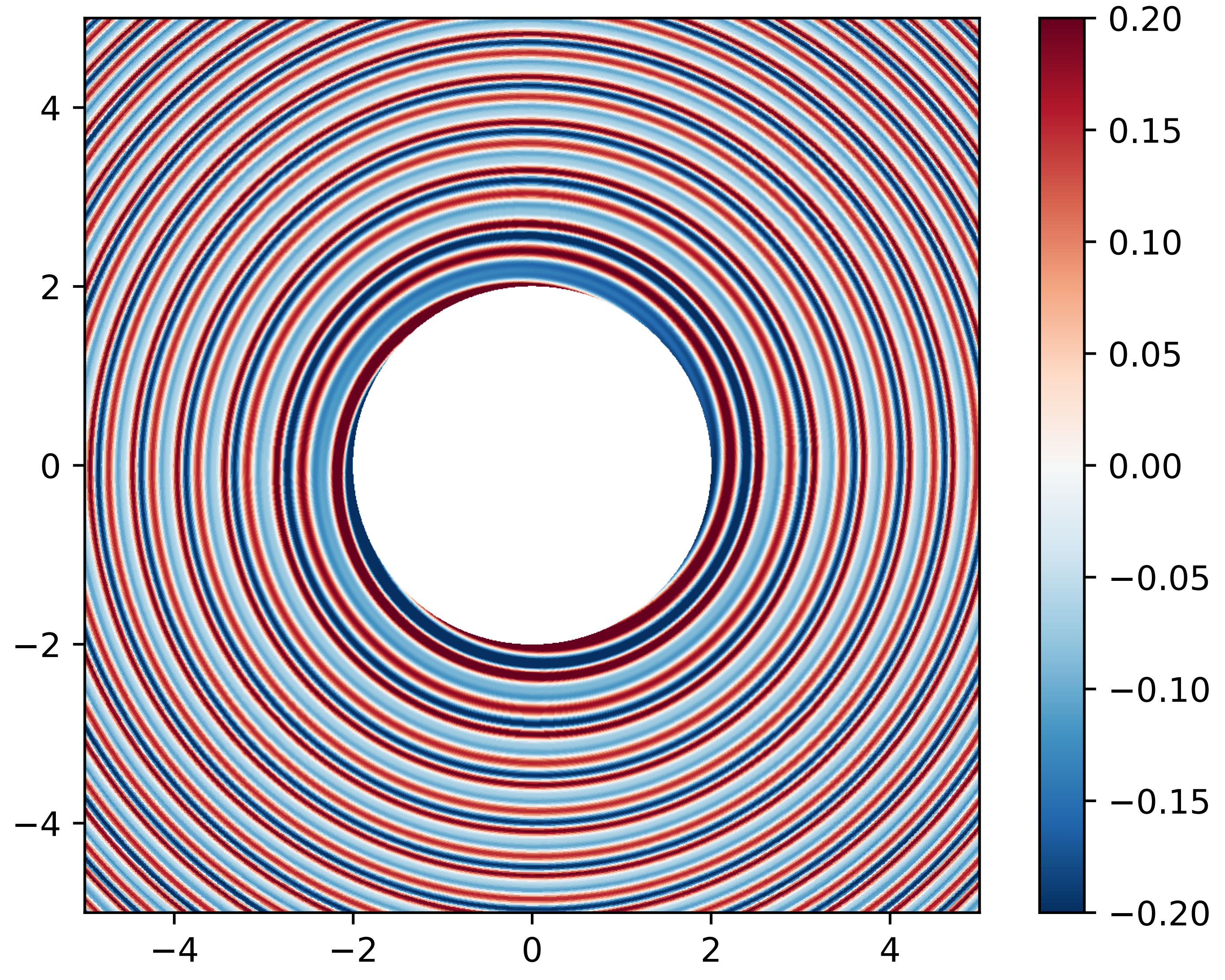


Surface Density

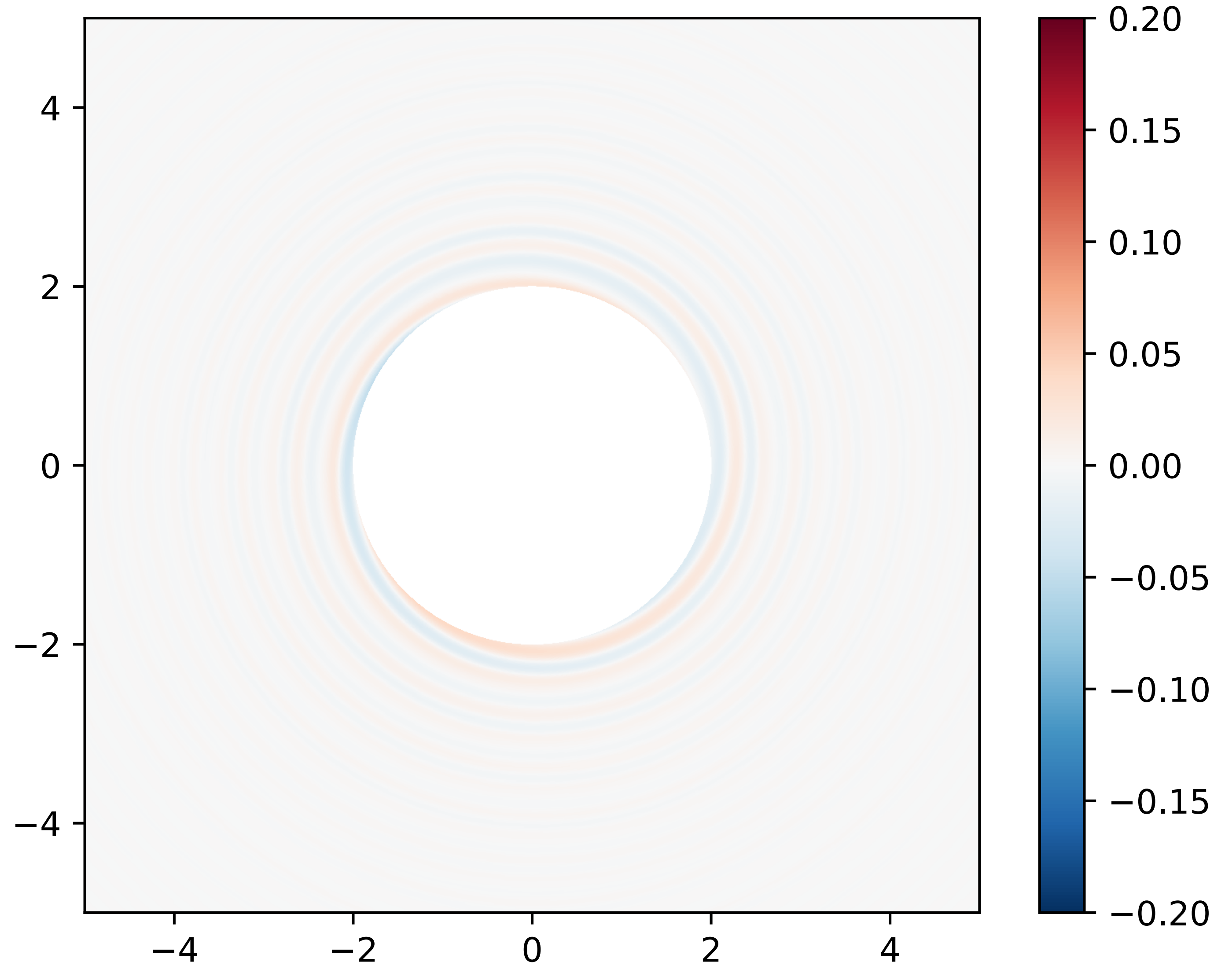
Azimuthal Velocity

Radial Velocity

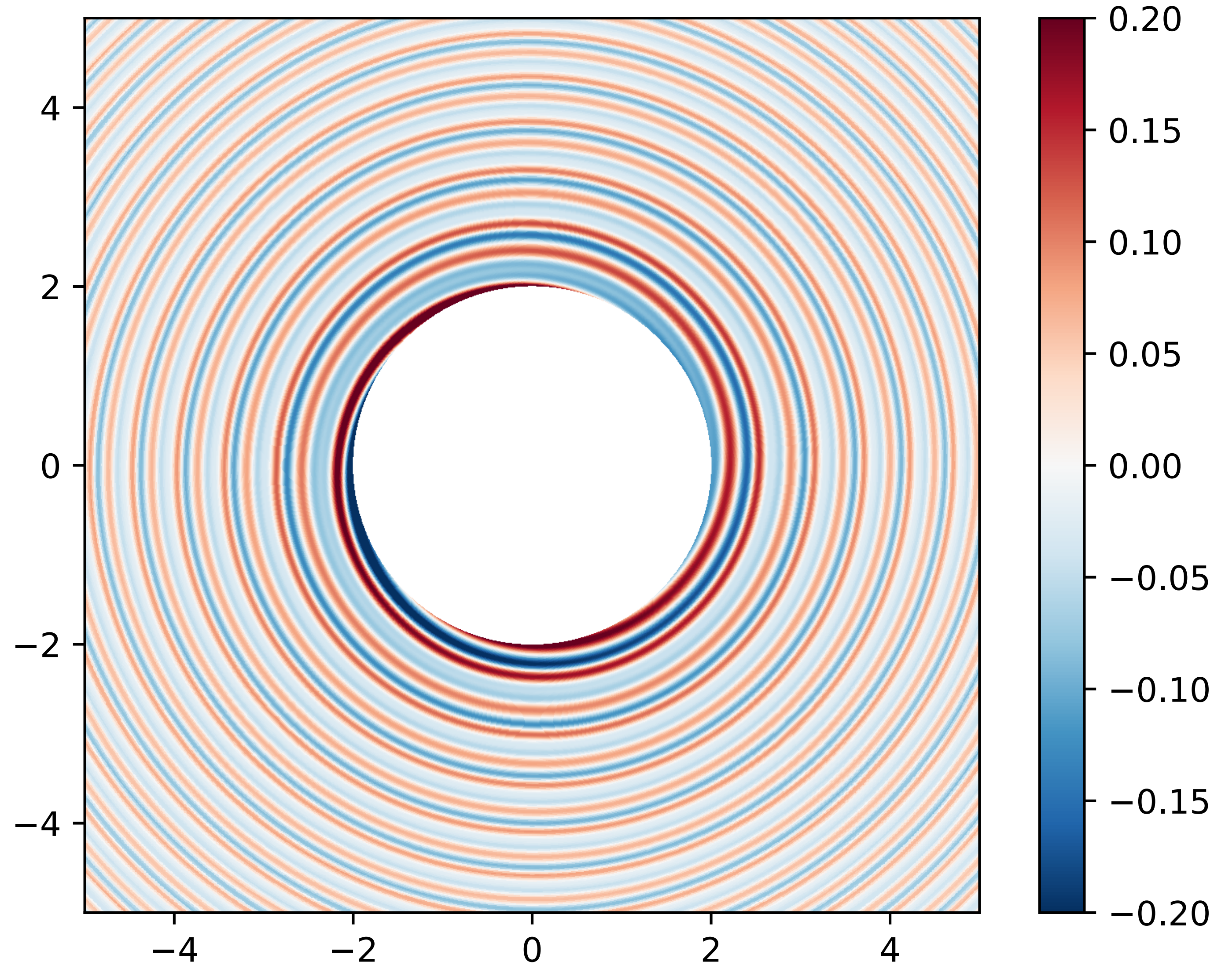
Linear Calculation Results: Adding up the Modes
m = 5



