## Lab Guides 2021

Lab 1 The latent heat of vaporization of liquid nitrogen ..... 1
Lab 2 Target hitting through collision ..... 7
Lab 3 Torsional pendulum ..... 12
Lab 4 Digital Storage Oscilloscopes ..... 18
Lab 5 Wheatstone bridge ..... 23
Lab 6 Magnetic field variation along the axis of a circular coil and a Helmholtz coil ..... 27
Lab 7 Converging Lens ..... 32
Lab 8 Frank-Hertz experiment with Neon ..... 37
Lab 9 X-ray experiment ..... 41

# Fundamental Physics Laboratory, Fudan University 



## Lab-I The latent heat of vaporization of liquid nitrogen

## Goal

1. Review the physical concepts and relationships associated with the flow of heat into and out of materials.
2. Determine the latent heat of liquid nitrogen (LN).

## Related topics

Heat transfer, Specific heat, Latent heat

## Introduction

When heat energy is added to a substance, its temperature usually rises except when a change of phase occurs (e.g., solid melts into liquid or liquid vaporizes into vapor). The phase change of most material occurs without increase or decrease in its temperature. The thermal energy is absorbed to change the state of the matter involved. The amount of energy required to change the phase of a matter is called its latent heat. Conversely, as the temperature drops to a point when the phase changes, the latent heat must be released. The latent heat associated with water vapor in the atmosphere is one of the most significant factors determining the weather. It moderates temperature drops at night. And when released through the formation of water droplets drives the winds associated with storms, and even hurricane.

The historical unit of heat energy, the calorie, was defined as the amount of heat energy needed to raise the temperature of one gram of water by one degree Celsius. The calorie is now defined in terms of the SI unit of energy, the joule, by:

$$
1 \mathrm{cal}=4.184 \mathrm{~J}
$$

### 2.1 Specific Heat

When heat flows into or out of an object, its temperature changes. The connection between the change in heat energy and the change in temperature is the specific heat. The heat energy $\Delta Q$ needed to raise the temperature of a substance is related to its mass according to the formula:

$$
\begin{equation*}
\Delta Q=m c \Delta T \tag{1}
\end{equation*}
$$

Where $\Delta Q$ is the quantity of heat entering the material, $m$ is the mass of the material, $c$ is the specific heat, and $\Delta T$ is the change in the temperature of the material. The specific heat $c$ is defined as the heat energy needed to raise the temperature of one gram of a substance by one degree Celsius $\left({ }^{\circ} \mathrm{C}\right)$. In general, the value of the specific heat of a solid substance is predominantly a function of temperature, though small variation of the specific heat occurs due to variation in pressure or volume. For relatively small temperature interval, the value of $c$ in Eq. (1) is taken as its average value over the temperature interval between its initial and final temperatures, $\mathrm{T}_{i}$ and $\mathrm{T}_{f}$ respectively.

The specific heat of substances varies with the temperature, as for example the average value of the specific heat of aluminum is $0.17 \mathrm{cal} / \mathrm{g} \cdot{ }^{\circ} \mathrm{C}$ between room temperature and liquid nitrogen
temperature while it remains essentially constant $\left(0.215 \mathrm{cal} / \mathrm{g} \cdot{ }^{\circ} \mathrm{C}\right)$ from room temperature to $100^{\circ} \mathrm{C}$. In contrast to this, the specific heat of water decreases from $1.00728 \mathrm{cal} / \mathrm{g} \cdot{ }^{\circ} \mathrm{C}$ to $0.99795 \mathrm{cal} / \mathrm{g} \cdot{ }^{\circ} \mathrm{C}$ in the temperature range 0 to $35^{\circ} \mathrm{C}$ and then increases to $1.00697 \mathrm{cal} / \mathrm{g} \cdot{ }^{\circ} \mathrm{C}$ at $100^{\circ} \mathrm{C}$.

### 2.2 Latent Heat

Under certain circumstances the heat supplied to (or removed from) a substance does not cause a change in its temperature, instead it causes a change of phase (e.g., boiling, melting, freezing or condensation). The relationship between the heat added and the amount of material that changes from one phase to another is,

$$
\begin{equation*}
\Delta Q=L \mathrm{~g} n \tag{2}
\end{equation*}
$$

Where:
a. $\Delta Q$ is the quantity of heat supplied to (or removed from) the material.
b. $\quad L$ is the latent heat associated with transformation in question, e.g., $L_{\mathrm{f}}$ (the latent heat of freezing), $L_{\mathrm{v}}$ (the latent heat of vaporization) or $L_{\mathrm{S}}$ (the latent heat of sublimation).
c. $\quad m$ is the mass of material which undergoes a transformation of state [liquid to its solid state (freezing) or into its vapor state (vaporization); or the transformation of solid directly into vapor (sublimation)].

### 2.3 Rate of Heat Flow

If we apply a temperature gradient to an object, one end is kept hot and the other end cold. Then heat flows from hot to cold. At a steady state, the rate $R$ (in calories/second or $\mathrm{J} / \mathrm{s}$ ) at which heat flows through the object is given by:

$$
\begin{equation*}
R=\left(\frac{k A}{\Delta x}\right) \Delta T \tag{3}
\end{equation*}
$$

Where $k$ is its thermal conductivity [e.g., for styrofoam, $k=6 \times 10^{-5} \mathrm{cal} /\left(\mathrm{cm} \cdot{ }^{\circ} \mathrm{C} \cdot \mathrm{s}\right)$ ], $A$ is the area of the material through which the heat transfer takes place, $\Delta x$ is its thickness, and $\Delta T$ is the temperature difference ( $T_{\text {hot }}-T_{\text {cold }}$ ).

### 2.4 Determination of the Latent Heat of Liquid Nitrogen

In this part of the experiment, the value of the latent heat of vaporization $L_{v}$ of liquid nitrogen (Reference value: 199.2 J/g) will be determined.

This part of the experiment will involve the immersion of a copper cylinder at room temperature into liquid nitrogen and measuring the quantity of the liquid nitrogen $\Delta m$ evaporated as the copper cylinder cools to the temperature of liquid nitrogen $\left(-195.8^{\circ} \mathrm{C}\right)$. The amount of the heat energy transferred to liquid nitrogen by the copper cylinder should be equal to the heat energy required to evaporate $\Delta m$ grams of the liquid nitrogen. That is:

$$
\begin{equation*}
L_{v} \Delta m_{L N}=\overline{C_{C u}} m_{C u} \Delta T \tag{4}
\end{equation*}
$$

Thus, we get:

$$
\begin{equation*}
L_{v}=\frac{\overline{C_{C u}} m_{C u} \Delta T_{C u}}{\Delta m_{L N}}=\frac{\overline{C_{C u}} m_{C u}\left(T_{i, C u}-T_{f, C u}\right)}{\Delta m_{L N}} \tag{5}
\end{equation*}
$$

where:
$L \mathrm{v}=$ Latent heat of vaporization of liquid nitrogen
$m_{\mathrm{Cu}}=$ Mass of the copper cylinder immersed into liquid nitrogen.
$\overline{C_{C u}}=$ Average specific heat of copper in the temperature range from room temperature to the temperature of liquid nitrogen $\left(-195.8^{\circ} \mathrm{C}\right)$
$T_{i, \mathrm{Cu}}=$ Initial temperature of the copper cylinder: room temperature.
$T_{f, \mathrm{Cu}}=$ Final temperature of the copper cylinder at the B.P.T. of LN $\left(-195.8^{\circ} \mathrm{C}\right)$.
$\Delta m_{L N}=$ Mass of liquid nitrogen evaporated by the immersion of the copper cylinder.
Since the specific heat of copper in this big range $\left(-195.8^{\circ} \mathrm{C} \sim\right.$ room temperature $)$ is not a constant, we can get this value through integration if we have the specific heat as a function of temperature. Here, we introduce another way to calculate the total heat energy transferred to liquid nitrogen by the copper cylinder.

After the first part of the lab, when the copper cylinder has the same temperature with the liquid nitrogen (i.e, they're in thermal equilibrium), we quickly move the copper cylinder into a cup of water(Why we should move the copper cylinder quickly?). As the copper now is at a very low temperature $\left(-195.8^{\circ} \mathrm{C}\right)$, it will absorb energy form the "hot" water and then they will be in thermal equilibrium. In order to avoid the background energy contribution, you can use the copper stirrer to speed up the energy transfer process, and the lowest value of the thermometer should be the thermal equilibrium temperature(Here, we should avoid that the thermometer has direct contact with the copper cylinder. Why? If there is direct contact, what will be seen?) The energy transferred from water to copper is as follow:

$$
\begin{equation*}
Q_{1}=\left(m_{w} c_{w}+m_{a} c_{a}+m_{c} c_{c}+h_{t}\right)\left(\theta_{2}-\theta_{3}\right)=m_{c u} \overline{c_{c u}}\left[\theta_{3}-(-195.8)\right] \tag{6}
\end{equation*}
$$

where:
$m_{w}=$ Mass of the water.
$c_{w}=$ Specific heat of water.
$m_{a}=$ Mass of the small aluminum cup.
$C_{a}=$ Specific heat of aluminum.
$m_{c}=$ mass of the stirrer(in our lab ,it is from aluminum).
$c_{c}=$ Specific heat of stirrer material(here aluminum) in the temperature range from $\theta_{2}$ to $\theta_{3}$, it's almost constant.
$\theta_{2}=$ Initial temperature of water and aluminum cup.
$\theta_{3}=$ Final temperature of the system (water, aluminum cup and copper cylinder).
$h_{t}=0, h_{t}$ means the thermal capacity of the part of the electric thermometer which is immersed under water. In our lab, this value can be considered to be small enough to be neglected.

The second part: the energy needed by the copper cylinder whose temperature increases from $\theta_{3}$ to $\theta_{1}$ (the initial temperature: room temperature)

$$
\begin{equation*}
Q_{2}=m_{c u} c_{c u}\left(\theta_{1}-\theta_{3}\right) \tag{7}
\end{equation*}
$$

where:
$m_{\mathrm{cu}}=$ Mass of the copper cylinder immersed into liquid nitrogen.
$c_{c u}=$ Specific heat of copper in the temperature range from $\theta_{1}$ to $\theta_{3}$, it's almost constant.
$\theta_{1}=$ Initial temperature of the copper cylinder: room temperature.
$\theta_{3}=$ Final temperature of the system (water, aluminum cup and copper cylinder).
So the Latent heat of vaporization of liquid nitrogen can be written as follow:

$$
\begin{equation*}
L_{v}=\frac{1}{\Delta m_{L N}}\left[\left(m_{w} c_{w}+m_{a} c_{a}+m_{c} c_{c}+h_{t}\right)\left(\theta_{2}-\theta_{3}\right)+m_{c u} c_{c u}\left(\theta_{1}-\theta_{3}\right)\right] \tag{8}
\end{equation*}
$$

## Experimental Apparatus

1. Electric thermometers
2. Copper cylinder
3. Water
4. Warm isolated cup
5. Stop watch
6. Electric balance
7. Liquid nitrogen

## Procedure

## 2) Measurement of the Latent Heat of Vaporization of Liquid Nitrogen

- Measure and record the mass ( $\boldsymbol{m}_{\boldsymbol{C u}}$ ) of the copper cylinder which will be placed in liquid nitrogen.
- Put liquid nitrogen into the warm isolated cup to $1 / 3-2 / 3$ full. Put the cup and the copper cylinder on the electron balance as shown in the figure right. Start the timer and record the time duration for each mass reduction of 0.5 gram, you should record at least six sets of data.

