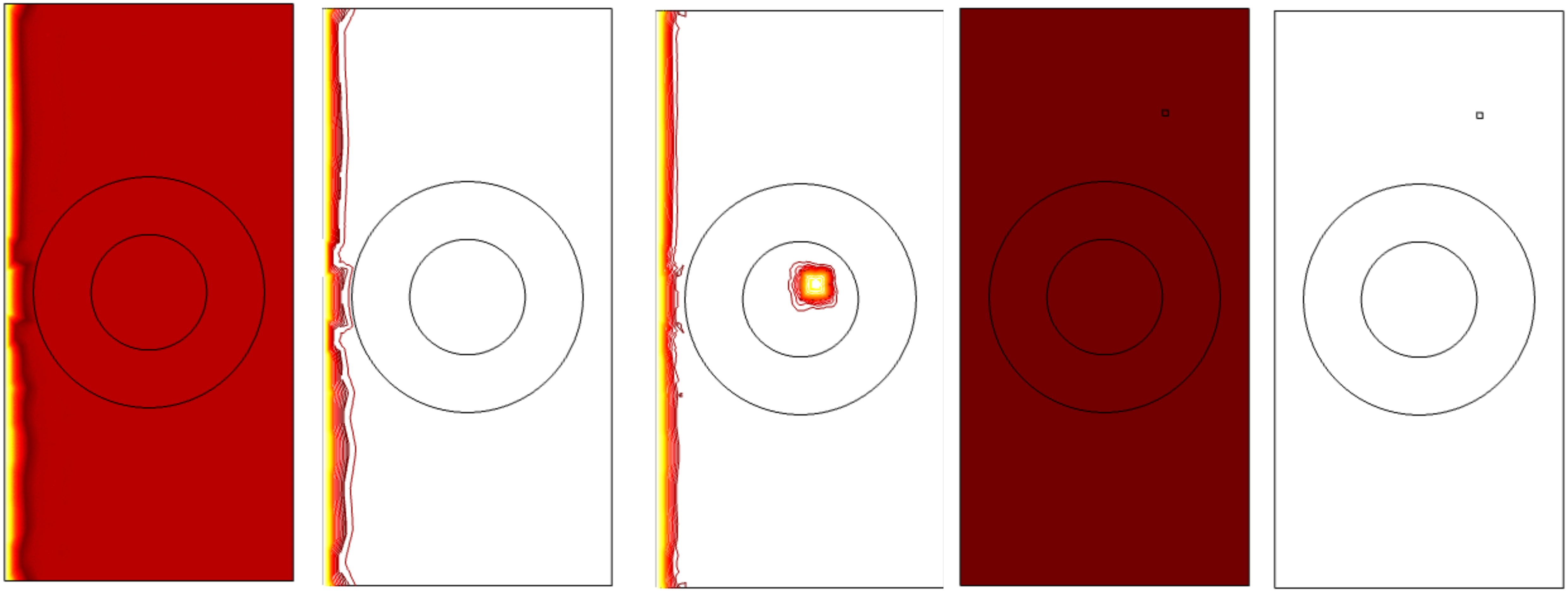
热隐身衣

HTML5 网页演示

舒驰

JUN.13,2024

COMSOL模拟



PART 01

理论分析

热传导方程

热传导方程：

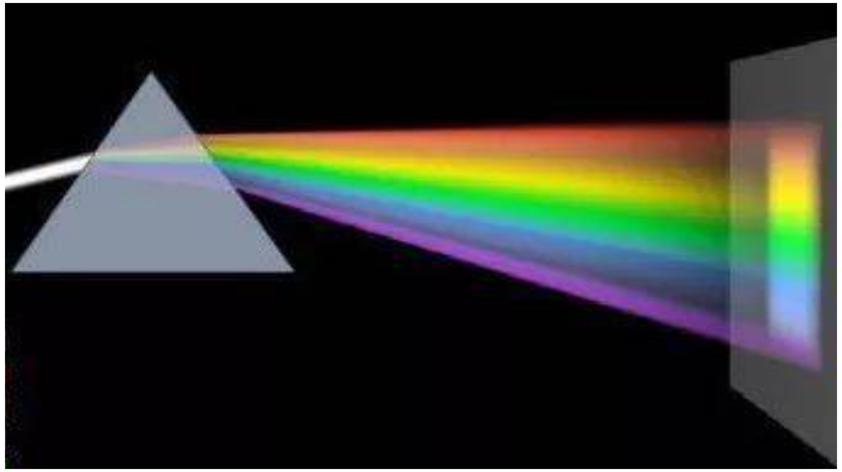
$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T)$$

