Stationary Waves in Liquid Jets

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ABSTRACT

Stationary waves are often set up in steady jets impinging on a liquid or a solid obstacle. Relations between the velocity of the jet, the diameter, the wave-lengths and the surface tension have been formulated.

In this paper vertical water jets striking a convex glass surface and vertical mercury jets striking a horizontal mercury surface have been studied.

The relation between the wave length and the diameter of the jet has been verified, but unknown factors seem to enter into the relation involving the surface tension. The work will be continued until the reasons for the variations in the surface tensions are discovered.

Undulations in liquid jets have been studied by Rayleigh (1), Pedersen (2) and Bohr (3). They experimented upon jets which were not circular in cross-section and worked out formulae giving the relationships between the surface tension, the shape and the cross-section of the jet and the other physical factors involved.

Smith and Moss (4), Tyler and Richardson (5), Tyler (6), Tyler and Watkin (7), and others have worked on the characteristics of jets and on instability in jets and have deduced the relationships between the length of the unbroken jet and other physical factors.

Maass (8) broke new ground in his paper on "A measurement of surface tension by means of a vertical jet." Using a circular orifice he directed a vertical jet into a dish of water. The impact sends waves up the jet, the water in the jet is travelling downwards, and the result is that a set of stationary ripples is set up for which the ripple velocity is equal to the jet velocity. Maass photographed several jets of water of about 2 mm. diameter, the rate of flow being a little less than 2 cc. per sec. On each of these four or five wavelengths and the mean diameter of the jet at the same places were measured. With a knowledge of the rate of flow and the area of cross-section the velocity of the jet at the different places was worked out and by insertion in the equation

$$\frac{T}{\rho} = \frac{v^2\lambda}{2\pi}$$

the value of the surface tension T was deduced. Quite good values of T were obtained. Brinkworth (9) independently performed a variation of the Maass experiment by allowing his vertical jet to impinge on the top of a glass sphere. His nozzles were about 1 cm. in diameter, the sphere was 5 cm. in diameter and the rate of flow about 1.5 gm. per sec. The jet was very beaded (much more so than in Maass's experiment) and he found that the stationary undulations set up were quite stable if the water could flow freely away from the region of impact.

His photographs were very good but apparently no measurements were made on them.

This paper describes (1) experiments with water, using the Brinkworth arrangement, and (2) experiments with mercury, using the Maass method.

Experiments with Water

The arrangement of apparatus is shown in Fig. 1. Tap-water filled a tank A to overflowing and supplied the jet I by means of a wide U-tube siphon U and a capillary nozzle N. The siphon tube gave a con-

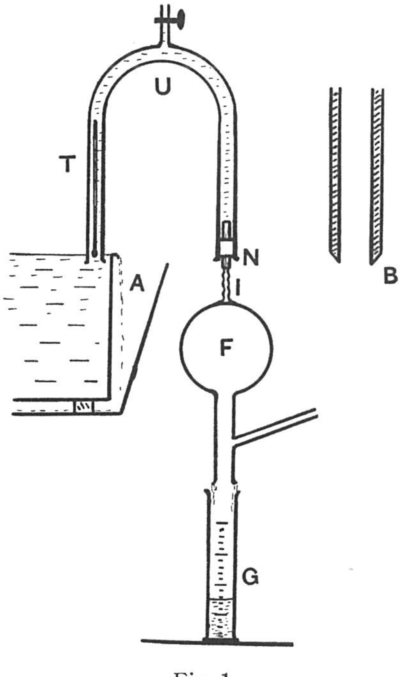

Fig. 1

venient support for N and by its elevation governed the rate of flow. The air bubbles which separated out of the water on the walls of the tube did not seriously affect the rate of flow, whereas if the nozzle N had been connected straight to the tap the air bubbles collecting in its narrow bore would have been very troublesome.

The water wets the whole bottom of the capillary tube, so that there must be some dead water at the top of the jet. To avoid this a tapering brass nozzle bevelled at the exit end (shown enlarged at B in Fig. 1) was occasionally used.

The jet impinges on the upturned convex bottom of a distillation flask F carried in a stand provided with rack and pinion so that the length of the jet could be altered. The rate of flow was measured by collection in a graduated cylinder G.

The temperature was read on a thermometer T.

Many jets were tried, variations being made in the diameter of the nozzle, the length of the jet, and the rate of flow.

They were photographed with 1/10 sec. exposure, using Ilford hypersensitive panchromatic plates, the camera being stopped down to f/5. One such photograph is shown in Fig. 2 on Plate I. The negatives were measured with a travelling microscope reading to 1/100 mm.

The diameter of the jet varied very little in successive waves and the crests and troughs were so minute that the mean diameter could be measured directly by a careful setting of the cross-hair of the microscope. The approximation used by Maass could therefore be applied.

The magnification was deduced from the actual and photographic sizes of the nozzle. In order to get good jets the rate of flow was made as small as possible. The average head was about ½ cm. of water. Smaller heads than this gave irregular effects.