- Put the copper cylinder into the liquid nitrogen (write down time of this moment as $t_{\mathrm{b}}$ ). Observe the phenomenon and record in detail. Do not stop the watch.
- After you have seen the second fog (write down the time of this moment as $t_{\mathrm{c}}$ ), measure and record the time duration for each mass reduction of 0.5 gram (at least six sets of data). The time duration should be recorded continuously together with the previous two steps so that you can draw a diagram of mass change in dependence of time duration as shown right.

- Determine the mass change of liquid nitrogen due to the input of the copper cylinder from the diagram you draw.

Data record for the measurement of the Latent Heat of Vaporization of Liquid Nitrogen

Table 1:

| $m_{c u}(\mathrm{~kg})$ |  | $c_{c u}\left(\mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)$ |  |
| :--- | :--- | :--- | :--- |
| $m_{c}(\mathrm{~kg})$ |  | $c_{c}\left(\mathrm{~J} / \mathrm{kg} \bullet{ }^{\circ} \mathrm{C}\right)$ |  |
| $m_{a}(\mathrm{~kg})$ |  | $c_{a}\left(\mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)$ |  |
| $m_{a+w}(\mathrm{~kg})$ |  | $c_{w}\left(\mathrm{~J} / \mathrm{kg} \cdot{ }^{\circ} \mathrm{C}\right)$ |  |
| $m_{w}(\mathrm{~kg})$ |  |  |  |

Table 2:

| $\theta_{1}\left({ }^{\circ} \mathrm{C}\right)$ |  |
| :--- | :--- |
| $\theta_{2}\left({ }^{\circ} \mathrm{C}\right)$ |  |
| $\theta_{3}\left({ }^{\circ} \mathrm{C}\right)$ |  |

Table 3:

| ab | 1 | 2 | 3 | 4 | 5 | 6 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| time(s) |  |  |  |  |  |  |
| Mass(g) |  |  |  |  |  |  |
| $t_{\mathrm{b}}$ : |  |  |  |  |  |  |
| $t_{c}$ |  |  |  |  |  |  |
| cd | 1 | 2 | 3 | 4 | 5 | 6 |
| time(s) |  |  |  |  |  |  |
| Mass(g) |  |  |  |  |  |  |

## Tasks:

Evaluate the value of the latent heat of LN under ambient condition. Compare your measured value with the reference value. Discuss the main possible reasons, that could make your measured value larger/smaller than the reference value.

## Questions

1. What have you learned from this lab?
2. If you are going to do the lab again, how can you make improvement?

## References

1. Adapted from "Experiment 3\# The Latent Heat of Vaporization of Liquid Nitrogen" http://www.physicslabs.umb.edu/Physics/spring06/Exp3_182_Spr06.pdf

## Lab 2 -Mechanics laboratory

## ——Target hitting through collision

## Goal

This experiment is designed for the review of mechanics. In this experiment, learn to predict, operate, observe, analysis, and then improve......

## Related topics

Projectile motion, collision, conservation law

## Introduction

In this experiment, you will be challenged to hit the target with a steel ball that is in projectile motion after being collided by another steel ball of the same mass at the bottom of its swing.

The followings are definitions of some related concepts:

- Projectile motion is the motion of an object moving through the air, subject to the acceleration of gravity. The object is called a projectile, and its path is called its trajectory.
- A conservative force is one, like gravity, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken.
- Conservation of mechanical energy: the total mechanical energy (i.e. sum of kinetic energy and potential energy) of a system that experiences only conservation forces is constant, can only changes forms among kinetic energy and various types of potential energy. Such a system is called a closed system.
- Conservation of momentum: the total momentum of any isolated system, with any numbers of bodies in it, is conserved. An isolated system is defined to be one for which the net external forces is zero.
- An elastic collision is one that conserves internal kinetic energy. Internal kinetic energy is the sum of the kinetic energies of the bodies in the system.
- An inelastic collision is one in which internal kinetic energy changes. This means the forces between colliding objects may remove or add internal kinetic energy.
For more details, please refer to your text book "College Physics", especially §3.4, §6.4-6, and §7.3-6.


Fig. 1 Schematic diagram of the experiment device

## Experiment device

Figure 1 is the sketch of the experiment device．During the experiment，very careful adjustment is necessary and helpful．Meanwhile，one should observe and analyze the forthcoming phenomenon roundly．

## Procedure

1．Place the target－sheet in an appropriate position，the centerline should coincide with the centerline of the bottom plate．

2．Place the projectile ball in its position
3．Adjust the system so that after release，the ball will move in the vertical plane as decided by the projectile ball and along the center of the target－sheet．
4．Measure the position of the center of the target－sheet．Calculate at which height the swing ball would be left free，so that the projectile ball will hit the center of the target－sheet．
5．Try the experiment once to verify your calculation．
6．If the center of the target－sheet is not hit，analyze the possible reason（s）and recalculate．Try again to verify the new calculation．
7．Procedure 6 may be repeated for several times until your＂score＂is better than $9^{\text {th }}$ circle （inclusive）．
Log your measured data，calculation，operation，phenomena and result in detail．Discuss the possible reasons which may result in energy lost in this experiment．

## Optional

If you have time，try this procedure again with a projectile ball of different mass，size，or composition．

## Questions

1．What is the main cause for energy lost in this experiment？
2．Write down the related equations you needed in this experiment．Specify the ranges of their application．

## References

1．Paul Peter Urone，College Physics，Chinese Machine Press，Beijing 2002
2．Physics 8A－－Fall 2000 Lab Manual，University of California at Berkley
3．沈元华，陆申龙 基础物理实验（Fundamental Physics Laboratory）高等教育出版社 北京 2004 pp．29－32

## Operation guide——Lab 2 Mechanics laboratory

## 1. Formulas derivation



Fig1: Schematic diagram
a) $\boldsymbol{h}_{0}$ : Initial height of the strike-ball.
b) $m_{1}:$ Mass of the strike ball.
c) $\boldsymbol{m}_{\mathbf{2}}$ : Mass of the struck ball.
d) $\boldsymbol{D}:$ Diameter of the ball, which should be measured for 5 times. Please read out from the vernier calipers correctly.
e) $y$ : Height of the adjustable stage.
f) $\boldsymbol{x}$ : Distance between the center of the target sheet and that of the adjustable stage.

## Please answer the following questions:

1) The initial gravitational potential energy of strike ball: $\underline{E_{0}}=$
2) Velocity of strike ball at the bottom of its swing: $\underline{v_{1}}=$
3) Elastic Collision:

Conservation of momentum: $\qquad$ .
Conservation of energy: $\qquad$ .
Velocity of struck ball after being hit: $\underline{v}_{2}=$ $\qquad$ .
If $\boldsymbol{m}_{1}=\boldsymbol{m}_{2}$, rewrite the velocity of struck ball: $\underline{v}_{2}=$ $\qquad$ .
4) Horizontal projectile motion:

Expression of drop height $\boldsymbol{y}$ and the distance $\boldsymbol{x}$ (which are functions of velocity $v_{2}$ and time $t$ ): $\boldsymbol{y}=$ , $\boldsymbol{x}=$ $\qquad$ .
5) The values of $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{m}_{1}$, and $\boldsymbol{m}_{2}$ can be measured easily in the lab. Please write the initial height of the strike ball by using the above quantities ( $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{m}_{1}$, and $\boldsymbol{m}_{2}$ ): $\underline{h_{0}=}$

If $\boldsymbol{m}_{1}=\boldsymbol{m}_{2}$, rewrite the initial height: $\underline{h}_{0}=$ $\qquad$ .
6) Since total energy of the system (strike ball \& struck ball) will be lost in the
actual experiment, the actual displacement of struck ball $\boldsymbol{x}$ ' is always smaller than the distance $\boldsymbol{x}$. Similar with the theoretical expression of initial height $h_{0}$, you can easily write down the theoretical expression of $\boldsymbol{h}$ ' which corresponds to the displacement $\boldsymbol{x}$ ': $\boldsymbol{h}^{\prime}=$

The lost energy in this trial is: $\Delta \boldsymbol{E}=$
7) In order to compensate the energy loss in previous trial, we should increase the initial gravitational potential energy of strike ball, in other words, the start height of the strike ball should be raised a little: $\boldsymbol{\Delta H}=$
8) Procedure 7) may be repeated for several times until your "score" is better than $9^{\text {th }}$ circle (inclusive).

## 2. Data Record

1) Do some adjustments in order to make sure your strike ball and struck ball had a head-on collision.
2) Try the experiment once to verify your calculation.
3) If the center of the target-sheet is not hit, analyze the possible reason(s) and make a new calculation. Readjust the device if necessary, especially the start height of the strike ball. (Write down the details of your observation, calculation and adjustment.) Try again to verify the new calculation.

## Data table No. 1:

(Pay attention to the significant figures of your values.)

| $D / \mathbf{c m}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |
| $m_{1} / \mathrm{g}$ | $m_{2} / \mathrm{g}$ | $y / \mathbf{c m}$ | $x / \mathrm{cm}$ | $h_{0}($ Calculated $) / \mathrm{cm}$ |  |
|  |  |  |  |  |  |

Data table No. 2:

| $\mathrm{H}_{0} / \mathrm{cm}$ | No. | Score | Shot position $\left(x^{\prime}, z^{\prime}\right) / \mathrm{cm}$ | Possible reasons for the "miss-hit" | $\Delta E_{1} / J$ (average) | $\Delta H_{1} / \mathrm{cm}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 |  |  |  |  |  |
|  | 2 |  |  |  |  |  |
|  | 3 |  |  |  |  |  |
| $\begin{gathered} H_{1}= \\ H_{0}+\Delta H_{I} \end{gathered}$ | No. | Score | Shot position $\left(x^{\prime}, z^{\prime}\right) / \mathrm{cm}$ | Possible reasons for the "miss-hit" | $\Delta E_{2} / J$ <br> (average) | $\Delta H_{2} / \mathrm{cm}$ |
|  | 1 |  |  |  |  |  |
|  | 2 |  |  |  |  |  |
|  | 3 |  |  |  |  |  |
| $\begin{gathered} H_{2}= \\ H_{1}+\Delta H_{2} \end{gathered}$ | No. | Score | Shot position $\left(x^{\prime}, z^{\prime}\right) / \mathrm{cm}$ | Possible reasons for the "miss-hit" | $\Delta E_{3} / J$ (average) | $\Delta H_{3} / \mathrm{cm}$ |
|  | 1 |  |  |  |  |  |
|  | 2 |  |  |  |  |  |
|  | 3 |  |  |  |  |  |
| Total energy loss: $\Delta E=$ |  |  |  |  |  |  |

$z^{\prime}$ is the distance of the shot position to the middle line of the target paper. It should be a small value.

