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Some Effects of Nonuniform Fields on Dielectrics*

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Some of the more interesting effects of nonuniform electric fields are described in this report. Experimental and theoretical studies show the effects to be rather striking for particles larger than molecular size. The results show that the effect can be used to produce a fairly efficient pumping action of nonconducting liquids, to cause continuous and easily measureable separations in coarse suspensions, to cause selective precipitation, and to produce mixing.

By this means, liquids may be thrown several feet into the air with an electromechanical efficiency of about 25%. A separation factor of at least 2.5 in continuous separatory operation may be produced in a suspension of polyvinyl chloride in carbon tetrachloride-benzene mixture. Suspensions of polar materials in less polar liquids may be either dispersed or precipitated. In one interesting "demonstration" type experiment, drops were "hung" in mid-air.

INTRODUCTION

THE way many common materials behave in strong, nonuniform electric fields often appears strange when judged by usual standards. These effects, however, have received but little attention in the past. Mueller,[1] Pohl,[2] and Loesche and Hultschig[3] independently studied the theory of the size and direction of the effects. Mueller, in a very penetrating analysis, concluded that the effects would not be marked for particles of molecular size. Loesche and Hultschig also concluded this from their study of the theory and from their careful experiments. On the other hand, it was shown by the author that these effects were indeed appreciable in the case of macroscopic particles or bodies of liquid. Debye, in a study of the theory of such effects on large polymer molecules, concluded that a grading by molecular size should be possible.

In practice, the study of the effects of nonuniform fields is made complex by the presence of a number of competing events. Of these, the effect known as "dielectrophoresis"[2] is the main concern of this discussion. Competing with this effect, may be conduction, thermal convection, diffusion, and the effect obtained from the sequence, charging, and electrostatic repulsion.

"Dielectrophoresis" is defined as the motion of matter caused by polarization effects in a nonuniform electric field. The most polar material moves toward the place of greatest field intensity. Unlike electrophoresis, this does not require charged particles. Instead, it depends upon the force felt by all polar material when in a nonuniform field. Such a force may be regarded as arising in the following way. Any dipole (induced or permanent) will have a finite separation of equal amounts of plus and minus charges in it. The electric field will cause a measure of alignment of the dipole with it. Because the field is nonuniform, one end of the dipole will be in a weaker field than the other. A net force will then result and the dipole will be pulled towards the place of greatest field intensity. It can be seen that the direction of the field can be reversed and still give rise to the original direction of travel of the dipole. That is, in dielectrophoresis, one way motion takes place in either direct or alternating fields. Not so in electrophoresis. The sketch in Fig. 1 shows how the nonuniform field may affect a discrete sphere lying in it.

THEORY

The translational force, $\mathbf{F}$, on a dipole $\mathfrak{y}$ (induced or permanent) in a nonuniform field is

$$\mathbf{F}=\mu\cdot\nabla\mathbf{E}, \tag{1}$$

where $\mathbf{E}$ is the field acting on the particle.

The dipole moment of a spherical particle embedded in a dielectric medium is

$$\mathfrak{y}=v\mathbf{P}=(4/3)\pi a^3\mathbf{P}, \tag{2}$$

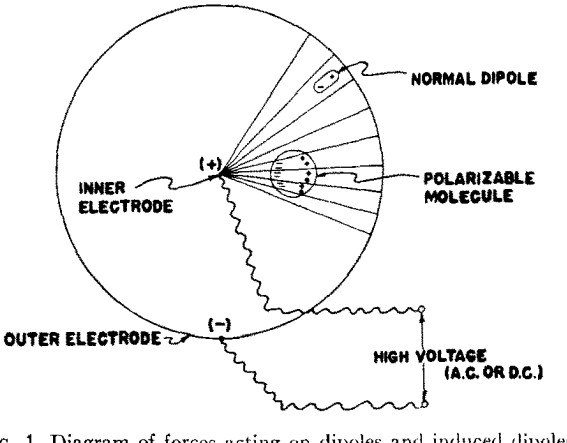

FIG. 1. Diagram of forces acting on dipoles and induced dipoles suspended in an inhomogeneous electric field.

* A portion of the research reported here was sponsored jointly by the Army Signal Corps and Office of Naval Research under Signal Corp Contract No. DA-36-039SC-70154; ONR 356-375.

[1] F. H. Mueller, Wiss. Veröffentl. Siemens Werken **17**, 20–36 (1938).