- ρ：介质的密度（单位：kg/m^3）
- c：介质的比热容（单位：$J/(kg \cdot K)$）
- T：温度（单位：K）
- t：时间（单位：s）
- κ：热导率（单位：$W/(m \cdot K)$）

在三维欧氏空间下，材料各项同性时：

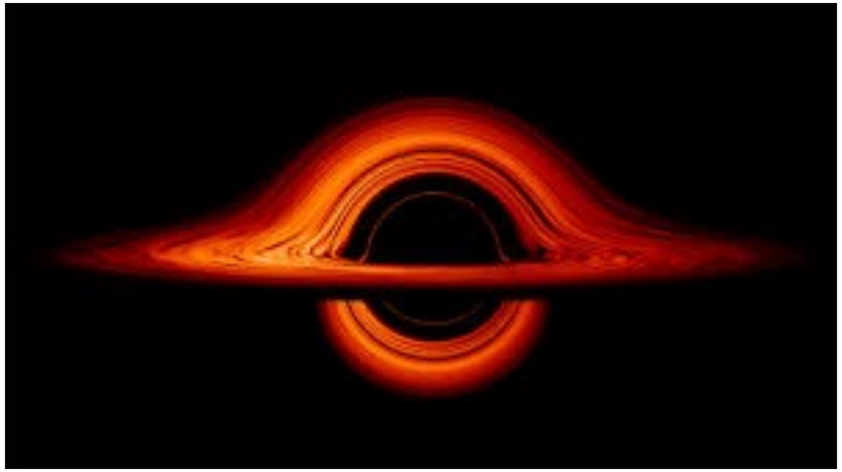
$$\rho c \frac{\partial T}{\partial t} = \kappa \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right)$$