## Lab 3 Torsional pendulum

## Goal

This experiment is designed for a review of the rotation of rigid body

## Related topics

Rotational motion, Oscillatory motion, Elasticity

## Introduction

A torsional pendulum, or torsional oscillator, consists usually of a disk-like mass suspended from a thin rod or wire (see Fig. 1). When the mass is twisted about the axis of the wire, the wire exerts a torque on the mass, tending to rotate it back to its original position. If twisted and released, the mass will oscillate back and forth, executing simple harmonic motion. This is the angular version of the bouncing mass hanging from a spring. This gives us an idea of moment of inertia. We will measure the moment of inertia of several different shaped objects. As comparison, these moment of inertia can also be calculated theoretically. We can also verify the parallel axis theorem. Given that the moment of inertia of one object is known, we can determine the torsional constant K.


Fig. 1 Schematic diagram of a torsional pendulum

This experiment is based on the torsional simple harmonic oscillation with the analogue of displacement replaced by angular displacement $\theta$, force by torque $M$, and the spring constant by torsional constant $\boldsymbol{K}$. For a given small twist $\theta$ (sufficiently small), the experienced reaction is given by

$$
\begin{equation*}
M=-K \theta \tag{1}
\end{equation*}
$$

This is just like the Hooke's law for the springs. If a mass with moment of inertia $\boldsymbol{I}$ is attached to the rod, the torque will give the mass an angular acceleration $\boldsymbol{\alpha}$ according to $M=I \alpha=I \frac{d^{2} \theta}{d t^{2}}$. Then we get the following equation:

$$
\begin{equation*}
\frac{d^{2} \theta}{d t^{2}}=-\frac{K}{I} \theta \tag{2}
\end{equation*}
$$

Hence on solving this second order differential equation we get:

$$
\begin{equation*}
\theta=A \cos (\omega t+\varphi) \tag{3}
\end{equation*}
$$

$$
\begin{gather*}
\omega=\sqrt{K / I}  \tag{4}\\
T=2 \pi \sqrt{I / K} \tag{5}
\end{gather*}
$$

Where, $\omega$ is the angular velocity of simple harmonic oscillation, and $T$ is the period. So, if the period $\boldsymbol{T}$ and the $\boldsymbol{K}$ are known, the inertia of the rotated object can be expressed as:

$$
\begin{equation*}
I=\frac{K}{4 \pi^{2}} T^{2} \tag{6}
\end{equation*}
$$

The additive property of inertia: The moment of inertia of a composite system is the sum of the moments of inertia of its component subsystems (all taken about the same axis):

$$
\begin{equation*}
I=\sum_{j} I_{j}(j=1,2,3 \ldots) \tag{7}
\end{equation*}
$$

## Experiment device



Fig. 2 Schematic diagram of the experiment device

## Procedure

- Familiarize yourself with the operation of the device. Adjust the device carefully so that it is ready for the measurement (i.e. the base is placed horizontally.).
- Determine the torsional constant $\boldsymbol{K}$ of the spiral spring with a plastic cylinder. Employ the theoretically calculated moment of inertia of the plastic cylinder as a known value.
- Measure the moment of inertia of differently shaped objects. Compare the measured results with the theoretically calculated values.


## Optional

- Verify the parallel axis theorem.


## Questions

1．On which factors is the moment of inertia dependent？
2．What are the causes that may bring error to our measurement？

## References

1．沈元华，陆申龙 基础物理实验（Fundamental Physics Laboratory）高等教育出版社 北京 2004 pp．100－103
2．Ravitej Uppu Torsional Pendulum http：／／www．cmi．ac．in／～ravitej／lab／4－torpen．pdf

## Appendix：

［1］Uncertainty of Plastic cylinder：

$$
\begin{aligned}
& I_{P C}=\frac{1}{2} m r^{2}=\frac{1}{8} m D^{2} \boldsymbol{\&} \quad u\left(I_{P C}\right)=\sqrt{\left[\frac{u(m)}{m}\right]^{2}+\left[2 \times \frac{u(\mathrm{D})}{D}\right]^{2}} \times I_{P C} \\
& \qquad u(m)=\sqrt{u_{B 1}^{2}(m)+u_{B 2}^{2}(m)} \\
& \text { Where, } u_{B 1}(m)=d=0.1 g \\
& \quad \begin{array}{l}
\quad \\
u_{B 2}(m)=\frac{a}{\sqrt{3}}=\frac{0.2}{\sqrt{3}} g
\end{array} u_{A}(D)=\sqrt{\frac{\sum\left(D_{i}^{2}-\bar{D}\right)^{2}}{n(n-1)}} . \\
& \\
& u_{B 2}(D)=\frac{a}{\sqrt{3}}=\frac{0.02}{\sqrt{3}} \mathrm{~mm}
\end{aligned}
$$

［2］Uncertainty of $K$ ．

$$
u(K)=\sqrt{\left[\frac{u\left(I_{P C}\right)}{I_{P C}}\right]^{2}+\left[\frac{u\left(T^{2}-T_{0}^{2}\right)}{T^{2}-T_{0}^{2}}\right]^{2}} \times K
$$

Please note：

$$
u\left(T^{2}-T_{0}^{2}\right)=\sqrt{\left[u\left(T^{2}\right)\right]^{2}+\left[u\left(T_{0}^{2}\right)\right]^{2}} \quad \& \quad u\left(T^{2}\right)=2 T \times u(T)
$$

Since we measure the time of 5 periods，so for each period，we have $T=\frac{1}{5} t$ and

$$
\begin{aligned}
u(t) & =\sqrt{u_{A}^{2}(t)+u_{B 2}^{2}(t)} \\
u(T)=\frac{1}{5} u(t) . \text { Where, } u_{A}(t) & =\sqrt{\frac{\sum\left(t_{i}-\bar{t}\right)^{2}}{n(n-1)}} . \\
u_{B 2}(t) & =\frac{a}{\sqrt{3}}=\frac{0.01}{\sqrt{3}} s
\end{aligned}
$$

## Operation Guide——Lab 3 Torsional Pendulum

## Contents

1. Familiarize yourself with the operation of the device. Adjust the device carefully so that it is ready for the measurement. Such as: learn how to operate the digital timer; adjust the stage so that the device is placed horizontally (Please discuss with your partner why we need to do so?).
2. Determine the torsional constant $\boldsymbol{K}$ of the spiral spring with a plastic cylinder.
1) Install the object holder. Adjust the chopping bar, so that it can block the light from the diode to the detector when it moves. Measure the time duration for $\mathbf{5}$ periods of the torsional pendulum with only the object holder $\boldsymbol{t}_{\boldsymbol{0}}$. So the period: $T_{0}=\frac{1}{5} t_{0}$.

## Data table No. 1:

| No. | $\mathbf{1}$ | $\mathbf{2}$ | 3 | Average |
| :---: | :---: | :---: | :---: | :---: |
| $t_{0} / \mathrm{s}$ |  |  |  |  |

2) Measure the mass of the plastic cylinder with an electric balance (one measurement is enough) and its diameter for $\mathbf{3}$ times with a vernier caliper. Install the plastic cylinder to the object holder. Measure the time duration for 5 periods of the torsional pendulum with the object plastic cylinder $t$. So the period: $T=\frac{1}{5} t$.

## Data table No. 2:

| No. | 1 | 2 | 3 | Average |
| :---: | :---: | :---: | :---: | :---: |
| $D / \mathrm{cm}$ |  |  |  |  |
| $t / \mathrm{s}$ |  |  |  |  |
| $m / \mathrm{g}$ |  |  |  |  |

$\diamond$ The experimental expression of inertia of the object holder $\boldsymbol{I}_{0}: I_{0}=\frac{K}{4 \pi^{2}} T_{0}^{2}$;
$\diamond$ The theoretical expression of inertia of the plastic cylinder $\boldsymbol{I}_{\mathbf{p c}}$ :

$$
I_{P C}=\frac{1}{2} m r^{2}=\frac{1}{8} m D^{2}
$$

$\diamond$ The total moment of inertia of the object holder and the plastic cylinder $\boldsymbol{I}$ :

$$
I=I_{0}+I_{p c}=\frac{K}{4 \pi^{2}} T^{2}
$$

The expression of the torsional constant $\boldsymbol{K}$ can be derived easily as:

$$
\underline{K}=
$$

Substitute the measurement results of the qualities of data table 2 into the above expression of $\boldsymbol{K}$, get the $\boldsymbol{K}$ value and estimate the Uncertainty. Write down the calculation process.
3. Measure the moment of inertia of differently shaped objects: a metal barrel, an iron ball and a long-thin metal bar (Optional). Compare the measured results with the theoretically calculated values.

1) The metal barrel:
a) Measure the time duration for $\mathbf{5}$ periods of the torsional pendulum with the object
metal barrel $t$. So the period: $T=\frac{1}{5} t$. The previously measured $T_{0}$ can be used again.
b) The theoretical equation for the moment of inertia of a barrel is $I_{t}=\frac{1}{8} m\left(D_{1}^{2}+D_{2}^{2}\right)$. Calculate the experimentally measured value $I$ and the theoretical value $\boldsymbol{I}_{t}$. Calculate the percentage difference between them.

## Data table No. 3:

| No. | $\mathbf{1}$ | $\mathbf{2}$ | 3 | Average |
| :---: | :--- | :--- | :--- | :--- |
| Inner diameter $\boldsymbol{D}_{I} / \mathrm{cm}$ |  |  |  |  |
| Outer diameter $\boldsymbol{D}_{2} / \mathrm{cm}$ |  |  |  |  |
| $\boldsymbol{t} / \mathrm{s}$ |  |  |  |  |
| $\mathrm{m} / \mathrm{g}$ |  |  |  |  |

2) The iron ball:
a) Measure the time duration for $\mathbf{5}$ periods of the torsional pendulum with the object iron ball $t$. So the period: $T=\frac{1}{5} t$. Here, the $\boldsymbol{T}_{0}$ should be measured again with the small holder only.
b) The theoretical equation for the moment of inertia of a ball is $I_{t}=\frac{2}{5} m r^{2}$. Carry out the similar evaluation as above.
Data table No. 4:

| No. | 1 | 2 | 3 | Average |  |
| :---: | :--- | :--- | :--- | :--- | :---: |
| $D / \mathrm{cm}$ |  |  |  |  |  |
| $t / \mathrm{s}$ |  |  |  |  |  |
| $t_{0}\left(5 \times \mathrm{T}_{0}\right) / \mathrm{s}$ |  |  |  |  |  |
| $m / \mathrm{g}$ |  |  |  |  |  |

3) The long-thin metal bar(Optional):

Data table No. 5:

| No. | $\mathbf{1}$ | 2 | 3 | Average |  |
| :---: | :--- | :--- | :--- | :--- | :---: |
| $D / \mathrm{cm}$ |  |  |  |  |  |
| $t / \mathrm{s}$ |  |  |  |  |  |
| $t_{0}\left(5 \times \mathbf{T}_{0}\right) / \mathrm{s}$ |  |  |  |  |  |
| $m / \mathrm{g}$ |  |  |  |  |  |
| $l / \mathrm{cm}$ |  |  |  |  |  |

4. Verify the parallel axis theorem (Optional)

Please design a lab to verify the parallel axis theorem experimentally.
Hints: Draw a diagram for $T^{2}$ versus $m_{1} d_{1}{ }^{2}+m_{2} d_{2}{ }^{2}$. Try to derive the $K^{\prime}$ value from the slope and understand the meaning of cross-section point between the obtained line and the $\mathbf{Y}$-axis.