[2] H. A. Pohl, J. Appl. Phys. **22**, 869–871 (1951).

[3] A. Loesche and H. Hultschig, Kolloid. Zhur. **141**, 177–187 (1955).

where

v = volume $= (4/3)\pi a^3$,
a = radius of particle,
$\mathbf{P}$ = polarization or moment per unit volume
$= (K_1'-1)\epsilon_0\mathbf{E}_{iz}$. (3)

The excess polarization, $\mathbf{P}_e$, in the sphere is

$$\mathbf{P}_e = (K_2'-K_1')\epsilon_0\mathbf{E}_{iz}, \tag{4}$$

where K_1', K_2' are the relative dielectric constants of the medium and sphere, respectively,

ϵ_0 = permittivity of free space,
$\mathbf{E}_{iz}$ = internal field in the sphere in the direction of the external field

$$= \frac{3K_1'}{K_2'+2K_1'}\mathbf{E}.$$

Accordingly, $\mathfrak{y}_e$, the effective dipole moment is

$$\mathfrak{y}_e = 4\pi a^3\left(\frac{K_2'-K_1'}{K_2'+2K_1'}\right)K_1'\epsilon_0\mathbf{E} \tag{5}$$

and the effective translational force, $\mathbf{F}_e$, is

$$\begin{aligned}\mathbf{F}_e &= \mu_e\cdot\nabla\mathbf{E}\\ &= 4\pi a^3K_1'\epsilon_0\left(\frac{K_2'-K_1'}{K_2'+2K_1'}\right)\mathbf{E}\cdot\nabla\mathbf{E}\\ &= 2\pi a^3K_1'\epsilon_0\left(\frac{K_2'-K_1'}{K_2'+2K_1'}\right)\nabla\mathbf{E}^2.\end{aligned} \tag{6}$$

The force is thus seen to be proportional to the cube of the particle radius, to the difference of its dielectric constant from that of the surrounding medium, and to the gradient of the *square* of the field strength. The latter statement contains the implication that the direction of the force is the same if the field be reversed in sign.

In an electrode system with cylindrical symmetry, with the outer cylinder grounded and of radius, r_2, the inner cylinder at potential V_1 and of radius r_1, the potential V, at a distance, r, from the axis, and lying between the electrodes is

$$V = \frac{V_1\ln(r/r_2)}{\ln(r_1/r_2)} \tag{7}$$

and the field, $\mathbf{E}$, is

$$\mathbf{E} = -\nabla V = -\frac{\mathbf{r}^0V_1}{\ln(r_1/r_2)}\cdot\left(\frac{1}{r}\right) = \mathbf{E}_1\cdot\frac{r_1}{r}, \tag{8}$$

where $\mathbf{r}^0$ is the unit radius vector. Hence

$$\nabla E^2 = \frac{-2r_0V_1^2}{r^3[\ln(r_1/r_2)]^2} \tag{9}$$

and

$$\mathbf{F}_e = -4\pi a^3\epsilon_0K_1'\cdot\frac{K_2'-K_1'}{K_2'+2K_1'}\cdot\frac{V_1^2}{r^3[\ln(r_1/r_2)]^2}\cdot\mathbf{r}^0. \tag{10}$$

The negative sign indicates that the motion will be toward the axis. The motion will be strictly radial.

For macroscopic particles suspended in a fluid medium the dielectrophoretic force, $\mathbf{F}_e$, will be opposed by a viscous drag and by back diffusion. The latter effect may be neglected here, however, as it is important only for very small particles. The magnitude of the viscous drag, according to the Stokes equation is

$$\mathbf{F}_d = 6\pi\eta\mathbf{a}\mathbf{v},$$

where η is the viscosity of the fluid medium, $\mathbf{a}$ is the particle radius, and $\mathbf{v}$ is the velocity of the particle through the medium.

Equating the magnitudes of the dielectrophoretic and viscous drag forces and solving for the velocity gives, for the cylindrical electrode case

$$\underset{(\text{mks})}{v} = \frac{2a^2\epsilon_0K_1'V_1^2}{3\eta r^3[\ln(r_1/r_2)]^2}\left(\frac{K_2'-K_1'}{K_2'+2K_1'}\right) \tag{11}$$

(mks units).