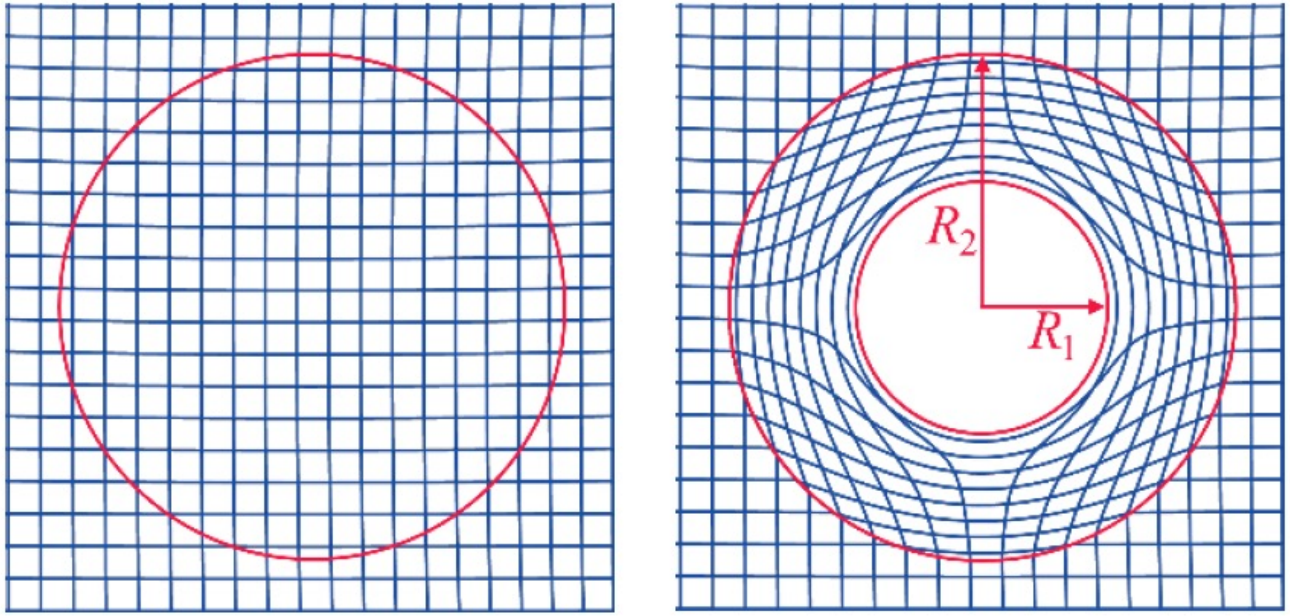
扩散方程



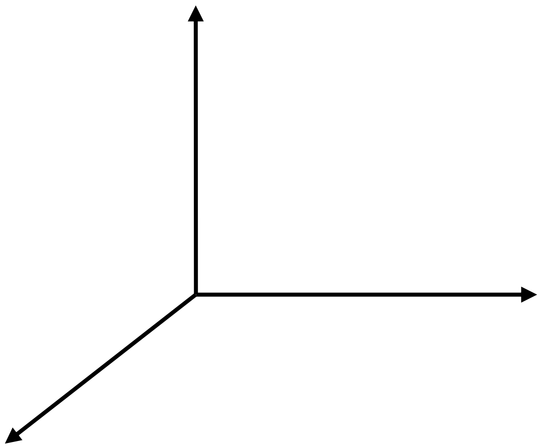
$$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T)$$

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{\sqrt{g}} \partial_\mu \left[\sqrt{g} \kappa^\mu_\alpha g^{\alpha\nu} \partial_\nu T \right]$$





Metric

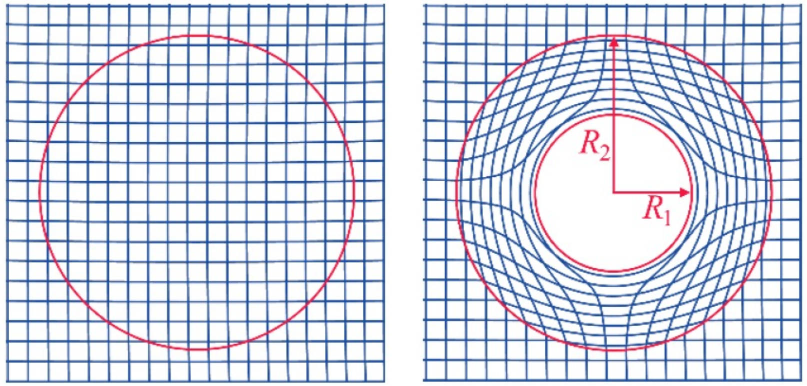


$$ds^2 = dx^2 + dy^2 + dz^2$$

$$J = \begin{pmatrix} \frac{\partial r}{\partial x} & \frac{\partial r}{\partial y} & \frac{\partial r}{\partial z} \\ \frac{\partial \theta}{\partial x} & \frac{\partial \theta}{\partial y} & \frac{\partial \theta}{\partial z} \\ \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} & \frac{\partial \phi}{\partial z} \end{pmatrix}$$

$$ds^2 = r^2 d\theta^2 + r^2 \sin^2\theta d\varphi^2$$

$$\rho c \frac{\partial T}{\partial t} = \frac{1}{\sqrt{g}} \partial_\mu \left[\sqrt{g} \kappa^\mu_\alpha g^{\alpha\nu} \partial_\nu T \right]$$