Surface Density

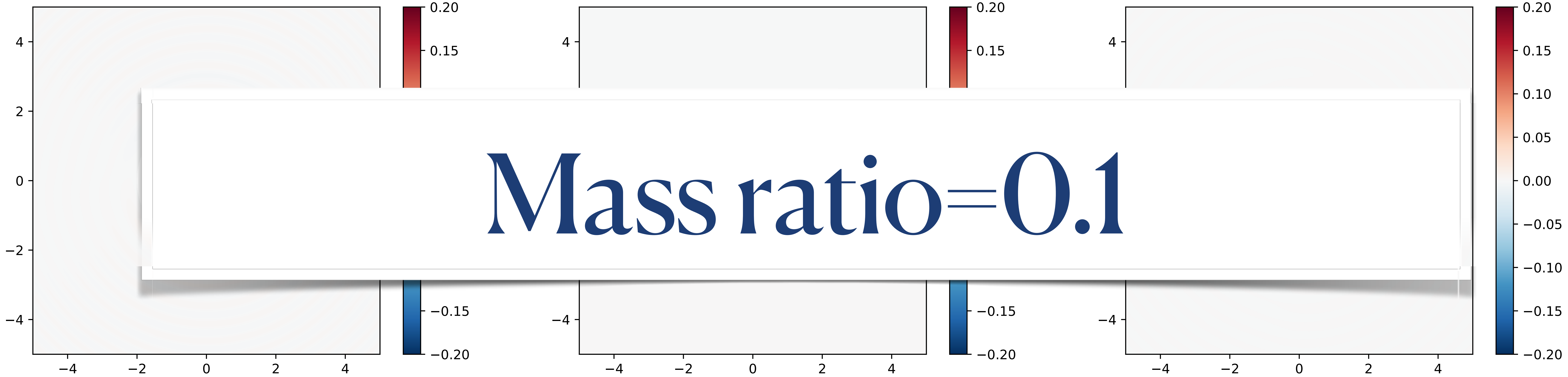


Azimuthal Velocity



Radial Velocity

Linear Calculation Results: Adding up the Modes
m=1



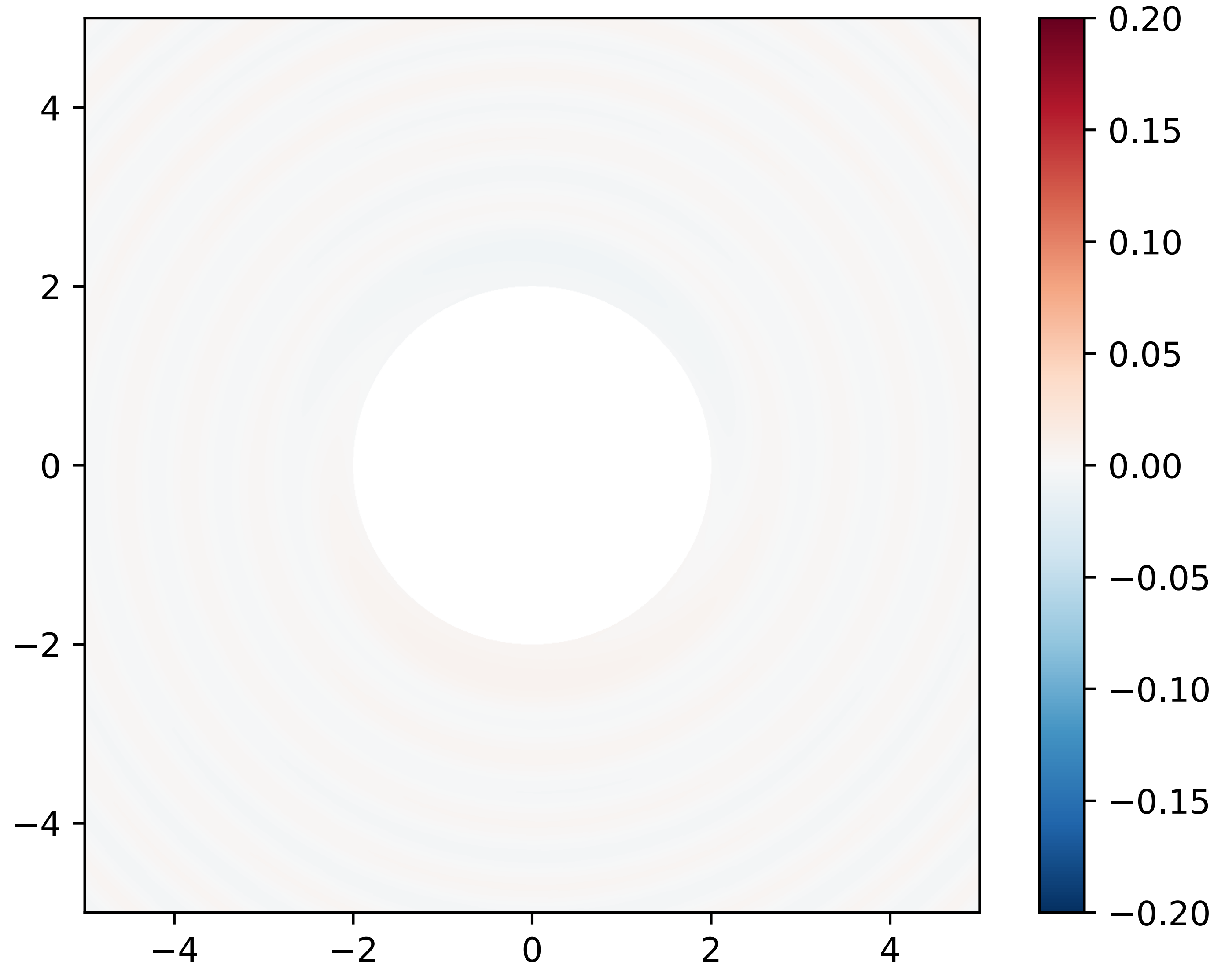