If $K_1'\cong K_2'\cong 1$, the velocity equation using Gaussian units is

$$\underset{(\text{cgs})}{v} = \frac{a^2V_1^2(K_2'-K_1')}{18\pi r^3[\ln(r_1/r_2)]^2} \tag{12}$$

(V_1 in statvolts).

In the case of a spherical condensor with the central sphere of radius r_1 and the outer concentric sphere of radius r_2, the potential at any point between the shells, if the outer is grounded and the inner is at potential V_1, is

$$V = \frac{V_1r_1}{r}\cdot\frac{r_2-r}{r_2-r_1}. \tag{13}$$

The field is

$$\mathbf{E} = -\nabla V = \frac{\mathbf{r}^0V_1r_2r_1}{r^2(r_2-r_1)} \tag{14}$$

and

$$\mathbf{E}\cdot\nabla\mathbf{E} = -\mathbf{r}^0\frac{2V_1^2r_1^2r_2^2}{r^5(r_2-r_1)^2}. \tag{15}$$

The dielectrophoretic force on a spherical particle at a distance r from the center is then

$$\mathbf{F}_e = -r^0\frac{8\pi a^3\epsilon_0K_1'V_1^2r_1^2r_2^2}{r^5(r_2-r_1)^2}\cdot\frac{K_2'-K_1'}{K_2'+2K_1'}. \tag{16}$$

Using Stokes equation for the viscous drag as before, the velocity of a spherical particle in such a field is

$$v = \frac{4a^2\epsilon_0K_1'V_1^2r_1^2r_2^2}{3r^5\eta(r_2-r_1)^2}\cdot\frac{K_2'-K_1'}{K_2'+2K_1'}. \tag{17}$$

To a good approximation, the field in the neighborhood of a wire electrode with a rounded tip, set at the radial center of a hemispherical cup will be that of a spherical condensor. The rounded tip will correspond to the innermost sphere, the hemispherical shell to the outermost sphere. When such is the case, we may apply the above equation for velocity in a spherical condensor to the tip-and-shell electrode system.

The velocity effect for the tip-and-shell electrode system will be greater than for the cylindrical at the region near the electrode. The former varies as $1/r^5$, the latter as $1/r^3$.

The direct dependence of the velocity upon the square of the particle radius limits the effect, in its practical large scale aspects, to that on larger particles. Particles such as material in suspension, especially coarse suspension, should show more readily apparent results than small molecules in solution. Further, the effect upon gross bodies such as visible liquid drops or other liquid bodies should be more easily noted than effects upon intramolecular velocities such as involved in molecular separations.

COMPARISON OF DIELECTROPHORESIS AND ELECTROPHORESIS

At this point it may be well to compare carefully the two phenomena, dielectrophoresis and electrophoresis.

Dielectrophoresis

Dielectrophoresis, arising because of the tendency of matter to become polarized and move into regions of highest field strength;

1. Produces motion of the particles in which the direction of motion is independent of the direction of the field, i.e., either dc or ac voltages can be employed.
2. Should be observable most readily in relatively coarse suspensions (e.g., particle diameter $\geqq 2\,\mu$).
3. Requires highly divergent fields. No motion should be observed in the nondivergent field between centers of parallel plates, for example.
4. Requires relatively high field strengths, e.g., 10 000 v across a 5-cm cell.
5. Would be most apparent in fluids of low viscosity (thin liquids, gases).
6. Generally requires a large difference in dielectric constant between solvent and solute, e.g., $(K_1-K_2) \cong 2-100$.
7. Will deposit weights of coarse sol in direct proportion to the voltage applied in equal times of deposition.
8. Is in general a weak effect easily observable only in strong fields and with coarse particles.

Electrophoresis

Electrophoresis, arising from the electrostatic attraction of charged electrodes for charged particles.

1. Produces motion of the particles in which the direction of the motion is dependent on the direction of the field. Reversal of the field reverses the direction of travel. Care must be taken in the application of this rule for distinction between electro- and dielectrophoresis, as occasionally motion of charged particles towards a sharp electrode can occur in rapidly alternating fields of high strength. The occurrence of partial rectification effects makes the applied alternating voltage appear as predominantly direct current.