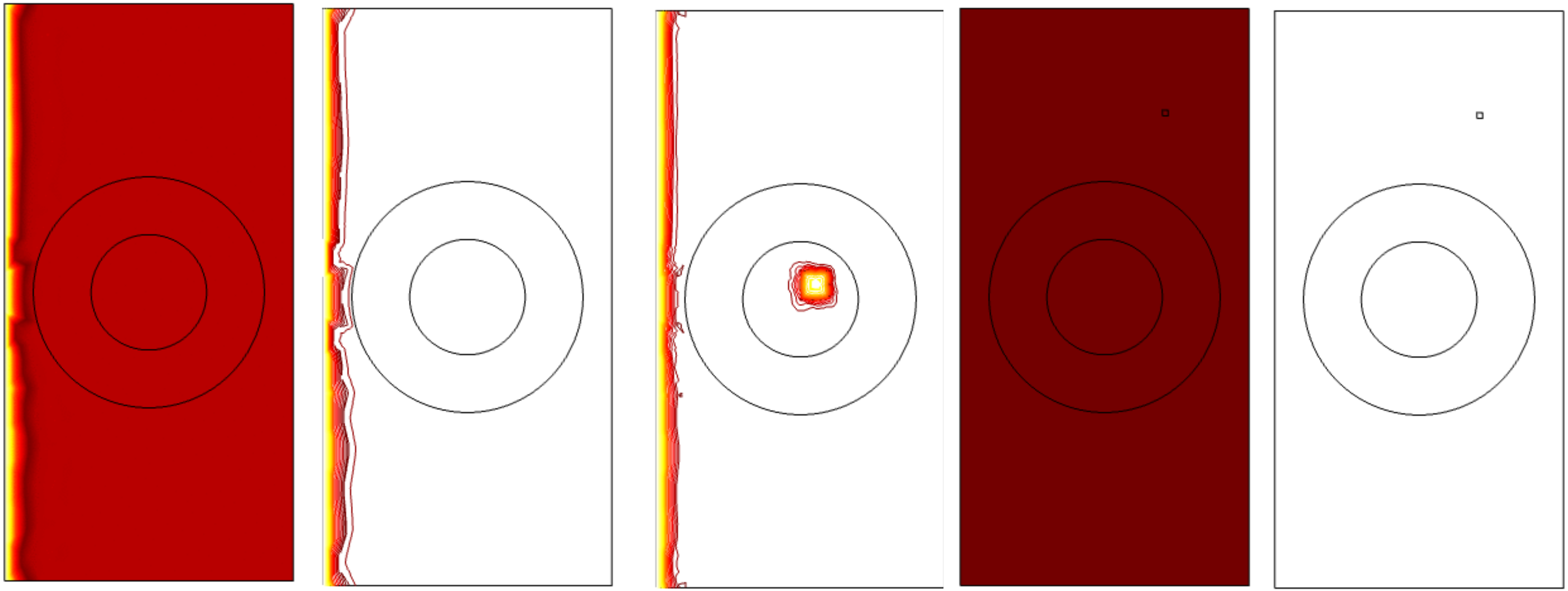


$$\sqrt{g} \kappa^\mu_\alpha g^{\alpha\nu} = \tilde{\kappa}^{\mu\nu}$$

$$\tilde{\kappa} = \frac{J^T \tilde{\kappa}' J}{\det J}$$

$$J = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \frac{R_2 - R_1}{R_2} & \\ & \frac{r}{r'} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

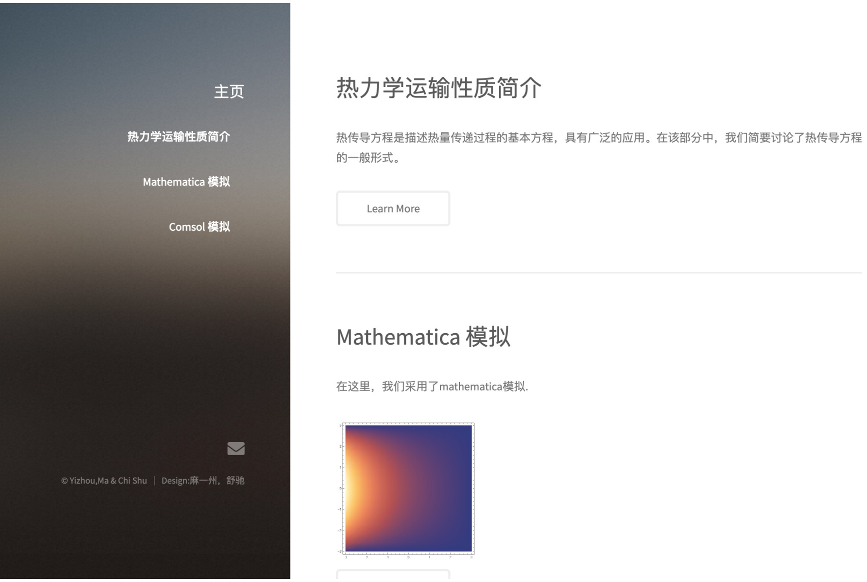
COMSOL模拟



PART 02

网页展示

主页



主页

热力学运输性质简介

Mathematica 模拟

Comsol 模拟

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热力学运输性质简介

热传导方程是描述热量传递过程的基本方程，具有广泛的应用。在该部分中，我们简要讨论了热传导方程的一般形式。

Learn More

Mathematica 模拟

在这里，我们采用了mathematica模拟.