Mass ratio=0.1

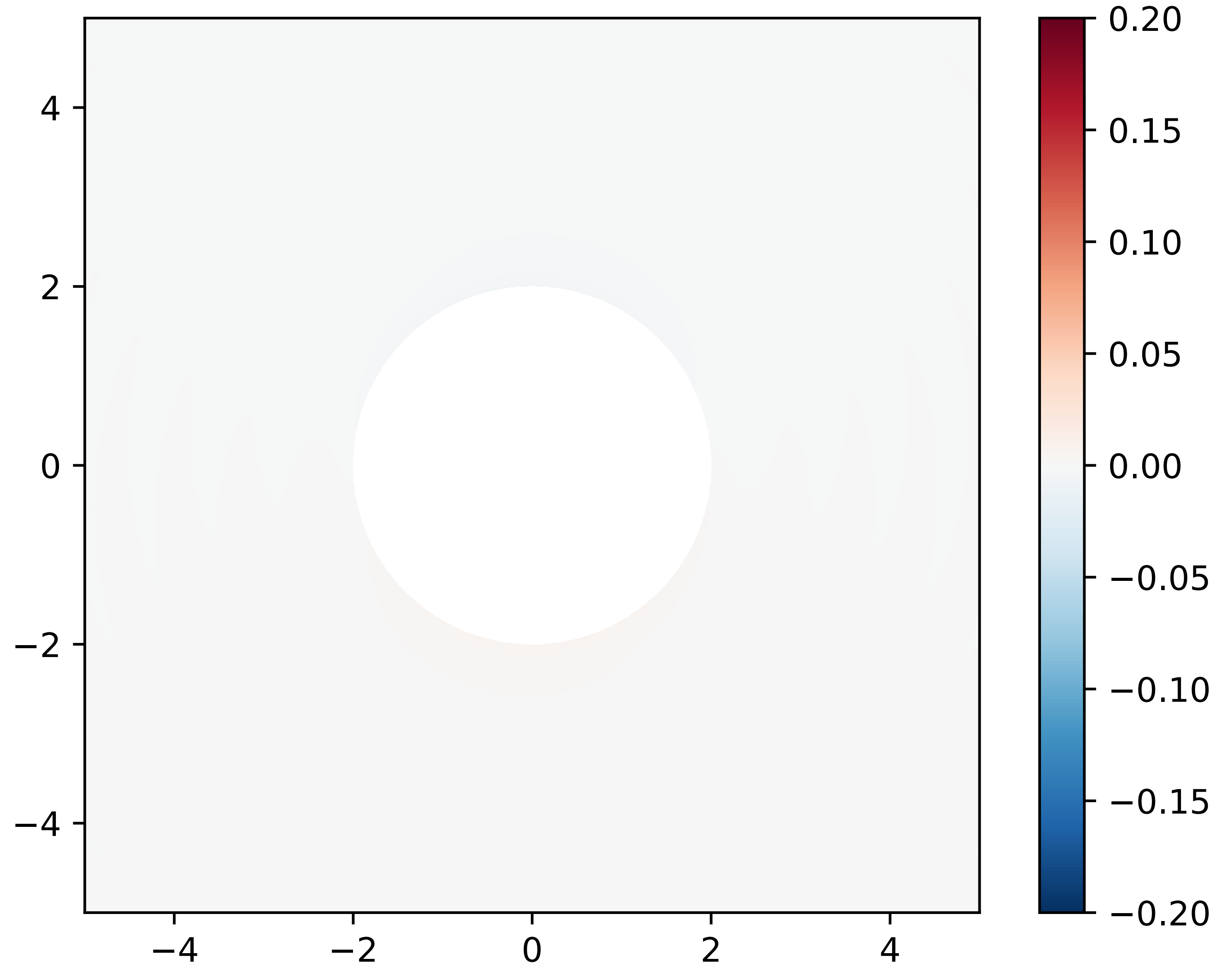
Surface Density

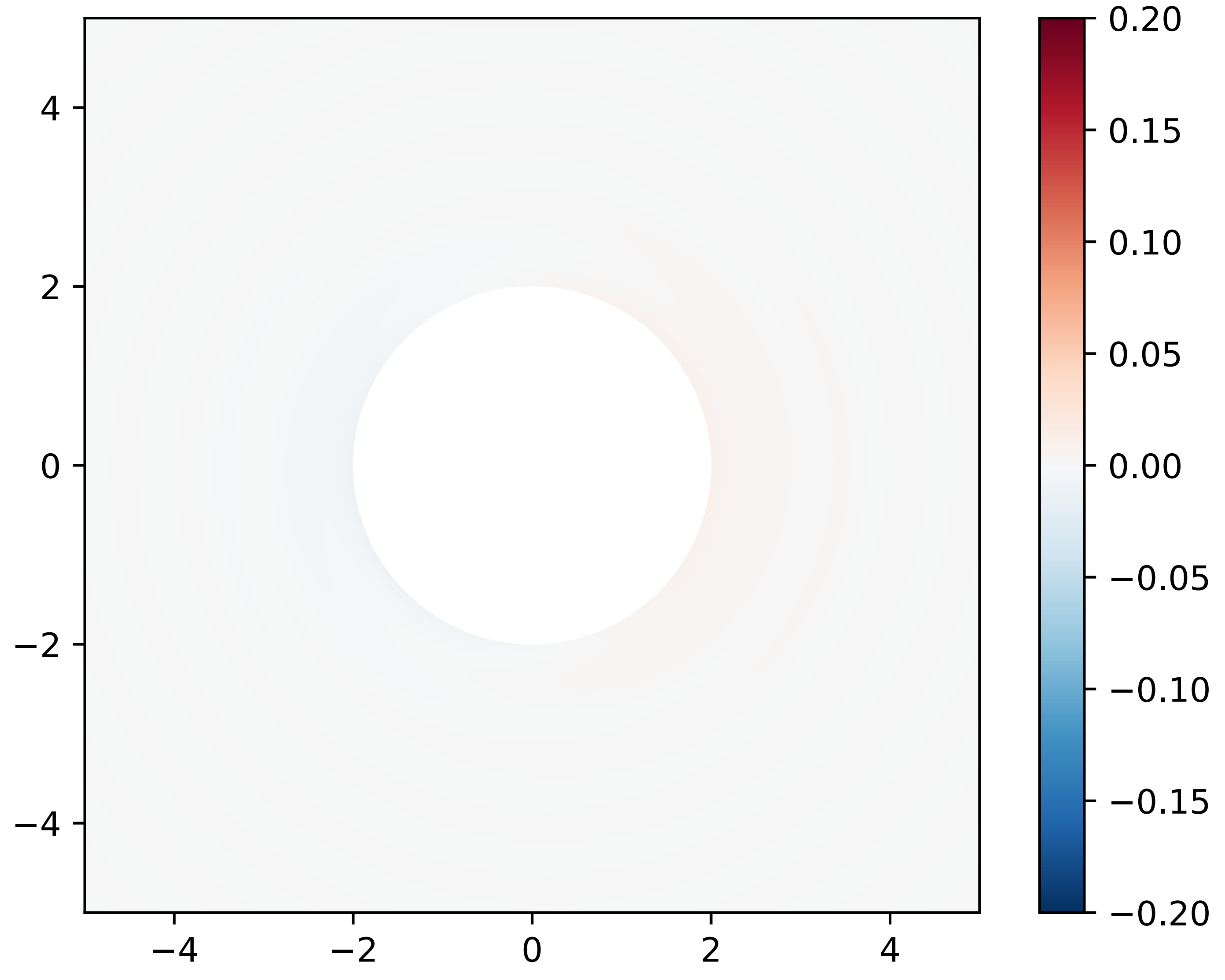
Azimuthal Velocity

Radial Velocity

Linear Calculation Results: Adding up the Modes m=1