We may note here that if

w_1 = weight of liquid delivered per second by the jet
d = diameter of the jet at any particular place
v = velocity of the jet at the same place

$$\text{then} \quad \frac{\pi d^2 v}{4} = \frac{w_1}{\rho}.$$

Hence substituting in $\frac{T}{\rho} = \frac{v^2\lambda}{2\pi}$ we get $T = \frac{8 w_1{}^2 \lambda}{\rho \pi^3 d^4}$ so that for the same jet the wavelength at the different places should be proportional to the fourth powers of the mean diameters at those places.

Table I gives the measurements and results for one jet of water among the many tried. The vertical order is the same as in the actual jet. The first two columns give the dimensions of successive wavelengths (λ') and diameters (d') on the photograph. To avoid irregularities due to inexact measurement of such small distances, smoothed curves between λ' and d' and the natural numbers were drawn, and better values of λ' and d' read off from the curves. Using the magnification factor these were reduced to actual λ's and d's in columns (3) and (4).

Column (5) indicates the constancy of $\frac{\lambda}{d^4}$, column (6) gives the jet velocity and the last column the deduced value of the surface tension.

TABLE I

WATER

Photograph shown in Plate I, Fig. 2
Temperature, 15.8°C.
Rate of flow, 2.81 gm. (or cc.) per sec.
Glass nozzle used

Wave-length on plate λ' cms.	Diameter on plate d' cms.	Actual wave-length λ cms.	Actual diameter d cms.	$\frac{\lambda}{d^4}$	Velocity of flow, cms. per second	Surface tension, T dynes/cm.
0.253	0.318	0.215	0.271	39.8	48.7	81.1
0.216	0.309	0.184	0.263	38.4	51.7	78.3
0.191	0.302	0.163	0.257	37.4	54.2	76.2
0.172	0.294	0.146	0.250	37.4	57.2	76.0
0.157	0.288	0.134	0.245	37.2	59.6	75.7
0.147	0.282	0.125	0.240	37.7	59.6	76.7
0.140	0.278	0.119	0.237	37.7	63.7	76.8

The values of the surface tension are high and variable. Other jets gave values from 75 to 84 dynes per cm.

EXPERIMENTS WITH MERCURY

A jet of mercury impinging on a convex surface refused to spread out evenly over the surface but wandered aimlessly around and the waves in the jet were irregular.

It was therefore decided to try the Maass' method.

A mercury circuit was built up as shown in Fig. 3 so that the same mercury was used over and over again. From a dish D on the floor a water-pump attached at P to an aspirator A sucked up a succession of mercury pellets. A tube from the aspirator led to an open dish B. From this a capillary tube siphon S formed the jet I. The jet fell into mercury in the thistle funnel F and then flowed away to another funnel G and back to the dish on the floor.

At first as the experiment proceeded the mercury surface in B became dirty. This is due, according to Professor Maass, to the use of india-rubber tubing in the circuit. The rubber tubing was therefore removed and an all-glass circuit made. The mercury jet and the surface of the mercury in F now remain perfectly clean. A little dust collects on the top of the mercury in A and in B but as the flow is always from the bottom of these vessels, the mercury going to the jet remains clean.

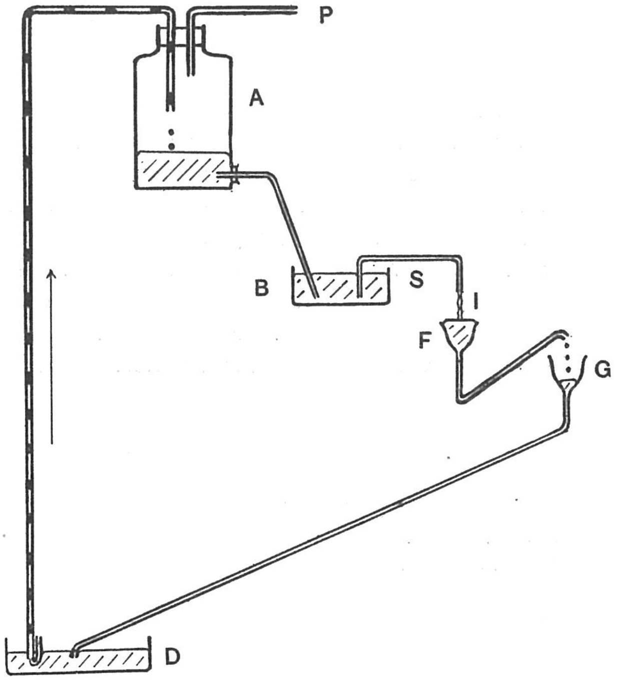

Fig. 3

By manipulating the water-pump and the height of the siphon the mercury surface could be kept above the top of the thistle funnel and maintained at a constant level for the fifteen or twenty minutes required for the photography and the measurement of the flow. The latter was made by collecting, in a beaker, the mercury dropping into

G. The short interruption of the circuit for this purpose did not alter appreciably the steady conditions in F.

The head of mercury driving the siphon was about half a centimetre.