Mass $m_{1}=$ $\qquad$ g, $m_{2}=$ $\qquad$ g.

Data table No. 6:

| Position / cm |  | 5.00 | 10.00 | 15.00 | 20.00 | 25.00 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} t / \mathrm{s} \\ (\mathbf{5} \times \mathrm{T}) \end{gathered}$ | 1 |  |  |  |  |  |
|  | 2 |  |  |  |  |  |
|  | 3 |  |  |  |  |  |
|  | Average |  |  |  |  |  |
| $\mathbf{T}^{2} / \mathbf{s}^{2}$ |  |  |  |  |  |  |
| $\begin{gathered} m_{1} d_{1}{ }^{2}+m_{2} d_{2}{ }^{2} \\ / 10^{4} \mathrm{~g} \cdot \mathrm{~cm}^{2} \end{gathered}$ |  |  |  |  |  |  |

## Lab 4 Converging Lens

## Goal

To determine the focal lengths of converging lenses by different methods, and to learn the structure of simple optical instrument such as microscope and telescope.

## Related topics

Geometrical optics, optical instrument

## Introduction

First, some definitions: a converging lens is defined as a lens with two convex surfaces. A diverging lens is a lens with two concave surfaces. An object is any article that is being viewed through an optical system; e.g., when one looks at a tree through a camera lens, telescope, or the naked eye (all are examples of optical systems), the tree is referred to an optical object.

There are many arrangements of lenses and mirrors that will redirect light in this manner. Telescopes that use lenses as their objective, or light-gathering element, are called refractive telescopes. The refractive telescope that Galileo constructed, for instance, uses two converging lenses in series. Telescopes that use mirrors as their objective are called reflective telescopes. Sir Issac Newton was the first to figure out that mirrors could be used to focus light instead of lenses. Over the years, reflective telescopes have come to dominate the field of astronomy, resulting in many improvements.

The lenses and mirrors that make up telescopic systems have two key characteristics. The first is the focal length ( $\boldsymbol{f}$ ) of a lens. Focal length may be loosely defined as the distance from a lens at which a distant object will produce an image. Object Distance ( $\boldsymbol{d}_{o}$ ) is the distance from an object to the lens. An image is the likeness of an object produced at a point in space by a lens or system of lenses, again, a lens, a telescope, a pair of binoculars or your eye. Image Distance ( $\boldsymbol{d}_{\boldsymbol{i}}$ ) is the distance from a lens or other optical element to an image.

Focal length $(f)$ is related to the object distance $\left(d_{o}\right)$ and the image distance $\left(d_{i}\right)$ by the following thin lens equation:

$$
\frac{1}{d_{i}}+\frac{1}{d_{o}}=\frac{1}{f}
$$

Rewriting the preceding equation, we can get Equation 1:

$$
\begin{equation*}
f=\frac{d_{i} \times d_{o}}{d_{i}+d_{o}} \tag{1}
\end{equation*}
$$

## Example:

Suppose that you have computed an average object distance for a lens of 90 cm , and an average image distance of 10 cm . In this case, $\boldsymbol{d}_{\boldsymbol{o}}$ (the object distance) is 90 cm and $\boldsymbol{d}_{\boldsymbol{i}}$ (the image distance) is 10 cm . You need to solve: $\frac{1}{90}+\frac{1}{10}=\frac{1}{f}$. Or in the alternative form: $f=\frac{90 \times 10}{90+10}$. The answer is the focal length of the lens. In this case, $f=9.0 \mathrm{~cm}$.

Once you measure the image distance, you can calculate the magnification of the object with the following equation:

$$
\begin{equation*}
\text { Magnification }=d_{i} / d_{o} . \tag{2}
\end{equation*}
$$

## Equipment

Optical bench, converging lens, light source with an arrow opening, image screen, mirror, diverging lens.

## Procedure

1. Focal length by using the thin lens equation:
1) Place the object (a light source with an arrow opening), the converging thin lens (with the $f \sim 10 \mathrm{~cm}$ ) and an image screen at proper positions of the optical bench.

2) For four different $d_{o}$ values find their related $d_{i}$ values when you see sharp images on the screen. Fill out the following table and finish the evaluation.

Table 1

| Position | Position | Object | Position | Image | Focal |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (object) | (lens) | distance | (image) | distance ( $\left.d_{i}\right)$ | (ength $f$ | Magnification |
| $/ \mathrm{mm}$ | $/ \mathrm{mm}$ | $\left(d_{o}\right) / \mathrm{mm}$ | $/ \mathrm{mm}$ | $/ \mathrm{mm}$ | $/ \mathrm{mm}$ | $(M)$ |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |

## Average Focal Length:

$\qquad$

Question 1: What happens to the magnification as the object distance decreases?
Question 2: How does your measured focal length change with the increasing $d_{o}$ ?
2. Autocollimation approach

1) Align the object, the converging thin lens and a mirror on the optical bench as illustrated by the figure on the right. Adjust the distance between the object and the thin lens, so that a
 sharp image is formed on the object plane. This distance between the object and the thin lens is the focal length $f$. Repeat the measurements for three times.
2) Rotate the converging thin lens by 180 degrees. Repeat the measurements for another three times.

Table 2

|  |  | Position <br> (Object) <br> /mm | Position <br> (Lens) <br> /mm | Average position <br> (Lens) <br> /mm | $\begin{gathered} f \\ / \mathrm{mm} \end{gathered}$ | Average $f$ <br> /mm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 |  |  |  |  |  |
| rotating | 2 |  |  |  |  |  |
|  | 3 |  |  |  |  |  |
| After | 1 |  |  |  |  |  |
| rotating | 2 |  |  |  |  |  |
| 180 degre | 3 |  |  |  |  |  |

Question 3: Can you understand this approach? How large is the magnification?

## 3. Bessel approach

Fix the distance between the object and the image screen as $D$, where $D$ should be larger than $4 f$. As shown
 in the figure on the right, move the converging thin lens
between the object and the image screen to find two positions for the lens(indicated by 'L P1' and 'L P2' in the figure) so that sharp images are formed on the screen. Write down the two positions of the lens and find its position shift $\boldsymbol{d}$.

Table 3

| Position (Object) $/ \mathrm{mm}$ | Position (Image) $/ \mathrm{mm}$ | $D / \mathrm{mm}$ |
| :---: | :---: | :---: |
|  |  |  |


| Position $\mathrm{d}_{\mathrm{o} 1}$ (Lens) $/ \mathrm{mm}$ | Position $\mathrm{d}_{\mathrm{o} 2}$ (Lens) $/ \mathrm{mm}$ | $d / \mathrm{mm}$ |
| :---: | :---: | :---: |
|  |  |  |

Calculate the focal length with the following equation:

$$
f=\frac{D^{2}-d^{2}}{4 D}
$$

## Focal Length:

$\qquad$

Question 4: Try to derive the above equation.

## 4. Measure the focal length of a diverging

## lens. (Optional)

Try the measurement with the method illustrated in the figure on the right.


## Table 4

$\left.\begin{array}{|c|c|c|c|c|}\hline \begin{array}{c}\text { Position } \\ \left(\mathrm{A}^{\prime}\right)\end{array} & \begin{array}{c}\text { Position } \\ \left(\mathrm{A}^{\prime \prime}\right)\end{array} & \begin{array}{c}\text { Position } \\ (\mathrm{L} 2)\end{array} & \begin{array}{c}u \\ / \mathrm{mm}\end{array} & / \mathrm{mm}\end{array}\right)$

Evaluate the focal length of the diverging lens $L_{2}$ with the follow equation:

$$
f=u v /(u+v)
$$

Note: $u$ is negative. Think about why.

Focal Length: $\qquad$

Question 5: Do you have another method for the determination of the focal length of a diverging thin lens?
5. Try to build a telescope with two converging lenses. (Optional)

## Discussion:

- What have you learned from this lab?
- How do you evaluate each method?


## Lab 5 Wheatstone bridge

## Goal

Learn the operation of the Wheatstone bridge.

## Related Topics

Electrics, electrical circuit

## Introduction

We have learned before how to measure the resistance using a voltmeter and a current meter. The accuracy of such measurements is limited by the internal resistances of these meters. Ideally, we need the internal resistance of a voltmeter to be infinity, and that of an ammeter to be zero. But that is not possible and all the measurements with these meters will have unavoidable instrumental errors.

In this experiment we will learn a way to measure the resistance precisely by using an equipment called Wheatstone Bridge, which is invented by Samuel Hunter Christie in 1833 and improved and popularized by Sir Charles Wheatstone in $1843^{[1,2]}$.


Fig. 1 Wheatstone Charles (1802-1875).


Fig. 2 The circuit diagram of a Wheatstone bridge

## 1. A Wheatstone bridge circuit

As shown in Fig. 2, the unknown resistor $R_{\mathrm{X}}$ and the other three adjustable resistors $R_{\mathrm{A}}, R_{\mathrm{B}}$ and $R_{\mathrm{S}}$ form a circuit called a "bridge". A voltmeter V is connected to two points, 2 and 4 . In operation, we adjust the resistance of $R_{\mathrm{S}}$ till the voltage between 2 and 4 is zero. This process is called "balancing" of the bridge. When the bridge is balanced, the current passing through the resistors $R_{\mathrm{A}}$, and $R_{\mathrm{B}}$ is the same, and that passing through $\mathrm{R}_{\mathrm{S}}$ and $\mathrm{R}_{\mathrm{X}}$ is the same. We then have the relationship:

$$
\begin{equation*}
R_{X}=\frac{R_{A} \cdot R_{S}}{R_{B}} \tag{1}
\end{equation*}
$$

## 2. Sensitivity of a Wheatstone bridge

In the measurement, the voltmeter would monitor the voltage between 2 and 4 . When the balance of the bridge circuit is reached, a tiny change of the resistor $R_{\mathrm{S}}$, described as $\delta R_{\mathrm{S}}$, will cause a deflection of the voltmeter $\delta V$. The Wheatstone bridge's sensitivity $M$ is defined by $M=\frac{\delta V / S_{V}}{\delta R_{S} / R_{S}}$, where, $S_{\mathrm{v}}$ is the minimum display value. Obviously, the larger the $M$ is, more accurately the $R_{\mathrm{S}}$ for the balance of the bridge circuit could be measured. Hence, $M$ can be used to describe the sensitivity of a Wheatstone bridge. $M$ is also deduced by

$$
\begin{equation*}
M=\frac{\varepsilon}{S_{V}} \bullet \frac{R_{A} / R_{B}}{\left(1+R_{A} / R_{B}\right)^{2}} \tag{2}
\end{equation*}
$$

where, $\varepsilon$ is the voltage provide by the DC power supply.