For example, a milky 20% suspension of Hycar rubber in methyl ethyl ketone was readily precipitated to form a thick adherent coat in a uniform field on parallel electrode plates 5 cm apart by dc but not by ac voltage of 2000 to 6000 v. This showed the suspension to consist mainly of charged particles capable of deposition following electrophoresis. In another experiment where the field was highly nonuniform, the suspension was also precipitated at the wire electrode of a wire-and-cylinder pair by either dc or 60-cycle ac voltage as above. The latter observation by itself might at first sight be considered as caused by dielectro-precipitation; however, further consideration shows this cannot be the case as the dielectric constant of the medium (18.4) exceeds that of the suspended particles (*ca* 15). This leaves deposition following electrophoresis as the more probable explanation with the assumption that partial rectification of the ac occured.

2. Is observable with particles of any molecular size.
3. Operates in either divergent or uniform fields.
4. Requires relatively low voltages.
5. Requires relatively small charges per unit volume of the particles.

SOME EXPERIMENTS WITH STRONG DIVERGENT FIELD

In this section is given a brief description of several means of using dielectrophoretic forces.

Early experiments on dielectrophoresis by the writer (Naval Research Laboratory Reports, 1943) showed the effects on macroscopic particles to be appreciable. A summary of these results was described in 1951[1] in the more open literature. Somewhat related, perhaps, are the results of Matsuda and Motoki[4] who reported the presence of strong electric fields to increase the tenacity of cotton and rayon fibers by about 12%.

Results of analogous character, but using inhomogeneous magnetic rather than electric fields, were obtained by Kolin and Leenov.[5,6] They showed that electromagnetophoresis using a field combination much like the one used in an electromagnetic pump[7] can produce an electromagnetic separatory action in inhomogeneous liquids. For example, a suspension of polystyrene spheres in a conducting liquid such as sodium chloride solution will show radial migration of

[4] N. Matsuda and M. Motoki, Tech. Repts. Eng. Research Inst. Kyoto Univ. **I**, 29–51 (1951).
[5] A. Kolin, J. Appl. Phys. **25**, 1065 (1954).
[6] D. Leenov and A. Kolin, J. Chem. Phys. **22**, 683–688 (1954).
[7] A. Kolin, Rev. Sci. Instr. **16**, 209–214 (1945).

the spheres if a current is made to flow in the axial direction through the solution while the whole is in a strong radially gradient magnetic field generated by a current-carrying wire. The process is termed electromagnetophoresis.

DIELECTROPHORETIC PRECIPITATION OF A SUSPENSION

Batch Precipitations

In these experiments, a fine wire was used as central electrode and either a metal dish or a foil-lined glass dish was used as an outer electrode. The general arrangement is shown in Fig. 2(a). One instance of how the technique might be used to aid polymer analysis is as follows. It often proves difficult for example to remove carbon black filler from polyvinyl chloride samples by means such as filtration or centrifugation. Removal was simply and rapidly accomplished in a powerful divergent electric field.

For example, a one-gram sample of the polymer sample was taken up in 50 ml of di-isopropyl ketone with the aid of gentle heating, then placed in the coagulation cell. The cell consisted of a Petri dish of about 10 cm diameter containing the electrodes. A 10-mil diameter, oxide-coated tungsten wire formed the central electrode and dipped vertically into the liquid. A band of tinfoil 8 mm high resting on the inner wall of the cell formed the outer electrode. A high voltage, either dc or ac of 10 000 v, was then applied. The liquid immediately underwent considerable action, small ripples formed, and the carbon particles underwent rapid migration toward the central wire to form a coating. The solution of polymer became water clear in a few minutes by what was very probably dielectro-precipitation. It was then removed for further analysis. Similar results were obtained with polyvinyl-chloride polyvinyl-acetate copolymers.

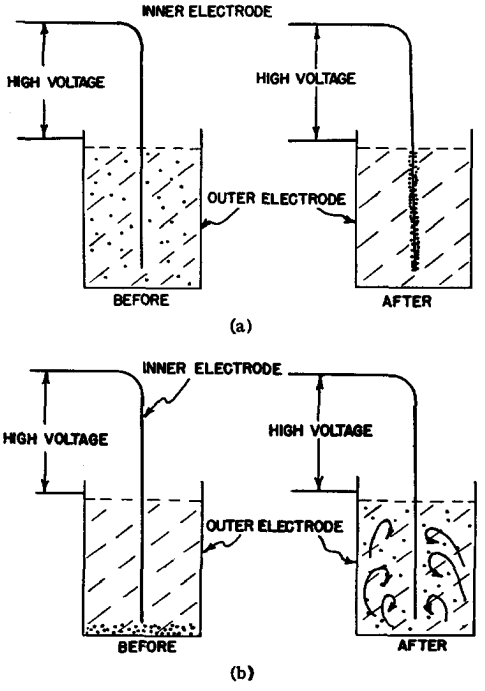

FIG. 2. (a) Dielectrophoretic precipitation of a suspension. (b) Dielectrophoretic dispersion of an agglomerated material.