It was found that the mercury in the circuit becomes strongly electrified. A voltmeter indicated a potential of 5,000 volts. An earthed wire was therefore immersed in the mercury in B and this possibility of trouble removed.

Several photographs were taken on different days. One is shown in Fig. 4 on Plate I. Its results are given in the upper part of Table II. The results in the lower part of Table II were taken with the same siphon but on a different day.

Many other photographs were taken and measured up. It will be noticed that although any one jet gives fairly consistent values for the surface tension of mercury, different jets give widely discordant values, although in all cases the jet relation, $\lambda/d^4 =$ a constant, is confirmed.

The cause of this wide variation has not been found. The same mercury was used throughout. It had not been specially purified but the jet and the surface in F looked always as clean as could be wished. The mercury was thoroughly aerated in its upward passage to the aspirator, and this might alter the surface tension. Any dirt present would lower the surface tension. However, abnormally high results

TABLE II

Results with Mercury

Photograph shown as Fig. 4 on Plate I
Temperature, 23.0°C.
Rate of flow, 9.34 gm. per sec.
Mercury not earthed, but as a graniteware dish was used in the circuit the electrification was very feeble

λ'	d'	λ	d	$\frac{\lambda}{d^4}$	v	T
0.158	0.180	0.137	0.156	231	36.2	386
0.137	0.174	0.119	0.151	229	38.6	381
0.123	0.169	0.107	0.147	229	40.8	383
0.113	0.165	0.098	0.143	234	43.1	391
0.105	0.162	0.091	0.141	230	44.3	384
0.099	0.159	0.086	0.138	237	46.3	396
0.094	0.157	0.082	0.136	240	47.6	399
0.089	0.155	0.077	0.135	232	48.3	386
0.085	0.154	0.074	0.134	230	49.0	382
0.081	0.152	0.070	0.132	231	50.6	385

ANOTHER JET

Temperature, 23.0°C.
Rate of flow, 15.4 gm. per sec.
Mercury earthed

λ'	d'	λ	d	$\frac{\lambda}{d^4}$	v	T
0.140	0.202	0.122	0.177	124	46.4	564
0.126	0.196	0.110	0.171	128	49.7	584
0.116	0.192	0.101	0.168	127	51.4	573
0.107	0.188	0.094	0.164	130	54.0	589
0.100	0.185	0.087	0.162	126	55.3	572
0.094	0.182	0.082	0.159	128	57.4	580
0.088	0.179	0.077	0.156	130	59.7	590
0.084	0.177	0.073	0.155	126	60.4	572

have been obtained as well as low results and the reason for the variation in T is still unknown.

The work on both the water and the mercury jets will be continued until the cause of the variation in the values of the surface tensions has been discovered.

University of Toronto,
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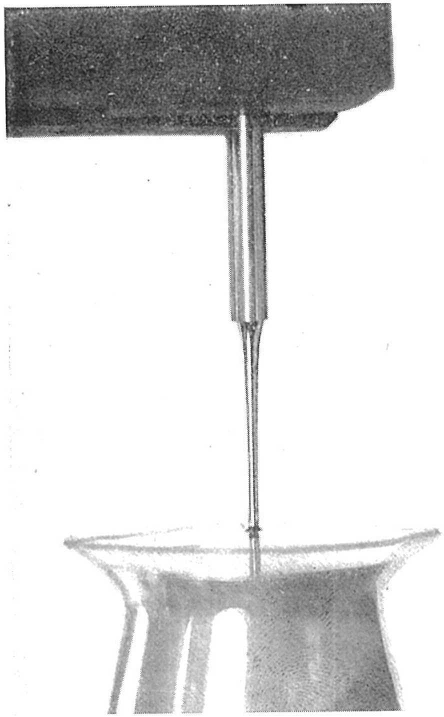

Fig. 4

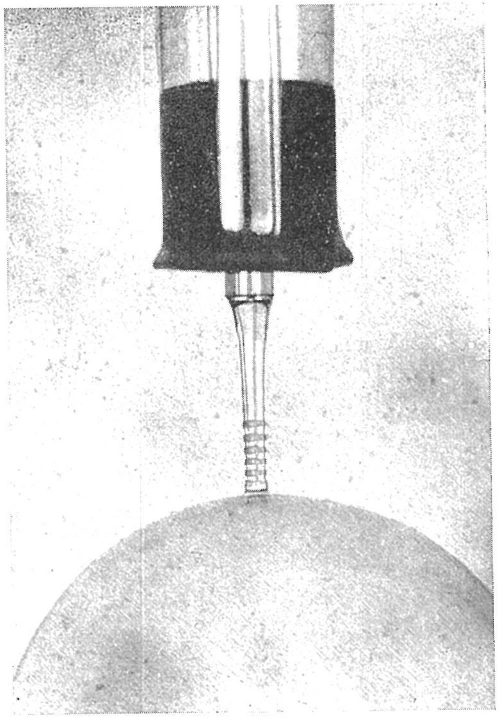

Fig. 2