## 3. Uncertainty of a Wheatstone bridge

In procedure, the resistor $R_{\mathrm{S}}$ is adjusted until the voltage between 2 and 4 is up to zero. Hence the uncertainty of the bridge circuit depends directly on this process. When the unbalance of the bridge cannot be detected, the measurement results will have a corresponding uncertainty. For example, a voltmeter with a minimum display index of 0.1 mV is used to test whether the bridge is balanced, to a certain $R_{\mathrm{a}}, R_{\mathrm{b}}, R_{\mathrm{x}}$, adjust $R_{\mathrm{S}}=5000 \Omega$ to reach balance bridge. When $R_{\mathrm{S}}$ is adjusted to $5010 \Omega$, the voltage between 2 and 4 is 1.0 mV . Therefore, when $R_{\mathrm{S}}$ is changed by $1 \Omega$, the voltage is changed by 0.1 mV , that is, the bridge is in an unbalanced state can just be identified. Thus, it can be considered that the measurement uncertainty of the measurement result $R_{\mathrm{S}}$ is $1 \Omega$.

That is to say,

$$
u_{B 1}\left(R_{S}\right)=S_{V} \cdot \frac{\delta R_{S}}{\delta V}
$$

In our experiment, $S_{\mathrm{v}}=0.1 \mathrm{mV}, \delta \mathrm{V}=1.0 \mathrm{mV}$, so, the measurement uncertainty $u_{\mathrm{B} 1}\left(R_{\mathrm{S}}\right)$ introduced by the bridge can be written as

$$
\begin{equation*}
u_{B 1}\left(R_{S}\right)=\frac{1}{10} \delta R_{S} \tag{3}
\end{equation*}
$$

## Experiment device

Three ZX21A resistors, an unknown resistor (about $2 \mathrm{k} \Omega$ ), a multimeter, a DC power supply, some connecting wires and two tapping keys.

## Procedure

1. Set up the circuit as shown in Fig. 2.
2. Measure $\boldsymbol{R}_{\mathbf{X}}$ at different values of $\boldsymbol{R}_{\boldsymbol{A}} / \boldsymbol{R}_{\boldsymbol{B}}$. (Set the voltage of power supply to 3 V .)

Set $R_{A}$ and $R_{B}$ according to $R_{A} / R_{B}$ values given in Table 1. Adjust $R_{\mathrm{S}}$ until the voltage between 2 and 4 equals to zero. Determine $R_{\mathrm{X}}$ at different $R_{A} / R_{B}$ values. $R_{\text {' }}^{\prime}$ is the measured resistance when the voltage is 1.0 mV . Fill out the table and find for which value of $R_{A} / R_{B}$, the sensitivity of Wheatstone bridge is largest.

Table 1 Data table when $R_{\mathrm{B}}=2000 \Omega$.

| $\boldsymbol{R}_{\mathbf{A}} / \boldsymbol{R}_{\mathbf{B}}$ | $\boldsymbol{R}_{\mathbf{A}} / \boldsymbol{\Omega}$ | $\boldsymbol{R}_{\mathbf{S}} / \boldsymbol{\Omega}$ | $\boldsymbol{R}^{\prime} / \boldsymbol{\Omega}$ | $S_{V} \cdot \frac{\delta R_{S}}{\delta V}$ | $M=\frac{\delta V / S_{V}}{\delta R_{S} / R_{S}}$ | $\boldsymbol{R}_{\mathbf{X}} / \boldsymbol{\Omega}$ |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| 20 |  |  |  |  |  |  |
| 10 |  |  |  |  |  |  |
| 5 |  |  |  |  |  |  |
| 1 |  |  |  |  |  |  |
| $1 / 5$ |  |  |  |  |  |  |
| $1 / 10$ |  |  |  |  |  |  |
| $1 / 20$ |  |  |  |  |  |  |

Here, $S_{\mathrm{v}}=0.1 \mathrm{mV}, \delta \mathrm{V}=1.0 \mathrm{mV}$

## 3. The measurement of $\boldsymbol{R}_{\mathbf{X}}$ at different power supply voltages.

According to Table 2, set $R_{\mathrm{A}}, R_{\mathrm{B}}$ and $R_{\mathrm{A}} / R_{\mathrm{X}}$ when the sensitivity $M$ is the largest. Carry out the measurement at different power supply voltages and fill out the Table 3. How is $M$ dependent on the power supply voltage?

$$
R_{A}=
$$

$\qquad$ , $R_{B}=$ $\qquad$ $R_{A} / R_{X}=$ $\qquad$ -

Table 3 Data table at different power supply voltage.

| Voltage/V | $\boldsymbol{R}_{\mathbf{S}} / \boldsymbol{\Omega}$ | $\boldsymbol{R}^{\prime} \mathrm{S} / \boldsymbol{\Omega}$ | $S_{V} \cdot \frac{\delta R_{S}}{\delta V}$ | $M=\frac{\delta V / S_{V}}{\delta R_{S} / R_{S}}$ |
| :---: | :--- | :--- | :--- | :--- |
| 1.0 |  |  |  |  |
| 3.0 |  |  |  |  |
| 5.0 |  |  |  |  |
| 7.0 |  |  |  |  |
| 9.0 |  |  |  |  |

## Questions

1. Derive the equation (1).
2. Determine the measurement uncertainty of $R_{\mathrm{X}}$ when the sensitivity $M$ is the largest in the table 3.
Supplementary information about the resistance box: how to calculate the uncertainty limits of a resistance box?
For example: if the indication of the ZX-21 type resistance box is $30362.5 \Omega$, and the zero resistance is $0.02 \Omega$, then the uncertainty limits a is:
$\mathrm{a}=30000 * 0.1 \%+0 * 0.1 \%+300 * 0.1 \%+60 * 0.1 \%+2 * 0.5 \%+0.5 * 2 \%+0.02$
3. Discuss how is measurement uncertainty of $R_{\mathrm{X}}$ on different values of $R_{\mathrm{A}} / R_{\mathrm{B}}, R_{\mathrm{A}} / R_{\mathrm{X}}$ and the power supply voltages.

## References

1．S．Hunter Christie，The Bakerian Lecture：Experimental Determination of the Laws of Magneto－electric Induction in different masses of the same metal，and its intensity in different metals，Philosophical Transactions of the Royal Society of London，vol．123，1833，pp．95－142．
2．Charles Wheatstone，The Bakerian Lecture：An Account of Several New Instruments and Processes for Determining the Constants of a Voltaic Circuit，Philosophical Transactions of the Royal Society of London，vol．133，1843，pp．303－－327．
3．沈元华，陆申龙 基础物理实验（Fundamental Physics Laboratory）高等教育出版社 北京 2004 pp．165－168．

# Lab 6 Magnetic field variation along the axis of a circular coil and a Helmholtz coil 

## Goal

1) To study the variation of magnetic field along the axis of a circular coil
2) To study the principle of superposition of magnetic field using a Helmholtz coil

## Related Topics

Magnetic field, Helmholtz coil, The principle of superposition of magnetic field

## Introduction

1. Circular coil

According to Biot-Savart Law, the intensity of magnetic field at a point ' P ', as shown in fig.1, lying on an $N$ turns identical circular current loops of radius $R$ at a distance ' $Z$ ' from the center ' O ' of the coil in S.I. units, is given by

$$
B=\frac{\mu_{0} N R^{2}}{2\left(R^{2}+x^{2}\right)^{3 / 2}} I
$$

Where $I$ is the current flowing through the coil, $\mu_{0}$ is the permeability of free space, which is equal to $4 \pi \cdot 10^{-7} \mathrm{H} / \mathrm{m}$.
The direction of the magnetic intensity at P is along OP if the current flows through the coil in the anti-clock-wise direction as seen from P .


Fig. 1 Geometry of a circular coil If the direction of the current is clockwise the field at P is along PO. The value of the magnetic intensity is maximum at the center of the coil and is given by

$$
B_{0}=\frac{\mu_{0}}{2 R} N I
$$

2. Helmholtz coil

A Helmholtz coil consists of two identical circular current loops of radius $R$, each having N turns. Each coil carries an equal electrical current flowing in the same direction. The coils are parallel and coaxial, and separated by distance $R$, as shown in Fig. 2.


Fig. 2 Geometry of the Helmholtz coil

The same current $I$ flows through both coils. As was the case for a single current loop, the field from a Helmholtz coil for an arbitrary point in space is shown in Fig. 3, a complicated function of the coordinates, but rotational symmetry simplifies the calculation on the field of the axis. To find the axial magnetic field due to a Helmholtz coil, we first note that, by superposition, the field from each loop of $N$ turns is simply $N$ times the field due to a coil of a single turn.
We choose the z axis along the axis of the coils, and choose the origin at the midpoint between the coils, as shown in Fig. 2 By superposition, the total field from the Helmholtz coil is the sum of the fields from each coil may be obtained from our expression for the single loop by applying


Fig. 3 Field lines for a Helmholtz Coil the coordinate transformation $Z \rightarrow Z-R / 2$ for the right hand coil, and $Z \rightarrow Z+R / 2$ for the left hand coil. Thus the axial field is given by

$$
B^{\prime}=\frac{1}{2} \mu_{0} N I R^{2}\left\{\left[R^{2}+\left(Z+\frac{R}{2}\right)^{2}\right]^{-3 / 2}+\left[R^{2}+\left(Z-\frac{R}{2}\right)^{2}\right]^{-3 / 2}\right\}
$$

Helmholtz coils are frequently used in the laboratory because they provide a reasonably large region free of material in which there is a uniform magnetic field (the region midway between the coils and along the axis). The uniform magnetic field between the coils is given by

$$
B_{0}^{\prime}=\frac{\mu_{0} N I}{R} \frac{8}{\sqrt{125}}
$$

## Experiment device

Constant current Power supply DC 50-200 mA, Digital Gauss meter with Axial Hall Probe (Transducer) $0-2.000 \mathrm{mT}$. Current carrying coil with 500 turns (N), Diameter 20.00 mm , support base and stand, and connecting leads, as shown in fig. 4 .


Fig. 4 Experimental Arrangement

## Procedure

2. Connect the DC supply, first circular coil in series as shown in the figure 4. Get the circuit checked by T.A. before switching on the power supply.
3. Switch 'ON' the DC supply. Set the output of the current is 100 mA . Note that current should be constant throughout the experiment. Check it before taking each reading.
4. You are provided with an axial Hall probe connected to a Gauss meter. Middle portion of the probe detects magnetic field. Move it to the axis of the coil and rotate the probe to check the direction at which magnetic field is maximum. Switch 'OFF' the DC supply and adjust the Gauss meter to zero before each measurement at different positions. (Why?)
5. Take readings of magnetic field for each 1.0 cm from 10 cm left to 10 cm right of the coil and tabulate. Magnetic field in Gauss meter fluctuates due to electronics used. At each step, wait for few seconds and note down the average value.
6. Switch off the supply and place the second coil at a distance equal to radius of the coil. Disconnect the first coil and make connections to second coil and repeat the measurement as above.
7. Third measurement is for Helmholtz coil setup. Make connections as in Figure 4.
8. Measure magnetic field from 10 cm left of the first coil to 10 cm right of the second coil for each 1.0 cm .
9. Draw the graphs between distance and magnetic field due to COIL 1, COIL 2 and BOTH along the axis of coils in a single graph.
10. Calculate the magnetic field at the center of the Helmholtz coil and compare it with the measured results.