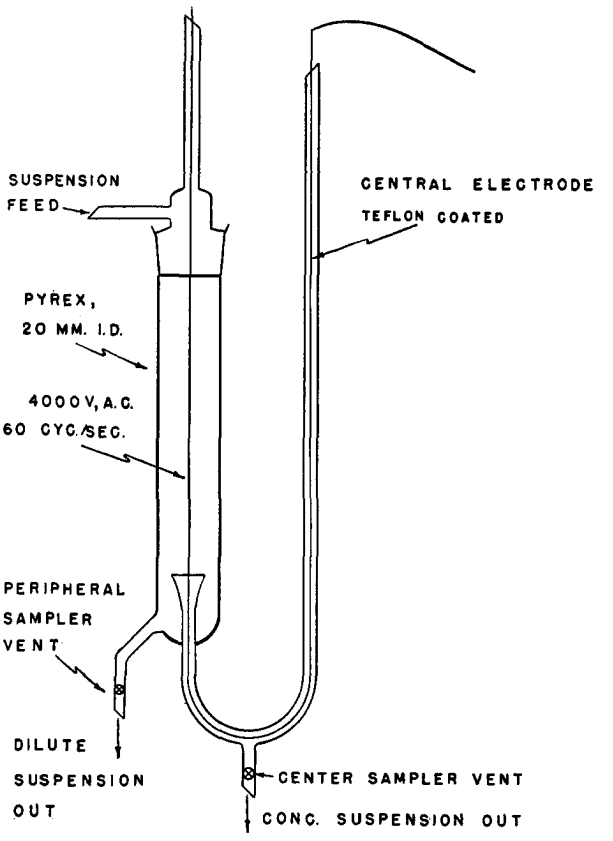

FIG. 3. Continuous dielectrophoretic separator.

When a bare copper wire, uninsulated, is used for the central electrode, only limited and temporary clumping occurs at it. Here the process of charging the particles and subsequent electrostatic repulsion interferes. It is therefore important that electrostatic charging and repulsion be minimized by use of insulation at the central electrode. When parallel plates, with carefully rounded edges to avoid high field gradients, were used instead of the wire and sheet combination, no precipitation was observable with either dc or ac. This adds confirmation to the conclusion that dielectrophoresis occurs under conditions in the wire-and-plate electrode cell.

Similar results were obtained with suspensions of coal dust and of charcoal dust in toluene.

Continuous Dielectro-Precipitation

Using the apparatus sketched in Fig. 3, a suspension of a polar polymer, ($K'=4.60$ at 100 cps) polyvinyl chloride (Exon 900A, a product of the Firestone Plastics Company) in a nonpolar ($K'=2.2$ at 100 cps)

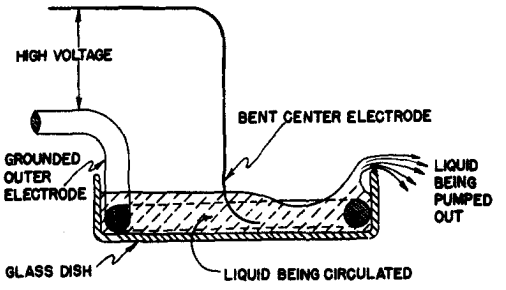

FIG. 4. Pumping and stirring action by strong divergent field.

1:1 v/v mixture of carbon tetrachloride and benzene was fed into the upper end of the dielectrophoresis cell. Continuous withdrawal of the liquid from the bottom was effected in two places: at the neighborhood of the central electrode, and at the periphery of the cell. In the absence of an electric field, the two exiting streams were identical in concentration of the powder. In the presence of a field, the powder flowed preferentially out the central electrode. An enrichment factor of over 2.5 was observed. It is expected that with apparatus of improved design much larger separation factors may be obtained. Clustering of the PVC powder at the central wire was very marked. Turning off the current caused the powder to fall off. In the presence of an alternating voltage field (60 cps), with the central electrode at mean voltages of 4000 v, the powder clinging to the central electrode was observed to shimmy and move slowly downwards toward the collecting port of the central electrode.