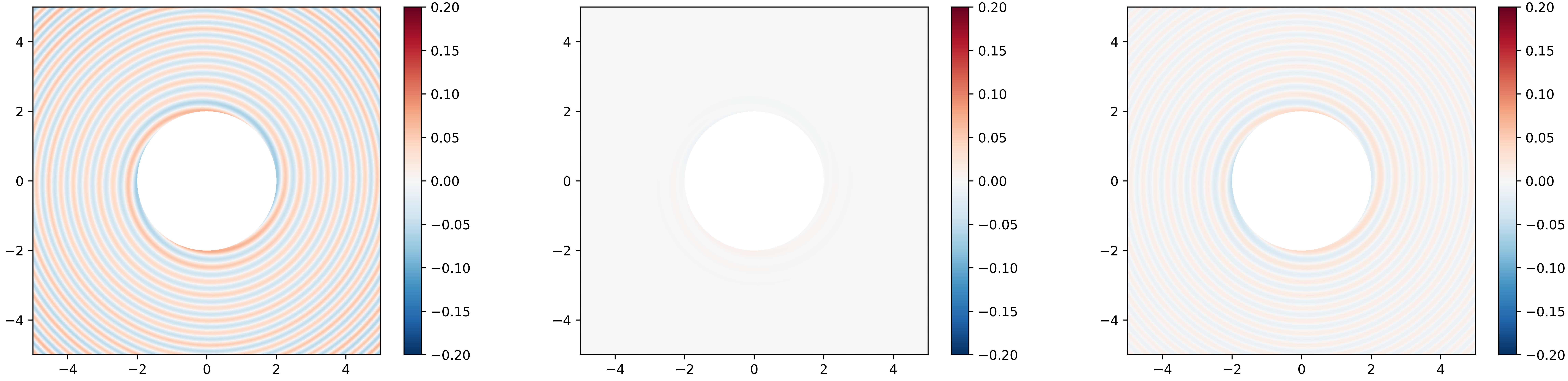


Surface Density

Azimuthal Velocity

Radial Velocity

Linear Calculation Results: Adding up the Modes m=2

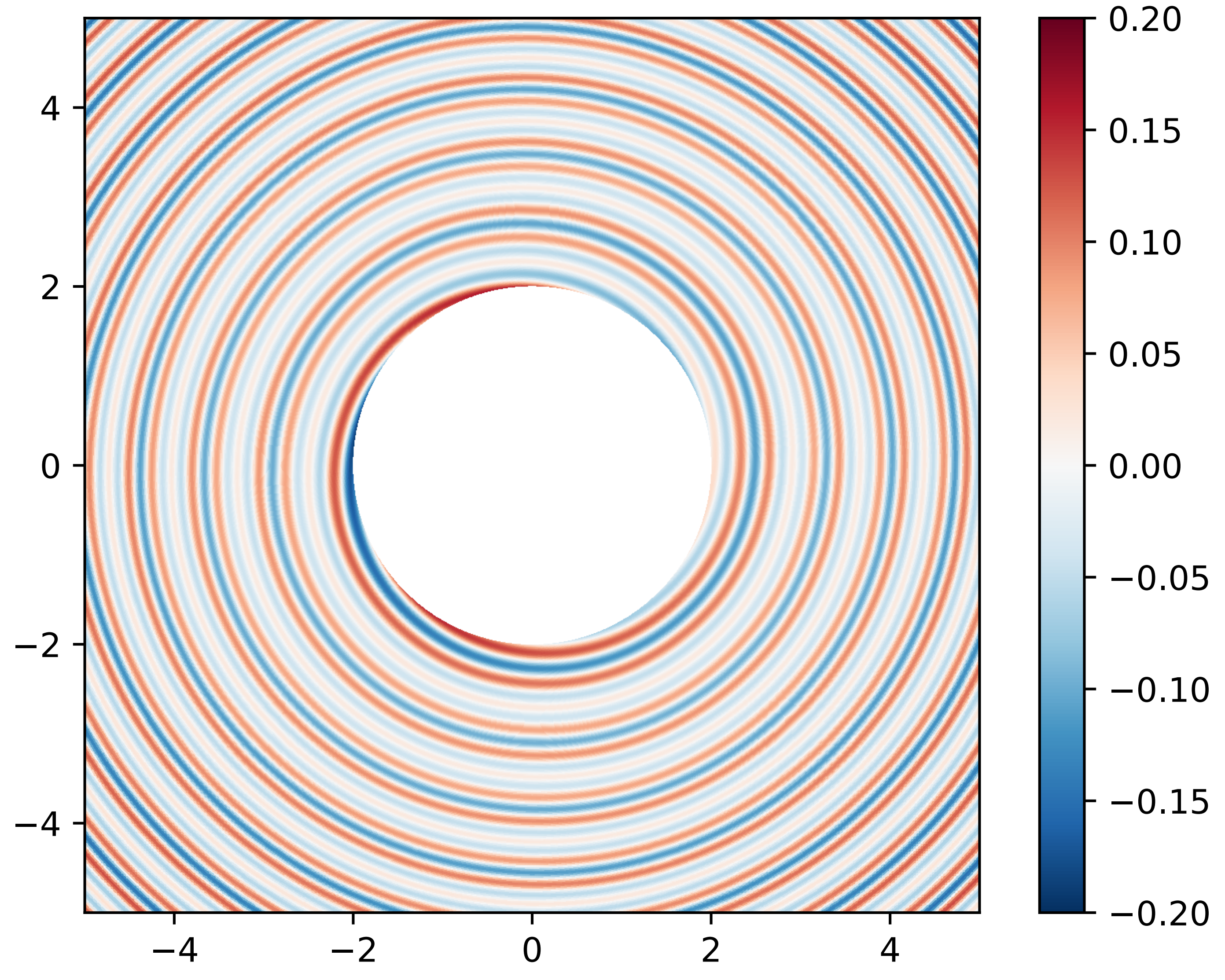