## Precautions

1．Care should be taken that there is no stray magnetic field or ferromagnetic material，such as keys，screwdriver etc．near the setup，while performing the experiment．
2．The Zero of the Gauss meter should be adjusted each time before beginning the experiments and verified after the completion of experiment by reducing the current in both the COILS to zero．

## Questions

1．What is the significance of the Helmholtz configuration？
2．What will happen to the axial profile of the magnetic field in the Helmholtz configurations if the coils are moved further apart？

## References

3．沈元华，陆申龙 基础物理实验（Fundamental Physics Laboratory）高等教育出版社 北京 2004 pp．189－193．
4．http：／／www．iiserpune．ac．in／～bhasbapat／bapat＿files／phy121＿2014＿manual．pdf
5．http：／／www．yorku．ca／marko／PHYS2211／Lab5．pdf
6．https：／／courses．physics．illinois．edu／phys401／sp2018／Files／Hall\％20Probe／E67＿Spr ing09．pdf
7．http：／／home．uni－leipzig．de／prakphys／pdf／VersucheIPSP／Electricity／E－07e－AUF．pdf

Table for lab 6
$I=$ $\qquad$ mA

Position of the coil:
$d=R, \mathrm{Za}=$ $\qquad$ $\mathrm{cm}, Z b=$ $\qquad$ cm

| $\mathrm{Z} / \mathrm{cm}$ | $\mathrm{Z}=R(10.00 \mathrm{~cm})$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $B(\mathrm{a}) / \mathrm{mT}$ | $B(\mathrm{~b}) / \mathrm{mT}$ | $B(\mathrm{a})+B(\mathrm{~b}) / \mathrm{mT}$ | $B(\mathrm{a}+\mathrm{b}) / \mathrm{mT}$ |
| -10.0 |  |  |  |  |
| -9.0 |  |  |  |  |
| -8.0 |  |  |  |  |
| -7.0 |  |  |  |  |
| -6.0 |  |  |  |  |
| -5.0 |  |  |  |  |
| -4.0 |  |  |  |  |
| -3.0 |  |  |  |  |
| -2.0 |  |  |  |  |
| -1.0 |  |  |  |  |
| 0.0 |  |  |  |  |
| 1.0 |  |  |  |  |
| 2.0 |  |  |  |  |
| 3.0 |  |  |  |  |
| 4.0 |  |  |  |  |
| 5.0 |  |  |  |  |
| 6.0 |  |  |  |  |
| 7.0 |  |  |  |  |
| 8.0 |  |  |  |  |
| 10.0 |  |  |  |  |

## Lab 7 Digital Storage Oscilloscopes

## Goal

Learn how to use the digital storage oscilloscope (DSO) and the function generator.

## Introduction

Along with the multimeter the oscilloscope is the second most useful electrical measuring instrument found in all scientific laboratories. The use of this instrument is so ubiquitous that any one embarking on a scientific career of any discipline must master its operation. Oscilloscopes are used in the sciences, medicine, engineering, and telecommunications industry. General-purpose instruments are used for maintenance of electronic equipment and laboratory work. Special-purpose oscilloscopes may be used for such purposes as analyzing an automotive ignition system or to display the waveform of the heartbeat as an electrocardiogram. Oscilloscopes are used to observe the change of an electrical signal over time, such that voltage and time describe a shape which is continuously graphed against a calibrated scale. The observed waveform can be analyzed for such properties as amplitude, frequency, rise time, time interval, distortion and others.

Oscilloscopes can be divided into two types: analog and digital. An analog oscilloscope works by directly applying a voltage being measured to an electron beam moving across the oscilloscope screen. The voltage deflects the beam up and down proportionally, tracing the waveform on the screen. This gives an immediate picture of the waveform.

In contrast, a digital oscilloscope samples the waveform and uses an analog-to-digital converter (ADC) to convert the voltage being measured into digital information. It then uses this digital information to reconstruct the waveform on the screen.

The basic oscilloscope, as shown in Fig. 1, typically divided into four sections: the display, vertical controls, horizontal controls and trigger controls.


Fig. 1 The panel of SDS1102CML.


Fig. 2. Digital Oscilloscope Block Diagram
When you connect an oscilloscope probe to a circuit, the vertical system adjusts the amplitude of the signal, the ADC in the acquisition system samples the signal at discrete points in time and converts the signal's voltage at these points to digital values called sample points. The horizontal system's sample clock determines how often the ADC takes a sample. The rate at which the clock "ticks" is called the sample rate and is measured in samples per second. The sample points from the ADC are stored in memory as waveform points. More than one sample point may make up one waveform point. Together, the waveform points make up one waveform record. The number of waveform points used to make a waveform record is called the record length. The trigger system determines the start and stop points of the record. The display receives these record points after being stored in memory. Depending on the capabilities of your oscilloscope, additional processing of the sample points may take place, enhancing the display.

Fundamentally, with a digital oscilloscope, you need to adjust the vertical, horizontal, and trigger settings to take a measurement.

## Waveform Measurement

The generic term for a pattern that repeats over time is a wave - sound waves, brain waves, ocean waves, and voltage waves are all repeating patterns. A waveform is a graphic representation of a wave. A voltage waveform shows time on the horizontal axis and voltage on the vertical axis. Waveform shapes tell you a great deal about a signal. Any time you see a change in the height of the waveform, you know the voltage has changed.

## 1. Frequency and period

If a signal repeats, it has a frequency. The frequency is measures in Hertz $(\mathrm{Hz})$ and equals the number of times the signal repeats itself in one second (the cycles per second). A repeating signal also has a period - this is the amount of time it takes the signal to complete one cycle. Period and frequency are reciprocals of each other, so that $1 /$ period equals the frequency and 1/frequency equals the period. So, for example, the sine wave in Fig. 3 has a frequency of 3 Hz and a period of $1 / 3$ second.


Fig. 3 Frequency and period
2. Voltage: Voltage is the amount of electric potential (a kind of signal strength) between two points in a circuit. Usually one of these points is ground (zero volts) but not always - you may want to measure the voltage from the maximum peak to the minimum peak of a waveform, referred to at the peak-to-peak voltage. The word amplitude commonly refers to the maximum voltage of a signal measured from ground or zero volts. The waveform shown in Fig. 4 has an amplitude of one volt and a peak-to-peak voltage of two volts.
3. Phase: Phase is best explained by looking at a sine wave. Sine waves are based on circular motion and a circle has $360^{\circ}$. One cycle of a sine wave has $360^{\circ}$, as shown in Fig. 4. Using degrees, you can refer to the phase angle of a sine wave when you want to describe how much of the period has elapsed.


Phase shift describes the difference in timing between two otherwise similar signals. In Fig.5, the waveform labeled "current" is said to be $90^{\circ}$ out of phase with the waveform labeled "voltage", since the waves reach similar points in their cycles exactly $1 / 4$ of a cycle apart $\left(360^{\circ} / 4=90^{\circ}\right)$. Phase shifts are common in electronics.

## Vertical Controls

Use the vertical controls to position and scale the waveform vertically. It lets you move the waveform up or down to exactly where you want it on the screen. The volts per division (usually written volts/div) setting varies the size of the waveform on the screen. The volts/div setting is a
scale factor. For example, if the volts/div setting is 5 volts, then each of the major vertical divisions represents 5 volts.

## Horizontal controls

Use the horizontal controls to position and scale the waveform horizontally. The horizontal position control moves the waveform from left and right to exactly where you want it on the screen. The seconds per division (usually written as sec/div) setting lets you select the rate at which the waveform is drawn across the screen (also known as the time base setting or sweep speed). This setting is a scale factor. For example, if the setting is 1 ms , each horizontal major division represents 1 ms . Changing the sec/div setting lets you look at longer or shorter time intervals of the input signal.

## Trigger controls

The trigger makes repeating waveforms appear static on the oscilloscope display. Imagine the jumble on the screen that would result if each sweep started at a different place on the signal (see Fig. 6).


Fig. 5 Untriggered display

## Input coupling

Coupling means the method used to connect an electrical signal from one circuit to another. In this case, the input coupling is the connection from your test circuit to the oscilloscope. The coupling can be set to $\mathrm{DC}, \mathrm{AC}$, or ground. DC coupling shows all of an input signal. AC coupling blocks the DC component of a signal so that you see the waveform centered at zero volts. Fig. 5 illustrates this difference. The AC coupling setting is handy when the entire signal (alternating plus constant components) is too large for the volts/div setting.


Fig. 6 AC and DC input coupling
The ground setting disconnects the input signal from the vertical system, which lets you see where zero volts is on the screen. With grounded input coupling and auto trigger mode, you see a horizontal line on the screen that represents zero volts. Switching from DC to ground and back again is a handy way of measuring signal voltage levels with respect to ground.

## XY mode

The oscilloscopes have the capability of displaying a second channel signal along the X －axis （instead of time）．This is called XY mode．This mode of operation opens up a whole new area of phase shift measurement techniques．This involves inputting one signal into the vertical system as usual and then another signal into the horizontal system．（This method only works if both signals are sine waves．）This set up is called an XY measurement because both the X and Y axis are tracing voltages．The waveform resulting from this arrangement is called a Lissajous pattern （named for French physicist Jules Antoine Lissajous and pronounced LEE－sa－zhoo）．From the shape of the Lissajous pattern，you can tell the phase difference between the two signals．You can also tell their frequency ratio．Fig． 7 shows Lissajous patterns for various frequency ratios and phase shifts．


Fig． 7 Lissajous patterns
Procedure（see the details in the special lab report for digital oscilloscope）
1．Auto measurement．
2．Save the measurement．
3．Manual measurement．
4．Lissajous patterns

## References

1．SDS 1000 CFL 系列数字存储示波器用户手册，深圳市鼎阳科技有限公司。——实验中心网页
2．XYZs of analog and digital oscilloscopes，www．tek．com

## Lab 8 Frank-Hertz experiment with Neon

## Goal

- Study of the quantized excitation of neon atom and to observe the light emission resulted from the de-excitation of excited atoms.
-To determine the first excitation potential of neon.


## Related Topics

Atomic structure, electron-molecular interaction.

## Introduction

The Franck-Hertz experiment was a physics experiment that provided support for the Bohr model of the atom, a precursor to quantum mechanics. In 1914, the German physicists James Franck and Gustav Ludwig Hertz sought to experimentally probe the energy levels of the atom. The now-famous Franck-Hertz experiment elegantly supported Niels Bohr's model of the atom, with electrons orbiting the nucleus with specific, discrete energies. Franck and Hertz were awarded the Nobel Prize in Physics in 1925 for this work.

In the early 20th century, experiments by Ernest Rutherford established that atoms consisted of a diffuse cloud of negatively charged electrons surrounding a small, dense, positively charged nucleus. Given this experimental data, Rutherford naturally considered a planetary-model atom. The laws of classical mechanics predict that the electron will release electromagnetic radiation while orbiting a nucleus. Because the electron would lose energy, it would gradually spiral inwards, collapsing into the nucleus. This atom model is disastrous, because it predicts that all matter is unstable.

Also, as the electron spirals inward, the emission would gradually increase in frequency as the orbit got smaller and faster. This would produce electromagnetic radiation with continuous spectrum. However, late 19th century experiments with electric discharges through various low-pressure gasses in evacuated glass tubes had shown that atoms will only emit light (that is, electromagnetic radiation) at certain discrete frequencies.