In a contrasting experiment using a nonpolar polymer, polyethylene, in a polar liquid, a 6:4 v/v mixture of acetone and nitrobenzene, the opposite effect was observed. Here, the more polar liquid of the suspension was attracted more strongly to the central electrode than were the polymer granules. The over-all effect was smaller and more difficult to observe. A separation factor of about 0.8 was observed. This lessening of the separation factor for the latter case is to be expected generally on the basis of the geometry of the system.

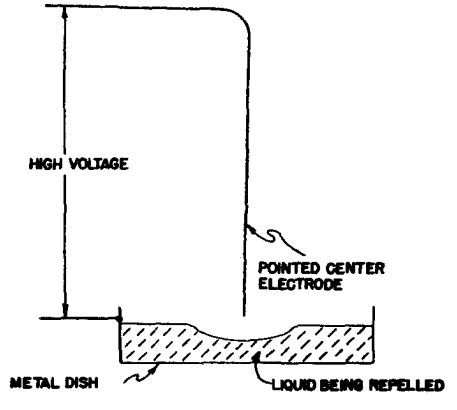

FIG. 5. Action under strong divergent field causing repulsion of liquid.

DIELECTROPHORETIC DISPERSION OF AGGLOMERATED MATERIAL

In an apparatus similar to that described above, and shown in Fig. 2(b), a potential was applied across the electrodes (center copper wire +) after the cell was filled with carbon tetrachloride, and one percent by volume of clumped carbon powder-alumina powder was dropped in. The clear liquid mixed with the powders which were dispersed rapidly. The combined action of both dielectro- and electrophoresis is present under such conditions.

STIRRING BY DIELECTROPHORESIS

In Fig. 4 is shown the action of the dielectrophoretic (and probably some electrophoretic) forces which can result when matters are arranged so as to maximize the effects. The apparatus is shown about half-scale size. A voltage of 10 000 v dc was applied, at a maximum

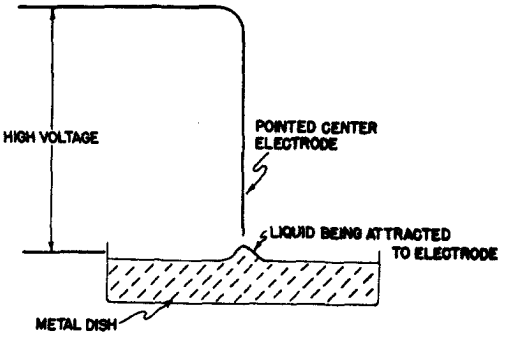

FIG. 6. Strong divergent field causing attraction for liquid.

power input of about 2 w. The indication appears to be that the electromechanical linkage is fairly efficient.[8]

The stirring effect has also been used by Senftleben and Gladisch[9] to aid in analysis of gas. The degree of stirring as measured by the cooling effect on a hot wire was used to analyze mixtures of propane and CO_2, and of acetylene and ethyl chloride.

"REPULSION" OF LIQUIDS BY DIVERGENT FIELD FORCES

In Fig. 5 is shown a sketch of the action of a properly placed, sharp-pointed electrode with its strongly divergent field. The field apparently repels the liquid below. In reality, this is but a secondary manifestation of the field effect upon the air in the neighborhood of the pointed electrode. The air molecules are attracted to the point by dielectrophoretic action, then charged at the electrode and subsequently repelled strongly.

[8] G. Ahsman and R. Kronig, Appl. Sci. Research **A3**, 83–84 (1951).
[9] H. Senftleben and H. Gladisch, Z. angew. Phys. **2**, 204 (1950).

The pressure of the "electric wind" pushes down the liquid below. By slightly changing conditions, as by adjusting the height of the wire, or using a wire loop instead of the point, the liquid may be made to rise to the wire instead of appearing to be repelled.

ATTRACTION OF LIQUIDS BY DIVERGENT FIELDS

In Fig. 6 is shown a sketch of properly arranged electrodes attracting the liquid (which may be CCl_4, benzene, nitro-benzene, etc.) This particular effect is capable of widely varied character. Differing arrangements of the electrodes will cause the liquid to move quietly or vigorously at the same applied voltage. By using a sharply pointed central electrode passing up through the liquid, the motion may be made to pump the liquid.