简介

Introduction

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Comsol 模拟

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热传导方程

热传导方程基本形式及其拓展

Fourier最早认为，单位时间内通过的热量J与温度梯度$\frac{\mathrm{d}T}{\mathrm{d}z}$成正比，即有

$$J = -\kappa \cdot S \cdot \frac{\mathrm{d}T}{\mathrm{d}z}$$

该公式考虑的是一维的情况，对于全空间场，我们不难得出热传导的微分形式为

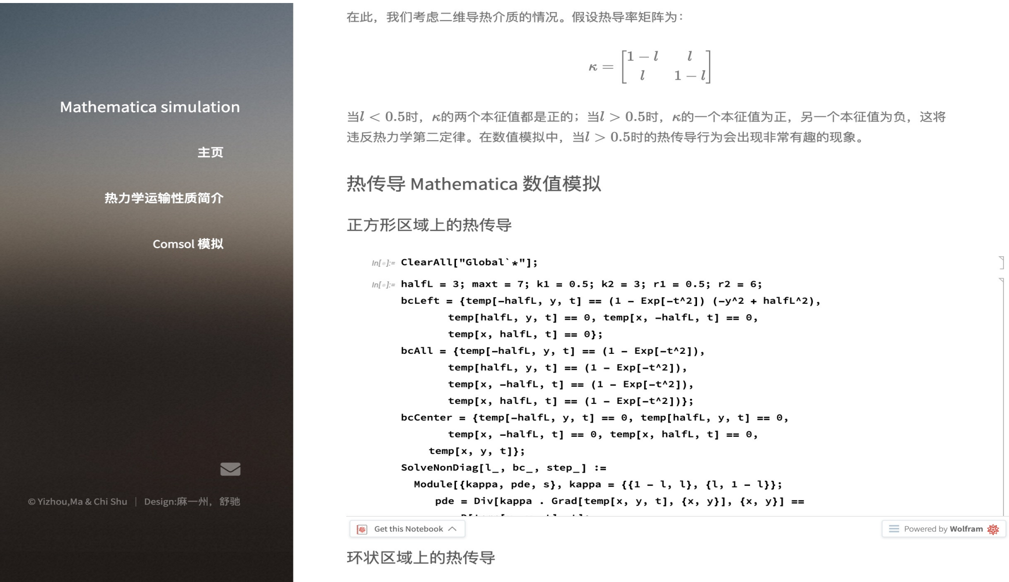
$$\dot{Q} = \nabla \cdot (\kappa \nabla T)$$

在三维空间中，$\nabla \cdot (\kappa \nabla T)$表示温度场的散度乘以导热系数的梯度。这个方程表示热量在空间中的扩散情况。进一步，我们可以有

$$\rho \cdot c_p \frac{\partial T}{\partial t} = \nabla \cdot (\kappa \nabla T)$$

其中，ρ表示介质的密度，c_p表示比热容，$\frac{\partial T}{\partial t}$表示温度随时间的变化率。该公式描述了在各向同性介质中热是如何传播的。这也被称为热传导方程，它反映了热量的扩散和传输机制。

Mathematica 模拟



Mathematica simulation

主页

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在此，我们考虑二维导热介质的情况。假设热导率矩阵为：

$$\kappa = \begin{bmatrix} 1-l & l \\ l & 1-l \end{bmatrix}$$

当$l < 0.5$时，κ的两个本征值都是正的；当$l > 0.5$时，κ的一个本征值为正，另一个本征值为负，这将违反热力学第二定律。在数值模拟中，当$l > 0.5$时的热传导行为会出现非常有趣的现象。

热传导 Mathematica 数值模拟

正方形区域上的热传导

```
In[•]:= ClearAll["Global`*"];

In[•]:= halfL = 3; maxt = 7; k1 = 0.5; k2 = 3; r1 = 0.5; r2 = 6;
        bcLeft = {temp[-halfL, y, t] == (1 - Exp[-t^2]) (-y^2 + halfL^2),
                temp[halfL, y, t] == 0, temp[x, -halfL, t] == 0,
                temp[x, halfL, t] == 0};
        bcAll = {temp[-halfL, y, t] == (1 - Exp[-t^2]),
                temp[halfL, y, t] == (1 - Exp[-t^2]),
                temp[x, -halfL, t] == (1 - Exp[-t^2]),
                temp[x, halfL, t] == (1 - Exp[-t^2])};
        bcCenter = {temp[-halfL, y, t] == 0, temp[halfL, y, t] == 0,
                temp[x, -halfL, t] == 0, temp[x, halfL, t] == 0,
            temp[x, y, t]};
        SolveNonDiag[l_, bc_, step_] :=
          Module[{kappa, pde, s}, kappa = {{1 - l, l}, {l, 1 - l}};
            pde = Div[kappa . Grad[temp[x, y, t], {x, y}], {x, y}] ==
```