Surface Density

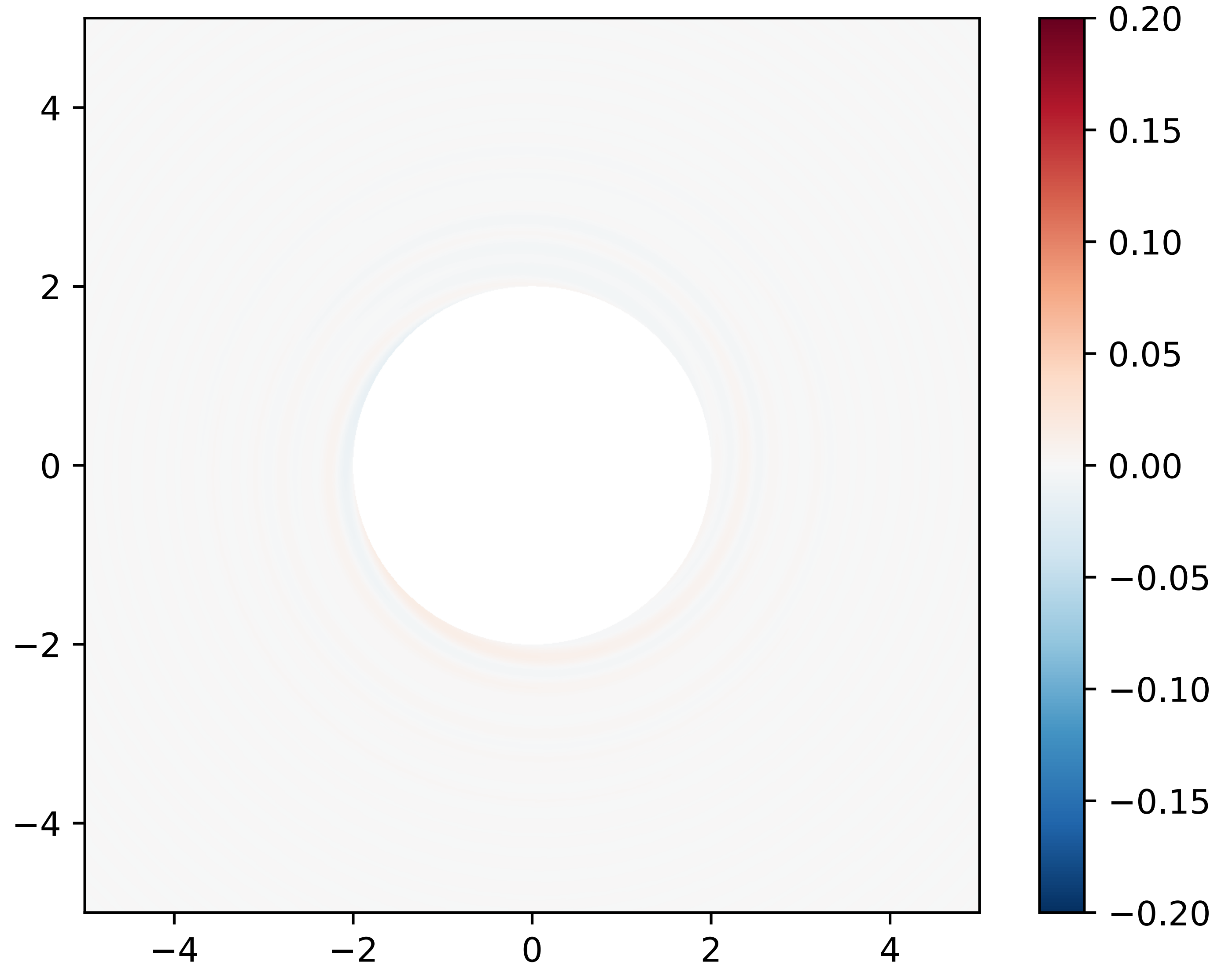
Azimuthal Velocity

Radial Velocity

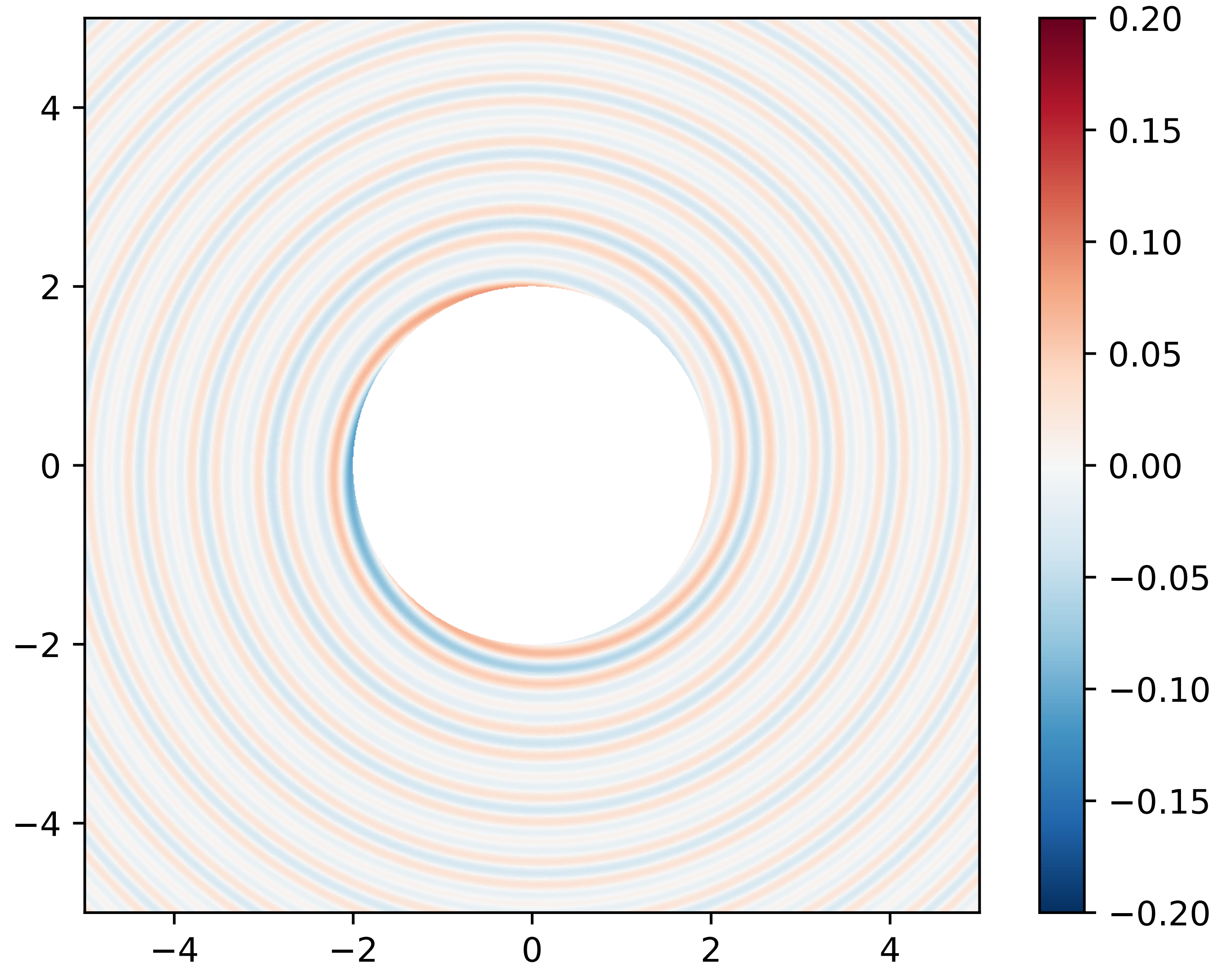
Linear Calculation Results: Adding up the Modes
m=3