To overcome this difficulty, Niels Bohr proposed in 1913 the now called Bohr model of the atom, which suggested that electrons could only have certain classical motions:

1) The electrons can only travel in specific orbits: at a certain discrete set of distances from the nucleus with specific energies.
2) The electrons do not continuously lose energy as they travel, instead, at constant energy.
3) They can only gain and lose energy by jumping from one allowed orbit to another, absorbing or emitting electromagnetic radiation with a frequency $v$,
determined by the energy difference of the levels according to Bohr's formula $h v$ $=\Delta E$, where $h$ is Planck's constant.
The electrons in atom can occupy different allowed orbitals, and the total energy of the electrons determines the "energy level", also called the "energy state", of the atom (the energy states of the nucleus is not considered here). As shown in Fig.1, for an atom, the level with the lowest energy is called the "ground state", and ones with higher energies are called "the first excitation state", "the second excitation state" and so on, in the order from the ground state. If the ground state and the first excitation state are described by $E_{0}$ and $E_{1}$, respectively, the first excitation potential is $U_{01}=\left(E_{1}-E_{0}\right) / \mathrm{e}$.


Fig. 1 Illustration of the atomic energy levels, excitation and de-excitation.
When free electrons collide with atoms, the collisions between electrons and atoms are classified into two types: elastic collisions and inelastic collisions. If the energies of the electrons are lower than the required value corresponding to the energy between two different atomic energy states, elastic collisions will occur when colliding with atoms. This is due to the prediction in quantum mechanics that an atom cannot absorb energy unless the collision energy exceeds that required to excite the atom to a higher energy state. That is to say, a free electron's kinetic energy could be converted into potential energy by raising the energy level of an electron bound in an atom. The excitation state is not stable, and the electron will return to the ground state after releasing the energy with radiation emission. If the wavelength of the radiation lies in the visible range, we will see it as light.

## Experiment device

An evacuated glass tube is filled with neon. The glass tube contains a planar system of four electrodes (see Fig. 2). The grid-type control electrode G is placed in close proximity to the cathode K ; the acceleration grid A is set up at a somewhat greater distance, and the collector electrode E is set up next to it. To prevent a potential differential along K , the cathode is heated independently by applying a voltage of $U_{\mathrm{F}}$. The heated electrode emits electrons to form a charge cloud. These electrons are attracted by the driving potential $U_{\mathrm{G}}$ between the cathode K and grid G . An acceleration voltage $U_{\mathrm{A}}$ is applied between grids G and A . A braking voltage $U_{\mathrm{E}}$ is present between grid A and the collector E. Only electrons with sufficient kinetic energy can reach the collector electrode and contribute to the collector current.


Fig. 2 Schematic diagram of a Frank-Hertz tube with Ne.

After adjusting the driving potential $U_{\mathrm{G}}$, the braking voltage $U_{\mathrm{E}}$ and the cathode voltage $U_{F}$ to proper values, these parameters can be kept constant. Then the acceleration voltage $U_{\mathrm{A}}$ is increased from 0 to 80 V , while the corresponding collector current $I_{\mathrm{E}}$ is then measured. This current initially increases, but reaches a maximum when the kinetic energy of the electrons in front of grid A is just sufficient to transfer the energy required to excite the neon atoms through collisions. After collision the electrons can no longer overcome the braking voltage $U_{\mathrm{E}}$ resulting in a collector current dropping off dramatically. As the acceleration voltage $U_{\mathrm{A}}$ increases, the electrons attain the energy level required for exciting the neon atoms at even greater distances from grid A. And after collision, they are accelerated once more and, absorb sufficient energy from the electrical field to excite neon atoms when the acceleration voltage is high enough. This results in the second maximum, which is at a greater voltage of $U_{\mathrm{A}}$, in the $I_{\mathrm{E}} \sim U_{\mathrm{A}}$ curve (as shown in Fig.3) . Along with this procedure, we can observe discrete red luminance layers between grids $G$ and


Fig.3. $I_{\mathrm{E}} \sim U_{\mathrm{A}}$ curve A when the acceleration voltage $U_{\mathrm{A}}$ is high enough.

## Tasks/Procedure

## 1. Light emission

1) Set all the control knobs of the Frank Hertz unit to the zero positions (turn counterclockwise to the end).
2) Connect the cables according to Fig. 2 and switch on the power of the equipment.
3) Set the initial parameters of $U_{\mathrm{F}}$ around $8.5 \mathrm{~V}, U_{\mathrm{G}}$ around 2 V , and $U_{\mathrm{E}}$ around 5 V .
4) Set the acceleration voltage $U_{\mathrm{A}}$ to "Man" mode. and gradually increase $U_{\mathrm{A}}$ to 65 V . Familiarize yourself with the function knobs of the device. Find out the setting of $U_{\mathrm{G}}$ so that you can see the light emission between the two grids G and A when you increase the acceleration voltage $U_{\mathrm{G}}$ gradually. Log the
detailed experimental phenomena，especially how the luminance layers look， and how can they be changed as $U_{\mathrm{G}}, U_{\mathrm{A}}$ or $U_{\mathrm{E}}$ is changed．What can you learn from these？

## 2．Obtain $I_{E}-U_{\mathrm{A}}$ curve using an oscilloscope

1）Connect the cable between the oscilloscope and the Frank－Hertz operating unit， with the X channel of the oscilloscope to $U_{\mathrm{A}} * 1 / 10$（noted at $U_{\mathrm{X}}$ ），and the Y channel to $U_{\mathrm{Y}}$ ，which is proportional to $I_{\mathrm{E}}\left(U_{\mathrm{Y}=} I_{\mathrm{E}}{ }^{*} V\right)$ ．
2）Set the acceleration voltage $U_{\mathrm{A}}$ to＂ramp＂mode．Set $U_{\mathrm{A}} \min$ and $U_{\mathrm{A}}$ max to 0 and 65 V ，respectively．Set $U_{\mathrm{E}}$ to 4 V ．Obtain the $U_{\mathrm{X}}$－t curve of the X channel and the $U_{\mathrm{Y}}$－t curve of the Y channel on the oscilloscope screen．Adjust $U_{\mathrm{G}}$ and the oscilloscope until you see a stable，properly sized $U_{\mathrm{X}}-\mathrm{t}$ curve and $U_{\mathrm{Y}}$－t curve．
3）Set $U_{\mathrm{E}}$ to $0 \mathrm{~V}, 2 \mathrm{~V}, 4 \mathrm{~V}, 8 \mathrm{~V}$ ，record the variation rule of $U_{\mathrm{Y}}$－t curve at each $U_{\mathrm{E}}$ ． Each time adjust the oscilloscope until you see a stable，properly sized $U_{\mathrm{X}}-\mathrm{t}$ curve and $U_{\mathrm{Y}}-\mathrm{t}$ curve．

## 3．Measuring the first excitation potential of neon．

1）Based on tasks 1 and 2 ，think about what $U_{G}$ and $U_{\mathrm{E}}$ values you should use to get an optimized $I_{\mathrm{E}}-U_{\mathrm{A}}$ curve．Set $U_{\mathrm{G}}$ and $U_{\mathrm{E}}$ to the optimal values．Make sure the collector current $I_{\mathrm{E}}$ just before its saturation when the acceleration voltage $U_{\mathrm{A}}=65$ V ．If not，slightly change $U_{\mathrm{G}}$ or $U_{\mathrm{E}}$ ．
2）Slowly increase $U_{\mathrm{A}}$ manually from 0 to 65 V ．Record $U_{\mathrm{A}}$ and $I_{\mathrm{E}}$ curve from the display．Measure the slope of $U_{\mathrm{A}}$ curve，measure the time when the $I_{\mathrm{E}}$ curve reaches its maxima（peak position）．To calculate the first excitation potential of Ne ．

## Questions

1．From what phenomena in this experiment，we can conclude that the energy of electrons in an atom is quantized？

2．Why is it necessary to apply a deceleration voltage between the collector electrode E and the anode grid A？
3．Discuss the reason（s）to why the current of the peaks and troughs of the curve increase when increasing the acceleration voltage $U_{\mathrm{A}}$ ．
4．What is the wavelength of the light emission between the two grids？Is it the emission from the de－excitation process of neon atoms from the first excitation state to the ground state？ （optional）

## References

1．沈元华，陆申龙 基础物理实验 高等教育出版社，北京 2003 326～332
2．http：／／www．niser．ac．in／sps／sites／default／files／basic＿page／Franck－Hertz\％20experim ent．pdf

## Lab 9 X-ray experiment

## Goal

a) Observe the image of an opaque object on the luminous screen, and to study how the image quality can be improved.
b) Investigate the lattice spacing of an NaCl monocrystal.

## Related Topics

Material science, Condensed matter physics

## Introduction

X-rays are a form of electromagnetic wave with the wavelengths ranging from $10^{-8}$ to $10^{-10} \mathrm{~m}$. X-rays are created when fast-moving electrons are rapidly decelerated in matter. According to the laws of classical electrodynamics, this deceleration gives rise to electromagnetic radiation, which is mainly radiated perpendicular to the direction of acceleration for energies below 50 keV , i.e. for an X-ray tube, perpendicular to the direction of the electrons striking the anode. For historical reasons, this x-ray component is referred to as "bremsstrahlung" after the German word for the deceleration process by which it occurs. The bremsstrahlung has a continuous spectrum which extends to a certain maximum frequency $\nu_{\max }$ or a minimum wavelength $\lambda_{\text {min }}$. If the energy of the electrons exceeds a critical value, the characteristic x-radiation is generated, which appears in the spectrum as individual lines in addition to the continuous bremsstrahlung spectrum. These lines are generated when high-energy electrons penetrate deep into the atomic shells of the anode material, and eject electrons from the innermost orbits by collision. The holes created in this process are filled by electrons from the outer orbits with emission of X-rays. The resulting X -radiation is characteristic of the anode material, and is roughly comparable to the optical line spectrum of a material in a gaseous, or vapor state.

Figure 1 serves to illustrate the nomenclature adopted for the orbital model of the atomic shell for the lines of the characteristic X-radiation: the individual orbits are characterized by a particular binding energy and are designated from the innermost to the outermost with the letters K, L, M, N, etc. Electrons can transit between the orbits in accordance with the laws of quantum mechanics; these transitions entail either the absorption or emission of radiation, depending on the direction of transitions. For example, radiation from transitions to the K-orbital occurs as a series of sequential lines designated $K_{\alpha}, K_{\beta}$,


Fig. 1 A Simplified term diagram of an atom and the definition of the K serie of the characteristic X-ray radiations.
etc. In this experiment, the anode is made of Molybdenum (Mo). Fig. 2. sketches out the emission spectrum of Mo target. The $K_{\alpha}$, and $K_{\beta}$ radiations of Mo have the wavelengths of $7.11 \times 10^{-2} \mathrm{~nm}$ and $6.32 \times 10^{-2} \mathrm{~nm}$, respectively.

X-rays are more energetic than visible light. Thus they can penetrate and travel through opaque materials. The ability of X-rays to pass through opaque materials and bodies makes them particularly useful in diagnostic applications. Depending on the composition of the irradiated object, the radiation is attenuated to a greater or lesser extent. That is why the images on the luminous screen reveal details of the internal structure of objects. In this experiment, this fact is demonstrated using a simple object, e.g. a pocket calculator, which has parts, made of materials with different absorption properties. This experiment investigates the effect of the emission current $I$ of the X-ray tube on the brightness, and the effect of


Fig. 2 A sketch of Mo emission spectrum with the limit wavelength $\lambda_{\text {min }}$ of the bremsstrahlung continuum and the characteristic $K_{\alpha}$, and $K_{\beta}$ the tube high voltage $U$ on the contrast of images on the luminous screen.