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环状区域上的热传导

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热隐身技术的实际应用

第一张图：热力学梯度

左侧图像展示了温度场的热力学梯度。图中颜色从红色到黄色渐变，红色区域表示温度较高，黄色区域表示温度较低。在图中间区域，我们可以看到一个圆形区域，该区域的温度均匀，没有明显的梯度变化。这表明该区域实现了热隐身，即使外部存在热源，内部的温度仍然保持均匀。

第二张图：等温线

右侧图像展示了等温线的分布。等温线表示相同温度点的连线，反映了温度场的分布情况。在外侧放置了一个热源，等温线在该热源周围弯曲，显示了热源对周围温度场的影响。然而，在中间的圆形区域内，等温线依然保持圆形，这表明该区域内部的温度不受外部热源的影响，实现了热隐身。

图片说明

参考文献

[1] Soboleva, V. Y., et al. "Cylindrical thermal invisibility cloak based on transformation thermodynamics." *Journal of Physics: Conference Series*. Vol. 1410. No. 1. IOP Publishing, 2019.

[2] 沈翔瀛,and 黄吉平."变换热学:热超构材料及其应用." *物理学报* 65.17(2016):106-132. doi:CNKI:SUN:WLXB.0.2016-17-008.

[3]M. Nakahara, “Geometry, Topology and physics”

[4]Ji-Ping Huang, “Transformation Thermotics and Extended Theories for Thermal Matamaterials”

Thanks

PART 04

Backup Slide

Details

说明：$\nabla\cdot X = \left(\frac{\partial X^\mu}{\partial x^\mu} + \Gamma^\mu_{\mu\nu} X^\nu\right) = \frac{1}{\sqrt{-g}}\left(\partial_\nu \sqrt{-g}\right)$

$$\Gamma^\alpha_{\rho\gamma} = \frac{1}{2} g^{\alpha k}\left(\partial_\gamma g_{\rho k} + \partial_\rho g_{\gamma k} - \partial_k g_{\rho\gamma}\right)$$

$$\Gamma^\mu_{\mu\nu} = \frac{1}{2} g^{\mu k}\left(\partial_\mu g_{\nu k} + \partial_\nu g_{\mu k} - \partial_k g_{\mu\nu}\right)$$

$$= \frac{1}{2} g^{\mu k}\partial_\mu g_{\nu k} - \frac{1}{2} g^{\mu k}\partial_k g_{\mu\nu} \left(\frac{1}{2} g^{k\mu}\partial_\mu g_{k\nu}\right) + \frac{1}{2} g^{\mu k}\partial_\nu g_{\mu k}$$

$$= \frac{1}{2} g^{\mu k}\partial_\nu g_{\mu k} = \frac{1}{\sqrt{-g}}\partial_\nu \sqrt{-g}$$

最后一步：$\delta \ln|\det M| \equiv \ln|\det(M+\delta M)| - \ln|\det M|$

$$= \ln \frac{\det(M+\delta M)}{\det M}$$

$$= \ln \det[M^{-1}(M+\delta M)]$$

$$= \ln \det[\mathbb{1} + M^{-1}\delta M]$$

$$= \ln(1 + \operatorname{Tr} M^{-1}\delta M)$$

$$= \operatorname{Tr} M^{-1}\delta M$$

$\therefore \delta \ln|\det g| = \operatorname{Tr} g^{-1}\delta g = g^{\alpha\beta}\delta g_{\alpha\beta}$

$$\partial_\nu \ln\sqrt{|\det g|} = \frac{1}{2} g^{\alpha\beta}\partial_\nu g_{\alpha\beta}$$

$$\frac{1}{\sqrt{\det g}}\partial_\nu \sqrt{|\det g|} = \frac{1}{2} g^{\alpha\beta}\partial_\nu g_{\alpha\beta}$$

$\therefore \Gamma^\mu_{\mu\nu} = \frac{1}{\sqrt{+g}}\cdot\partial_\nu\sqrt{+g}$

$$\nabla\cdot X = \frac{1}{\sqrt{+g}}\cdot\frac{\partial(\sqrt{+g}\,X^\mu)}{\partial x^\mu}$$