Surface Density

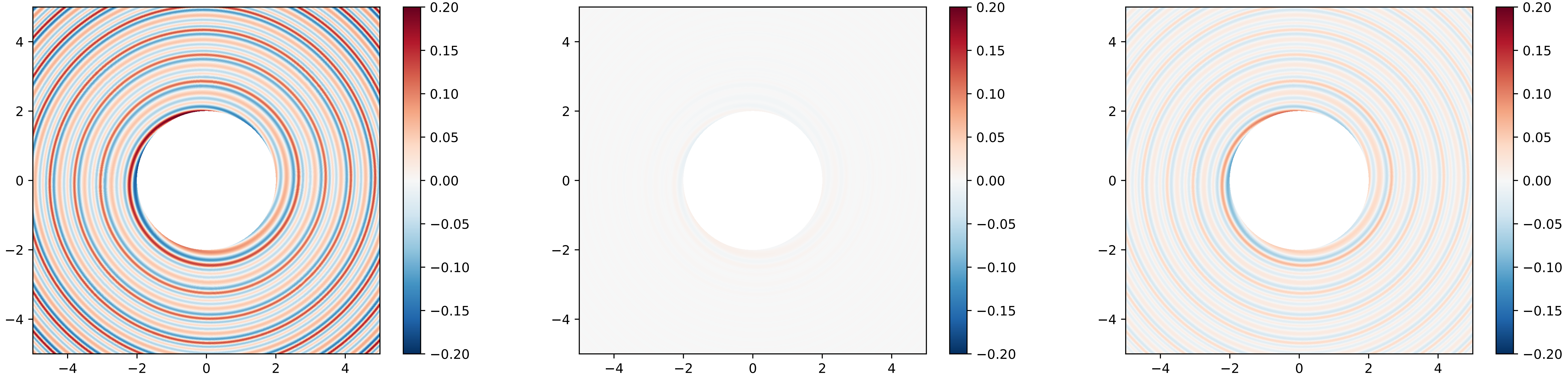


Azimuthal Velocity



Radial Velocity

Linear Calculation Results: Adding up the Modes m=4

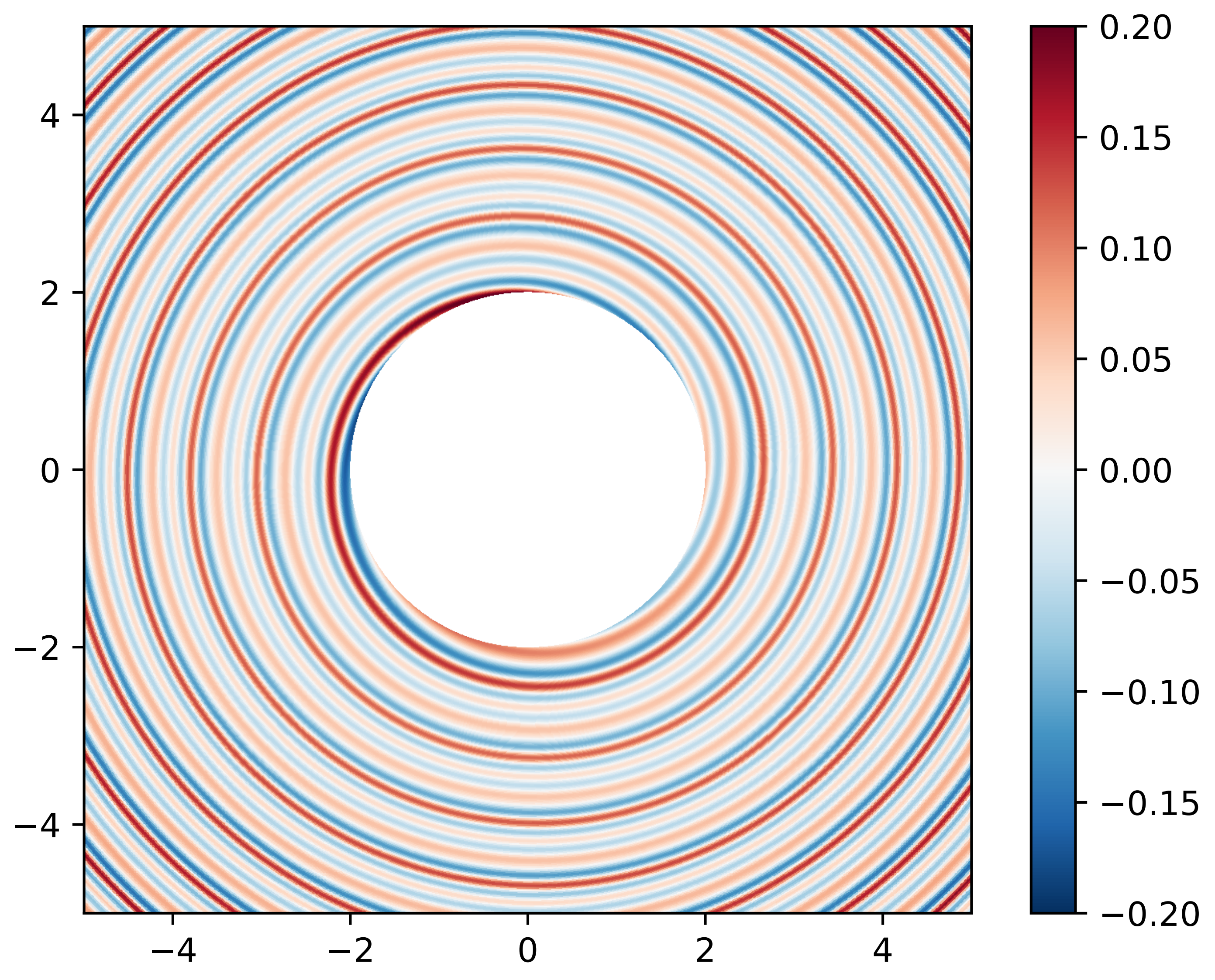