X-rays can be used to investigate the microstructure of crystals since their wavelengths are in the same order of magnitude as the distance between atoms. In the crystal, atoms are arrayed regularly in a lattice formation. The spacing between the two adjacent lattice planes is called the lattice spacing (as shown in Fig.3). In 1913, H. W. Bragg and W. L. Bragg proved that the constructive interference arised in the rays reflected at the individual lattice planes when their path differences $\Delta$ were integral


Fig. 3 X-ray reflection on a crystal lattice. multiples of the wavelength $\lambda$. That is:

$$
\begin{equation*}
\Delta=2 \mathrm{~d} \sin \beta=\mathrm{k} \lambda, \quad \mathrm{k}=1,2,3 \ldots \tag{1}
\end{equation*}
$$

The meanings of $d, \beta$ and $\lambda$ are illustrated in Fig.3.
In this experiment, we investigate the lattice spacing of NaCl monocrystal the characteristic X-radiation of Mo target.

## Experiment device

Figure 4 shows the X-ray apparatus used in this experiment. There are two lead glass sliding doors on the front of the X-ray apparatus. So we can see the tube and experiment chamber clearly with the protection of the lead glass. This apparatus is divided into three areas. They are the control area, the tube chamber and the experiment chamber (from left to right).

The tube's structure is shown in Fig.5. It is a high vacuum quartz tube. The part " 1 " refers to a grounded emission grid. When it's electrified, it will emit electrons. The " 2 " refers to a Mo target (It is surrounded by copper block. " 3 " is a copper block, which helps in heat dissipation.) . When high voltage is supplied, usually around tens of kilovolt, the accelerated electrons will
collide with the target, and X-ray will be generated.


Fig. 4 An overview of the X-ray apparatus


Fig. 5 A sketch of the X-ray tube


Fig. 6 The target holder

The experiment chamber is at the right side of the equipment (see Fig. 4). Related parts are:

## A1: Collimator.

A2: Target holder, as shown in Fig.6.

## A3: Sensor.

The X-ray apparatus is controlled using 1) a number of keys to set the parameters and select the operating mode; 2) a knob for setting the desired value of the selected parameter; and 3) a display panel for the set values. For each pushbutton, an LED indicates the corresponding parameter selection as shown in Fig.7.

## B1: Display panel

The upper area displays the current counting rate.
The lower area displays the quantity selected using a key from $\mathbf{B 3}$ or $\mathbf{B 4}$.

## B2: Adjustment knob

For setting the desired values. This incremental control knob can be turned in both directions and has a dynamic response, i.e. the values change by greater increments when the knob is turned rapidly. The apparatus accepts the displayed values when a key is pressed.

## B3: Parameter selecting keys

- Key $\mathbf{U}$ activates the display and setting of tube high voltage $U$.
- Key I activates the display and setting of the emission current $I$.
- Key $\Delta t$ activates the display and setting of the measuring time (per angular step) $\Delta t$.
- Key $\Delta \boldsymbol{\beta}$ activates the display and setting of the angular step width $\Delta \beta$ of the "auto-scan mode".
- Key $\boldsymbol{\beta}$-LIMIT activates the display and definition of the upper and lower limit angles of the auto-scan mode. A measurement cannot be initiated when the upper limit is smaller than the lower limit. The display will flash until this error is corrected. The first time this key is pressed, the display panel shows the
symbol $\downarrow$. At this point, the lower limit angle can now be set. After the second press of this key, the display panel shows the symbol $\uparrow$ to allow the upper limit to be set. The symbol b in the display panel indicates the setting $\Delta \beta=0.0^{\circ}$ which indicates the deactivation of the "auto-scan mode"..


## B4: scanning-mode keys.

- Key SENSOR activates "sensor scanning" in both the auto-scan or manual scan modes. You can define the limit angles of the sensor arm for auto-scan. The sensor arm movement can be controlled either manually or automatically. The lower display area shows the angular position of the sensor.
- Key TARGET activates "target scanning" in auto-scan or manual scan modes. You can define the limit angles of the target arm for auto-scan. The target arm movement can be controlled either manually or automatically. The lower display area shows the angular position of the target.
- Key COUPLED activates "coupled scanning" in auto-scan or manual scan modes. You can define the limit angles of the target arm for auto-scan. The sensor and target arms can be moved manually or automatically with an angular coupling of $2: 1$. In manually controlled movement, the reference point for the $2: 1$ coupling is the angular position of both the target and the sensor before the COUPLED key is pressed. For automatic movement, the reference point is the zero point of the measuring system. The lower display area shows the angular position of the target. The COUPLED key toggles the upper display area between the counting rate and the angular position of the sensor.
- Key ZERO causes the target and sensor arms to move to the zero position of the measuring system.


## B5: operational keys.

- Key RESET: when the goniometer (554 83) is mounted, this key causes the target and sensor arms to move to the zero position of the measuring system and resets all parameters to the default settings. The tube high voltage is switched off.
- Key REPLAY: activates readout of the stored measured values.
- Key SCAN ON/OFF: switches the tube high voltage on/off and activates/stops the measuring program (provided that the safety circuit is closed). The measured values are stored internally in the device. In auto-scan mode, the device first travels to the zero position of the measuring system, and then to the lower limit angle. The tube voltage is then switched on. The scan initiates as soon as the tube high voltage is present and an emission current is flowing. The starting and end points are the upper and lower limit angles set with the $\boldsymbol{\beta}$-LIMIT.
- Key HV ON/OFF: switches the tube voltage on and off. The tube high voltage can only be switched on when the safety circuits are closed. High voltage indicator lamp flashes when the tube high voltage is switched on. The tube high voltage can be switched on using either the SCAN or HV ON/OFF key.
The software specific for this equipment "X-ray Apparatus" has been installed in the computer. Double click on the icon, and a measurement window will appear (Fig. 8 shows such a typical measurement). The window consists of the main menu, data field and graph field. (You are suggested to read the "Help" document of the software on our Wiki web to familiarize yourself with the software operation.)


## Tasks/Procedure

## 1. Observe the image of an opaque object on the luminous screen

1) Switch on the power of the equipment.


Fig. 8 A typical measurement result.
2) Press the SENSOR key, then set the sensor to about $50^{\circ}$ with $\mathbf{B 2}$.
3) Open the lead glass sliding door, remove the collimator from the experiment chamber, and place an opaque object, e.g. pocket calculator, in the experiment chamber in front of the luminous screen.
4) Set the emission current $I=1.00 \mathrm{~mA}$, the tube high voltage $U=35.0 \mathrm{kV}$. and switch on the high voltage with the HV ON/OFF key. Reduce the emission current $I$ in steps, and observe the change of the image on the luminous screen. Now set the current $I=1.00 \mathrm{~mA}$ again, Reduce the tube high voltage in steps, and observe the change of the image.
Log the detailed experimental phenomena, especially how the image brightness and contrast change with the parameters. What can you learn from these?
2. Measure the lattice spacing of the NaCl monocrystal using the characteristic X-ray radiation of molybdenum, and confirm Bragg's law of reflection.

1) Mount the collimator in the collimator mount.
2) Attach the target holder with the target stage.
3) Loosen the knurled screw, and lay a NaCl crystal flat on the target stage. Carefully raise the stage as far as it will go beneath the flange, and tighten the knurled screw with care (press against the screw lightly to prevent it from stripping).
(Notes: NaCl crystals are hygroscopic and fragile. Store the crystals in a dry vessel. Avoid mechanical stress on the crystal. When handling, please wear gloves to avoid touching the crystal directly. Do be patient.)
4) Calibration of the zero position of the goniometer (see appendix).
5) Start the software "X-ray Apparatus" on your computer. Please make sure that the X-ray apparatus is properly connected.
6) Set the X-ray high voltage $U=35.0 \mathrm{kV}$, emission current $I=1.00 \mathrm{~mA}$, measuring time per angular step $\Delta t=3 \sim 5 \mathrm{~s}$, and the angular step width $\Delta \beta=0.1^{\circ}$.
7) Press the COUPLED key on the device to enable $2 \theta$-coupling of the target and sensor. Set the lower limit value of the target angle to $3.0^{\circ}$, and the upper limit to $25.0^{\circ}$.

8）Press the SCAN key to start the measurement．The measured data will be transferred to the computer automatically．A $R \sim \beta$（count rate per second versus the angle position of the sensor in degree，which is usually called $2 \theta$ ）curve will be plotted simultaneously on the computer monitor．
9）When the measurement is finished，save the measurement series to a file under a suitable name．

What are the features of the $R \sim \beta$ curve？What do the peaks represent？Write down the measured data（both angle position and count values）of each peak．Determine $\beta_{\min }$ from the curve． Print the $R \sim \beta$ curve（Ensure only the curve is printed and not the data！），for further evaluation and the lab report．

After your experiment，determine the lattice spacing of the NaCl crystal and $\lambda_{\text {min }}$ ．Prove that the Bragg＇s law of reflection holds．

## Questions

1．How are X－rays generated？What is the difference between the bremsstrahlung and the characteristic X－radiation？

2．Why is the image of the obejct no more visible on the screen when the high voltage is reduced to about $60 \%$ of the maximum？Why is the image still clear even when the current is reduced to about 0.1 mA ？
3．What is the＂coupled scan mode＂？Why do we use the＂coupled mode＂for the measurement of crystal lattice spacing？

## References

1．沈元华，陆申龙 基础物理实验（Fundamental Physics Laboratory）高等教育出版社 北京 2004 pp．343－351
2．Instruction sheet of X－ray apparatus，Leybold Diactic GmBH
3．Leybold physics leaflets for X－ray apparatus http：／／leybold－didactic．de／literatur／

## Appendix：

## Calibration of the zero position of the goniometer

If the counting rate is too low，you should adjust the zero position of the measuring system． You can adjust the zero position of the measuring system most precisely using the experiment ＂Bragg reflection＂；the adjustment is then with reference to the lattice planes of the NaCl monocrystal．For NaCl ，whose lattice space is 0.283 nm ，the maximum counting rate should appear at the angle of $7.2^{\circ}$ ．The steps are as follows：

1）Press the ZERO key to return the target and sensor arms to the current zero position．
2）Mount the NaCl crystal and the counter．
3）In coupled scanning mode，set the target to about $7.2^{\circ}$ using the ADJUST knob．
4）Set the tube high voltage $U=35.0 \mathrm{kV}$ and emission current $I=1.00 \mathrm{~mA}$ and switch on the tube high voltage with HV ON／OFF．
5) Switch between sensor and target scanning modes and locate the maximum counting rate for the first reflection maximum of the NaCl monocrystal under manual control .
6) In coupled scanning mode, move the target back by $7.2^{\circ}$ (even if this takes you into the negative range!).
7) Save the positions of the target and the sensor as the zero position of the measuring system by pressing TARGET, COUPLED and $\boldsymbol{\beta}$ LIMITS simultaneously.
8) To verify your adjustments, assume the angle $7.2^{\circ}$ in coupled scanning mode and check the maximum counting rate.