PUMPING ACTION IN STRONG NONUNIFORM FIELDS

In Figs. 7 and 8 are shown sketches of several arrangements in which liquid is made to leave the

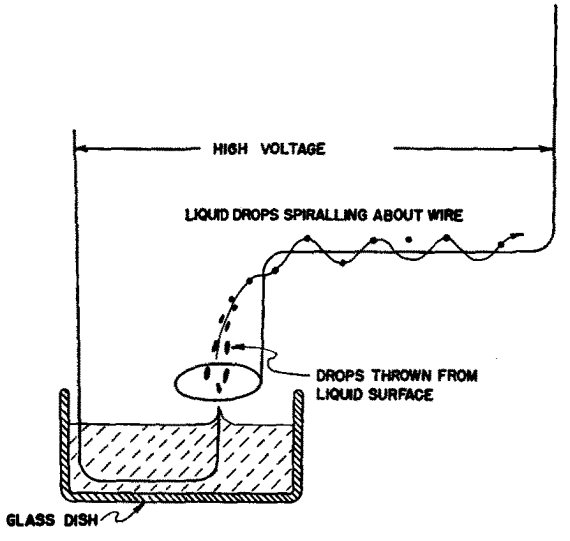

FIG. 7. The "hanging drops experiment."

main body of the liquid at rather high velocities. Figure 7 shows the drops leaving the dish and "hanging" in the air around the electrode. Occasionally the individual drops will remain suspended or circling around the lead-in wire for as long as 15 sec. It would appear that most of these drops have become charged, hence the effect is a combination of dielectro- and electrophoresis. The voltage applied in this experiment was 11 000 v, dc. In another experiment, shown diagrammatically in Fig. 8, where CCl_4 liquid was thrown over four feet up into the air at about 50 cc/sec, the voltage was applied from below by a small Van de Graaf generator. The voltage applied was approximately 200 000 v (negative) at the fine wire electrode. The electrical power input, at 20 μa was therefore

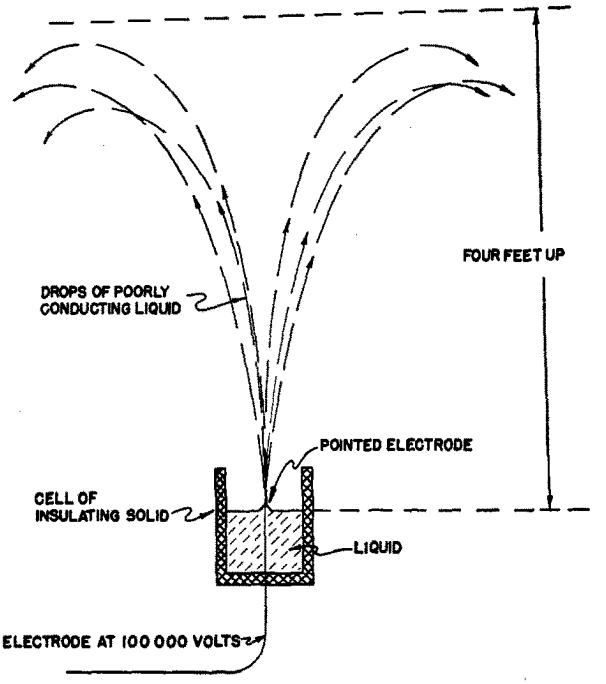

FIG. 8. Pumping action of strong divergent electric field.

about four w. The power expended in the liquid rising at the indicated rate and height is about one joule/sec, indicating the electromechanical "pump" to be about 20 to 25% efficient.

Similar results were observed using fine powders instead of liquid. Powdered $CuSO_4,5H_2O$, polyvinyl-chloride, talc, NaCl, alumina, and silica were pumpable in this fashion. This indicates that the procedure may have some wider applications, as in the drying or processing of powders.

CONCLUSIONS

From these investigations, the following conclusions may be drawn.

1. The effects of nonuniform electric fields become large and quite noticeable, perhaps eventually even quite useful, when larger and larger particles are the subject of the effect. The effect upon very small particles of the size assigned to usual molecules, is normally negligible. The effects upon suspensions and coarse colloids is marked, and is striking when applied to bodies of liquid or solid particles.