Surface Density

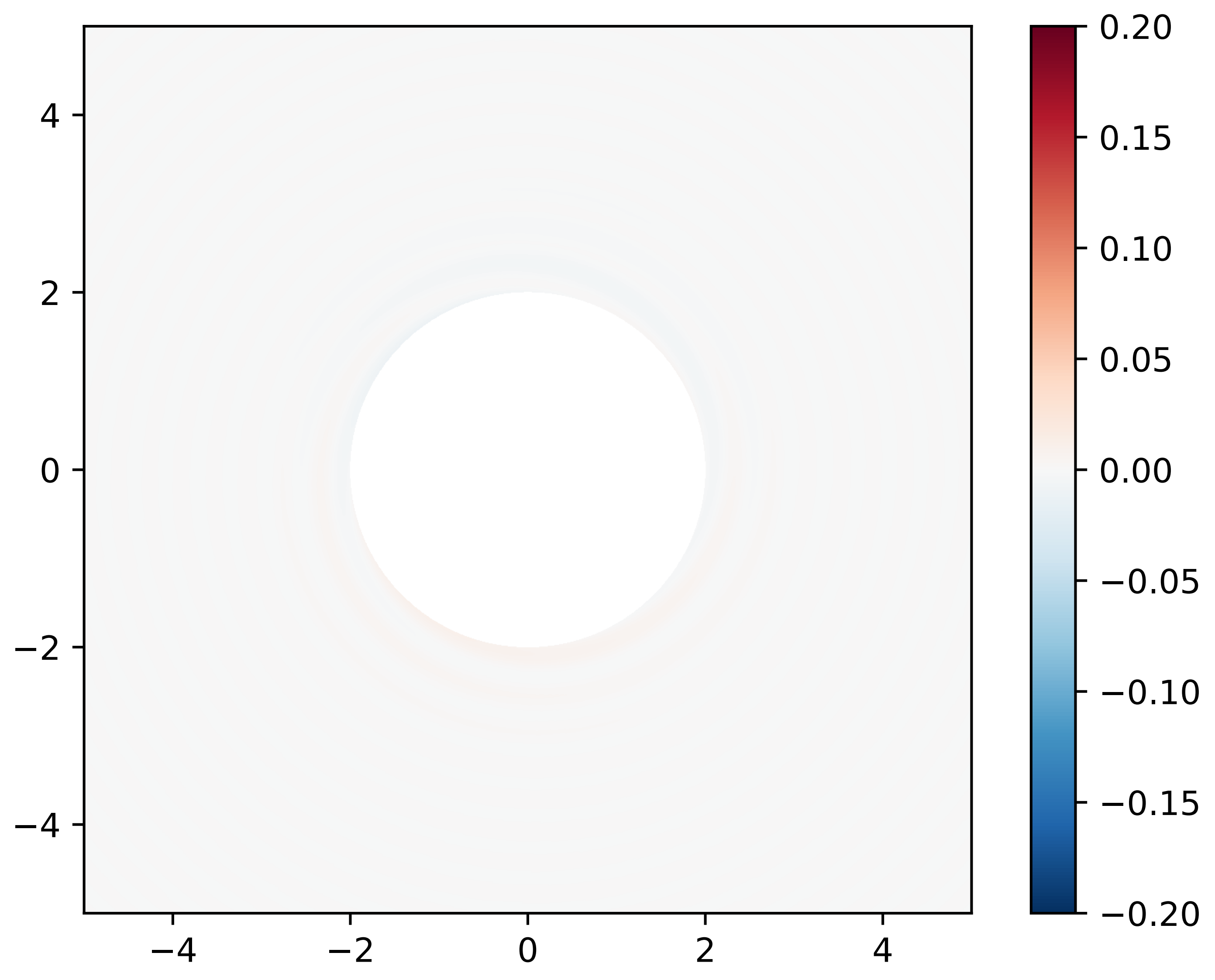
Azimuthal Velocity

Radial Velocity

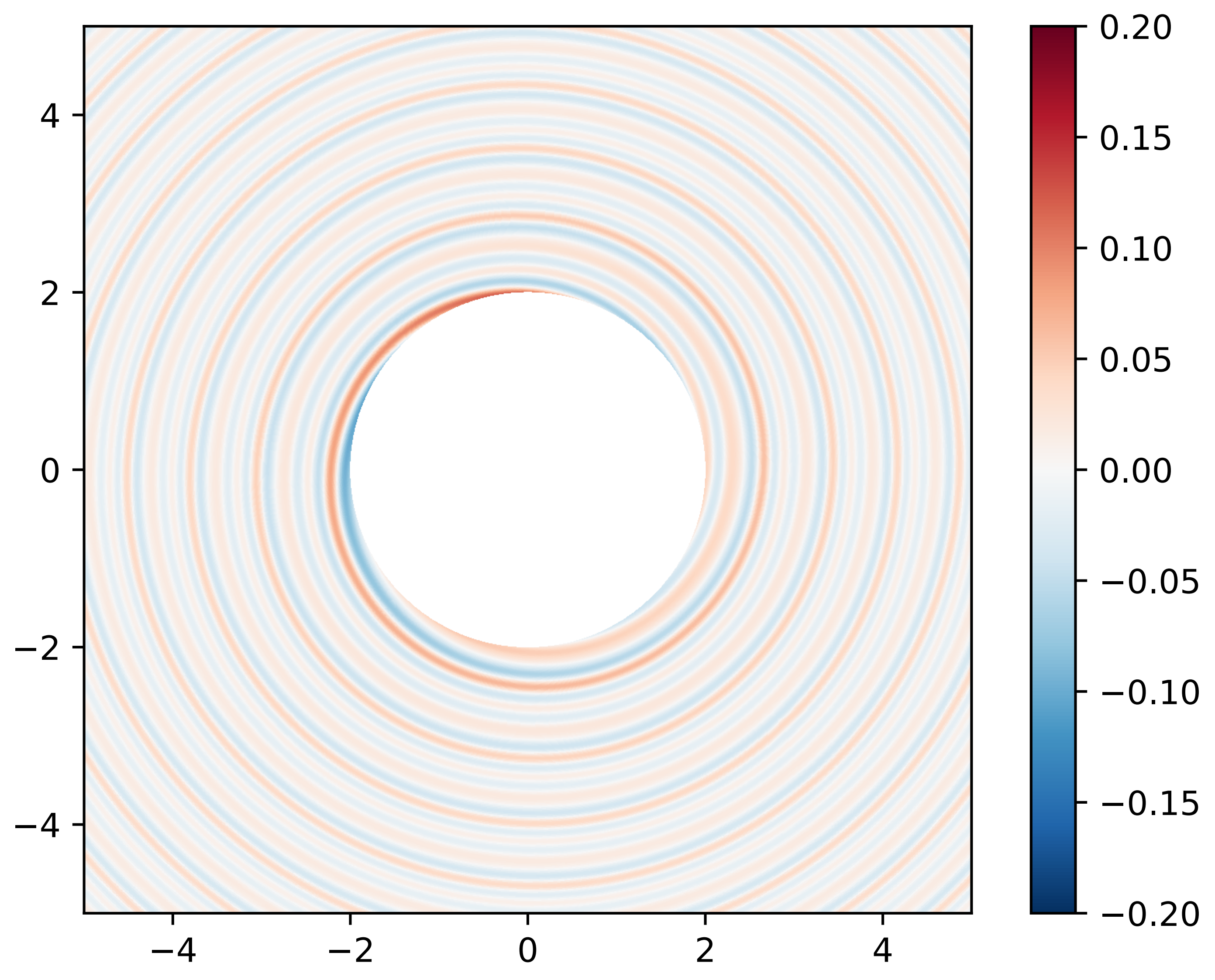
Linear Calculation Results: Adding up the Modes
m=5