得到任意流形下的热传导方程

$$\rho C \frac{\partial T}{\partial t} = \nabla\cdot(K\cdot\nabla T)$$

$$R.H.S = \nabla\cdot\left(K^\alpha_\beta \frac{\partial}{\partial x^\alpha}\otimes dx^\beta \otimes \left(g^{\mu\nu}\frac{\partial T}{\partial x^\mu}\frac{\partial}{\partial x^\nu}\right)\right)$$

$$= \nabla\cdot\left(K^\alpha_\nu g^{\mu\nu}\frac{\partial T}{\partial x^\mu}\cdot\frac{\partial}{\partial x^\alpha}\right)$$

$$= \frac{1}{\sqrt{g}}\partial_\alpha\left(\sqrt{g}\,K^\alpha_\nu g^{\mu\nu}\frac{\partial T}{\partial x^\mu}\right)$$

$$\Rightarrow \rho C\cdot\frac{\partial T}{\partial t} = \frac{1}{\sqrt{g}}\partial_\alpha\left(\sqrt{g}\,K^\alpha_\nu g^{\mu\nu}\frac{\partial T}{\partial x^\mu}\right)$$

不同坐标下 $\tilde{K}^{\mu\nu}$ 的变换关系

$$K^{\mu\nu} = K'^{\alpha\beta}\frac{\partial x^\mu}{\partial x'^\alpha}\cdot\frac{\partial x^\nu}{\partial x'^\beta}$$

$$\frac{1}{\sqrt{g}}\tilde{K}^{\mu\nu} = \frac{1}{\sqrt{g'}}\tilde{K}'^{\alpha\beta} J^\mu_\alpha J^\nu_\beta$$

$$g'_{\alpha\beta} = g_{\mu\nu}\frac{\partial x^\mu}{\partial x'^\alpha}\cdot\frac{\partial x^\nu}{\partial x'^\beta} = g_{\mu\nu}J^\mu_\alpha J^\nu_\beta$$

$$\Rightarrow \tilde{K}^{\mu\nu} = \frac{J^T \tilde{K}'^{\alpha\beta} J}{\det J}$$

J: $x' \to x$ 的 Jacobian

关于 $\tilde{K}$ 的计算

$$(x,y) \to (r,\theta) \to (r',\theta') \to (x',y')$$

$$J^{-1} = \frac{D(x',y')}{D(r',\theta')}\cdot\frac{D(r',\theta')}{D(r,\theta)}\cdot\frac{D(r,\theta)}{D(x,y)}$$

$$= \begin{pmatrix}\cos\theta & -r'\sin\theta \\ \sin\theta & r'\cos\theta\end{pmatrix}\begin{pmatrix}\frac{R_2}{R_2-R_1} & 0 \\ 0 & 1\end{pmatrix}\begin{pmatrix}\frac{x}{\sqrt{x^2+y^2}} & \frac{y}{\sqrt{x^2+y^2}} \\ -\frac{y}{x^2+y^2} & \frac{x}{x^2+y^2}\end{pmatrix}$$

$$J^{-1} = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & r'\end{pmatrix}\begin{pmatrix}\frac{R_2}{R_2-R_1} & 0 \\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & \frac{1}{r}\end{pmatrix}\begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}$$

$$\det|J| = \frac{r}{r'}\cdot\frac{R_2-R_1}{R_2}$$

令 $\tilde{K}' = \mathbb{1}$

$$J^T\tilde{K}'J = R_\theta\begin{pmatrix}1 & 0 \\ 0 & \frac{1}{r}\end{pmatrix}\begin{pmatrix}\frac{R_2}{R_2-R_1} & 0 \\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & r'^2\end{pmatrix}\begin{pmatrix}\frac{R_2}{R_2-R_1} & 0 \\ 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 \\ 0 & \frac{1}{r}\end{pmatrix}R_{-\theta}$$

$$\therefore \tilde{K} = \begin{pmatrix}\cos\theta & -\sin\theta \\ \sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\frac{r-R_1}{r} & \\ & \frac{r}{r-R_1}\end{pmatrix}\begin{pmatrix}\cos\theta & \sin\theta \\ -\sin\theta & \cos\theta\end{pmatrix}$$