2. Used as a pump, the nonuniform field of a small electrostatic generator may be made to throw non-conducting liquid or powder several feet into the air. The electromechanical efficiency was about 25% in these experiments.

3. Continuous separation of the components of a suspension was achieved. A suspension of a polar solid, polyvinyl chloride, in a less polar liquid (1:1 mix of CCl_4 and benzene) was run through a dielectrophoresis

cell and caused to separate into two fractions, one rich in solids, the other poor in solids content. The ratio of the concentrations (the separation factor) was observed to be 2.5. (higher factors should be obtainable with improved design).

4. Suspensions of several polar solids in less polar liquids were precipitated out using the effect of the nonuniform field.

5. Equations derived for the effect showed the force to depend upon the fifth power of the radius of the central electrode for spherical geometries, and upon the third power of the radius for cylindrical geometries. It was also shown that the velocity of a particle in such fields is proportional to the square of the particle radius, hence, the dependence noted above upon particle size.

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Note on Diffusive Separation of Gas Mixtures in Flow Fields*

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(Received February 10, 1958)

This paper discusses the effect of pressure-diffusion flux upon the concentration distribution of gas mixtures in flow fields. The equation of concentration is formulated for a binary gas mixture in which the mass ratio is large and the concentration of the lighter gas is very small. An asymptotic solution to the steady-state equation of concentration is given for an irrotational and incompressible flow.

As an illustration, the diffusive separation (i.e., deviation from the original homogeneous state) of a mixture of helium and nitrogen along streamlines at the entrance to a long straight channel is calculated. It is assumed that the pressure inside the channel is 90% of that in the free stream, and that the diffusion coefficient of the mixture corresponds with the atmospheric conditions at 80-km altitude.

I. INTRODUCTION

IN Chapman's recent discussion[1] of thermal diffusion in gases, he reviewed a newly proposed method by this author of separating gas mixtures via suction of the thermal-diffusion boundary layer along a heated surface (referring to an unpublished report[2]). A paper dealing with Part I of this report is now being published.[3] The present paper summarizes Part II of the unpublished report which deals with mixture separation in a flow field due to existing pressure-diffusion flux.

It is well known that mixture flow of Knudsen gases is diffusively separative. As the mean free path relative to a characteristic dimension of the flow decreases, intermolecular collisions, which cause momentum transfer, become increasingly important. In a continuous medium, the flow is almost always nonseparative. The physical laws governing the intermediate range of flow remain the least known. The basic phenomenon of interaction between mass flow and pressure diffusion leading to diffusive separation of gas mixtures will be treated here.

II. ANALYSIS

Consider a binary mixture in which both the volume concentration of the lighter gas n_{10} and the molecular mass ratio m_1/m_2 are much less than unity. The equation governing n_{10} in an incompressible and isothermal flow field can be formulated from the application of the equation of continuity to the total flow and to the light component flow. If we adopt the formula for diffusion flux given by Chapman and Cowling[4] and neglect the body force and thermal-diffusion terms, the diffusion equation in a two-dimensional flow field (u,v)† can be obtained.

$$u\frac{\partial n_{10}}{\partial x}+v\frac{\partial n_{10}}{\partial y}=D_{12}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)n_{10}+D_{12}\left(\frac{\partial G}{\partial x}\frac{\partial n_{10}}{\partial x}+\frac{\partial G}{\partial y}\frac{\partial n_{10}}{\partial y}\right)+D_{12}n_{10}\left(\frac{\partial^2 G}{\partial x^2}+\frac{\partial^2 G}{\partial y^2}\right), \quad (1)$$

where $m_2G=(m_2-m_1)\ln p$. Equation (1) can be simplified by transformation from the x, y plane to the ϕ, ψ plane where ϕ denotes velocity potential ($u=\partial\phi/\partial x$, $v=\partial\phi/\partial y$), and ψ, stream function ($u=\partial\psi/\partial y$, $v=-\partial\psi/\partial x$). The transformed equation, after substitution of $n_{10}=N_{10}\exp[-\frac{1}{2}(G-\phi/D_{12})]$,

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[1] S. Chapman, Second Biennial Gas Dynamics Symposium, Tech. Inst. Northwestern University (August, 1957).

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[3] V. C. Liu, Quart. J. Mech. Appl. Math. (to be published).

[4] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-Uniform Gases* (Cambridge University Press, New York, 1953), Eq. (7), p. 140.

† The notation used in reference 4 is followed.