Surface Density

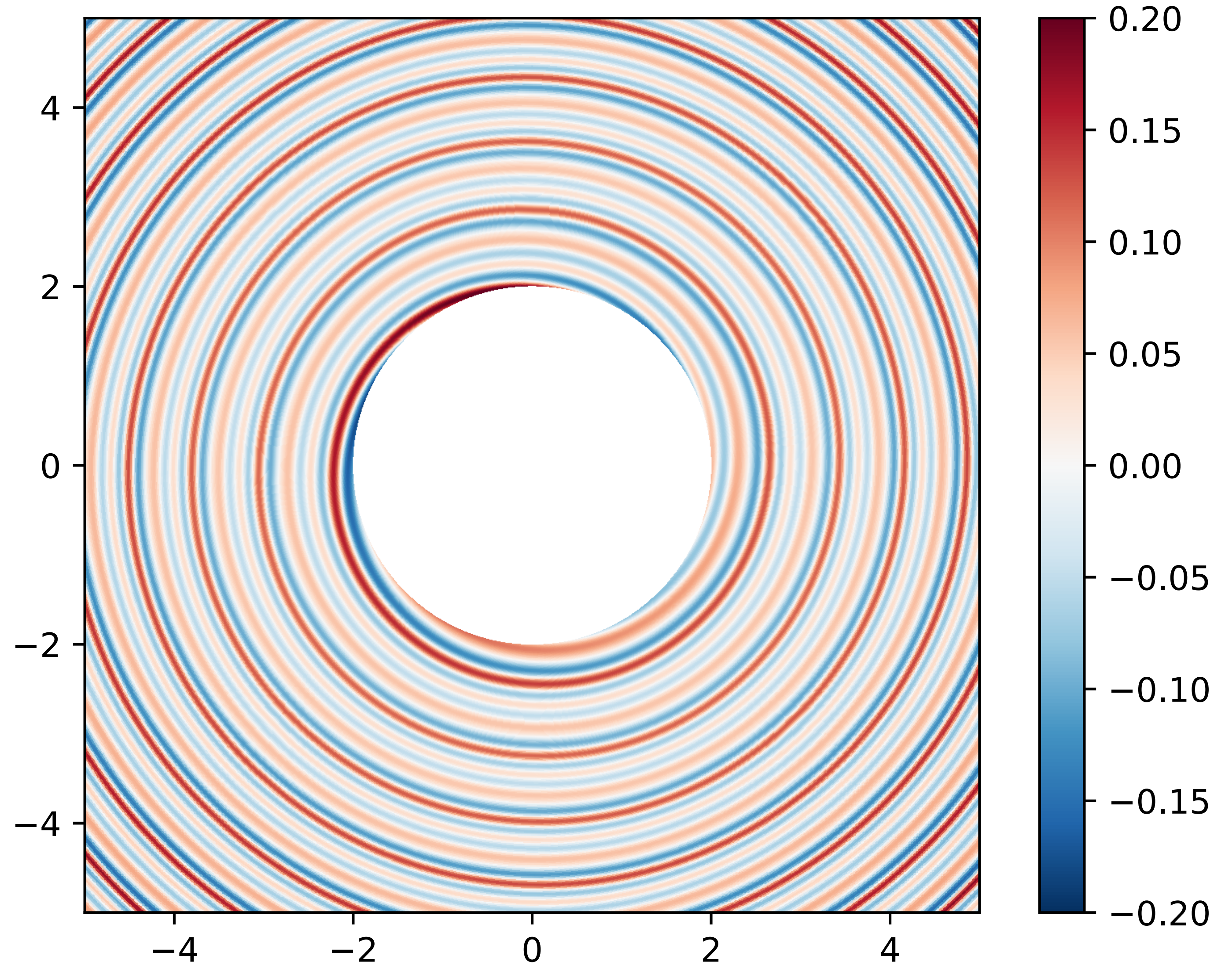


Azimuthal Velocity

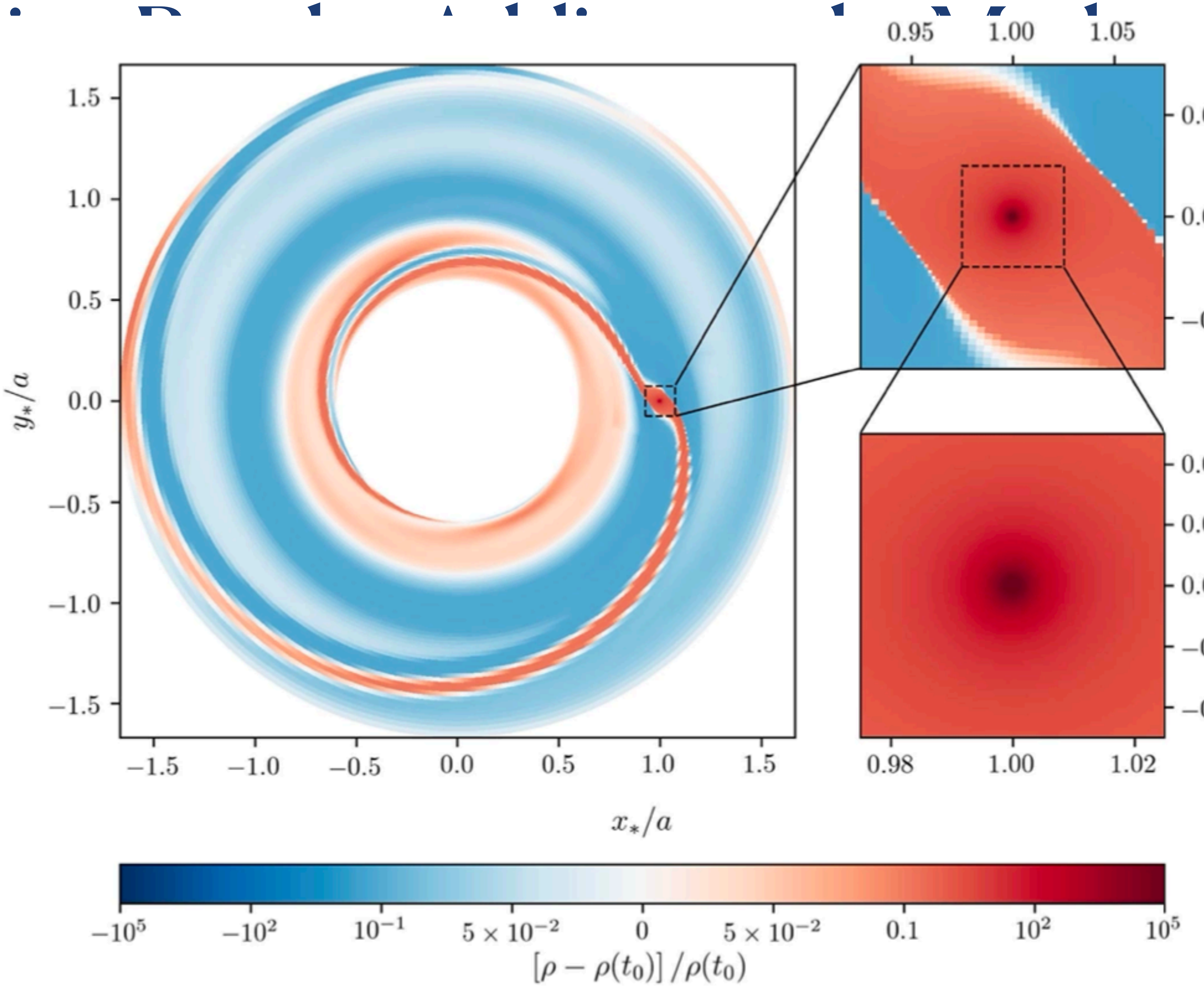


Radial Velocity

Linear Calcula



Surface Density



Conclusion

1. One of the first attempts at analytical calculation of **Binary-Disk-Interaction**

2. **m=2 to m=1** (azimuthal mode) from Binary-Disk-Interaction to Planet-Disk-Interaction

3. “Cheap Simulation”